Card Games in an Undergraduate Geometry Course

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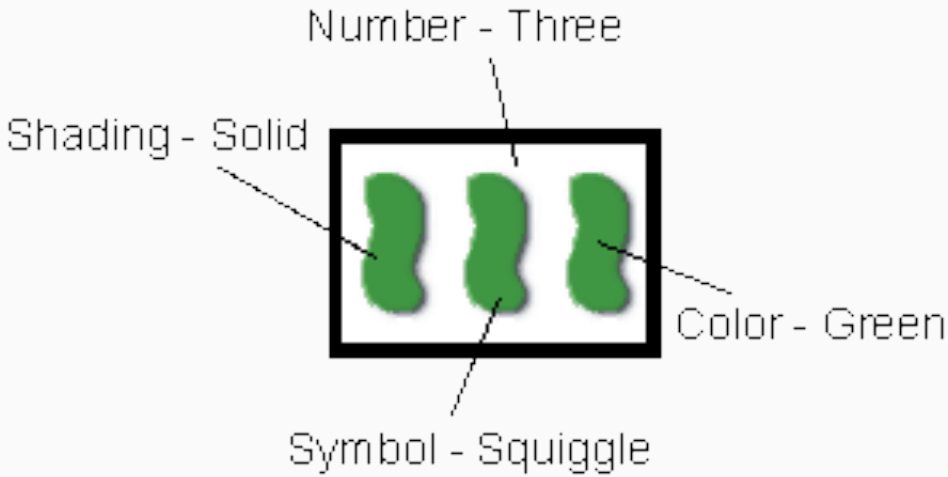
Presentation Overview

The Game of SET

Brief Introduction to the Game of SET

SET Cards:

- 81 cards
- 4 attributes:
  - Color
  - Number
  - Symbol
  - Shading
- 3 possibilities for each attribute:
  - Red, Green, Purple
  - 1, 2, 3
  - Diamond, Oval, Squiggle
  - Solid, Open, Striped



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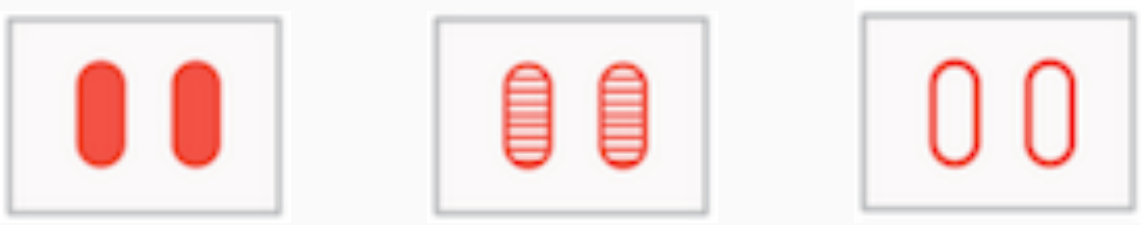

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Incidence Geometry and the Game of SET

Incidence Geometry:

- Undefined terms: point, line, incidence
- Axiom 1: For every pair of distinct points P and Q there exists a unique line incident with P and Q.
- Axiom 2: For every line l there exist at least two distinct points that are incident with l.
- Axiom 3: There exist three distinct points such that no line is incident with all three of them.

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SET is a model of incidence geometry:

- Points $\rightarrow$ Cards
- Lines $\rightarrow$ SETs
- Incidence $\rightarrow$ A card is incident with a SET if it is contained in that SET.

Incidence Geometry and the Game of SET

Hyperbolic parallel property - Given a line l and a point not incident with l, there exist at least two lines parallel to l incident with P.

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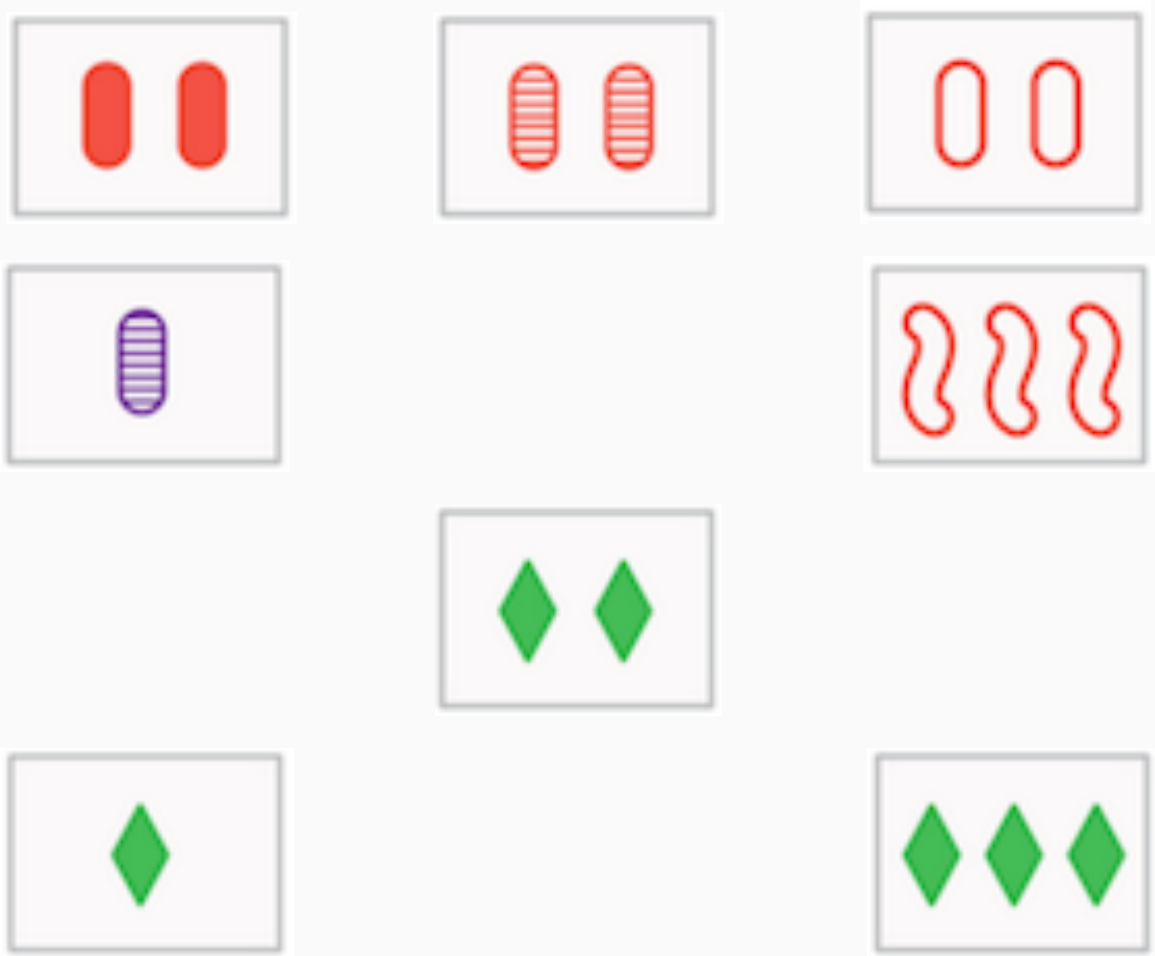

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We can use results from incidence geometry to learn about the game of SET:

- Result in incidence geometry: For every point, there exist at least two distinct lines incident with it.
- Corresponding result in SET: For every card, there exist at least two distinct SETs that contain it.

The Game of Spot It!

Brief Introduction to the Game Spot It!

Spot It! cards:

- 55 cards
- 57 symbols
- 8 symbols on each card
- Every pair of cards has exactly one symbol in common.

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Spot It! is an example of (a subset of) a projective plane:

- Points $\rightarrow$ Symbols
- Lines $\rightarrow$ Cards
- Incidence $\rightarrow$ A symbol is incident with a card if it is on the card.

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- A projective plane of order 7 has:
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We can use what we know about projective planes to analyze the game Spot It!:

- A projective plane of order 7 has:
  - 57 points
  - 57 lines
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- You could add two more cards to the game of Spot It! to get the projective plane of order 7.

Conclusion

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If Euclid failed to kindle your youthful enthusiasm, then you were not born to be a scientific thinker. - Albert Einstein