

GEM (Guided Explorations in
Mathematics) Seminar at State
College High School

January 2011, New Orleans

People: Mark Levi + ST (Penn State), and Greg Somers (State High).

Format: Biweekly 2 hour lecture (with a break for food), 20–40 participants, based on the existing Math Club. Home problems given.

Funding: MAA Dolciani Mathematics Enrichment Grant (2 year, 6K\$), starting fall 2010.

see Google site

statehighmathclub/gem-seminars

for lecture notes, problem sets and solutions.

Main idea is to cover “under-represented” areas: solid geometry, physical reasoning in mathematics, geometric ideas in calculus and algebra.

Sources:

S. Tabachnikov. *Geometry and billiards* Amer. Math. Soc., 2005.

D. Fuchs, S. Tabachnikov. *Mathematical Omnibus*, Amer. Math. Soc., 2007

M. Levi. *The mathematical mechanic: using physical reasoning to solve problems*. Princeton, 2009

1st GEMinar

Hosted by SCAHS Math Club

Eat *food* while learning about...

Benford's Law!

With PSU Math Professors

Dr. Mark Levi and Dr. Sergei Tabachnikov

Tuesday, October 5th

3:45 – 5:45PM

Room 218 North

Problem set to Benford's Law

- What is the last digit of 7^{100} ?
- What are the last 2 digits of 7^{100} ?
- Let p and q be coprime integers. Show that there exist integers a and b such that $ap + bq = 1$.
- How many different n -pointed stars are there for (a) $n = 7$, (b) $n = 8$, and (c) $n = 30$?
- Derive the distribution of second digits in the sequence $1, 2, 4, 8, \dots, 2^n$.

Topics, so far

- Benford's Law (ST)
- Bicycle Wheel Math (ML)
- Geometry of Equations (ST)
- Regression and Mechanics (ML)
- Sums of Squares (J. Roe, guest speaker)
- Knot Theory (ST)
- Optical Properties of Eyes (ML)

Plans for future

Why is rainbow an arc of a circle? (after Descartes and Newton)

Optical properties of conics (from Archimedes to Poncelet and Jacobi)

Gravitational attraction of ellipsoids (from I. Newton to J. Ivory to V. Arnold)

Fundamental Theorem of Algebra (a topological proof)

Chebyshev polynomials (an algebraic variational problem solved geometrically)

Spherical geometry, curvature, and Foucault pendulum

Hyperbolic geometry

Fewnomials, or how many roots does a polynomial have?

Straight lines on curved surfaces

Solving equations of degree three and four

Equations of degree five

Compass and ruler constructions of regular polygons (after Gauss)

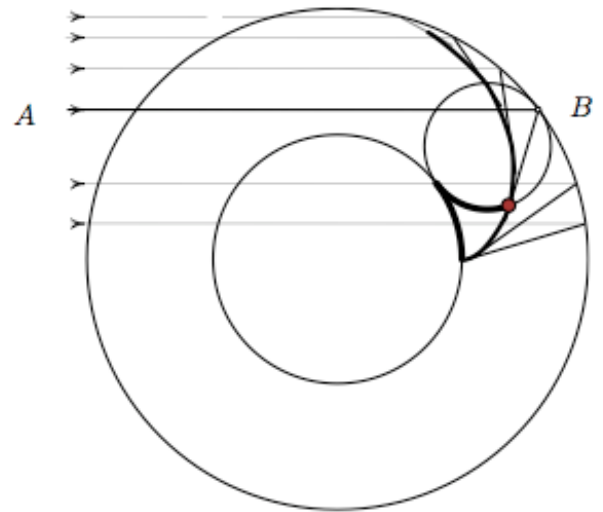
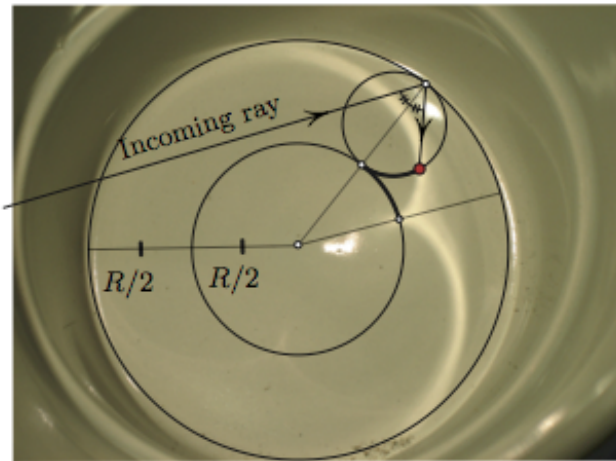
Tilings, possible and impossible

Not all infinities are equal (Cantor's Theorem)

$$1^k + 2^k + \dots + n^k = ?$$

Model building

Example: the caustic is a hypercycloid



The caustic is the bright curve on the bottom of the pot. The reflected ray is tangent to the caustic at the red point. The incident rays are nearly parallel.

Other projects

- Parallel problem-solving session for middle school students
- Math Wrangle
- Elementary school circle?
- Teachers enrichment (in-service)