A Favorite Math Circle Problem: The Check is in the Mail

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Why is this a favorite problem?

- All involved.
- Subgroups breaking off to try different approaches.
- Very little direction.
- Much discussion about organizing information.
- Understanding the questions.
- Much constructive Tantonic flailing.
- Low threshold, high ceiling.
- Try a simpler problem absolutely necessary.

Setting

Math Teachers' Circle targeting probability and statistics supported by Improving Teacher Quality Grants Program $51,881.04,
February 2013 – May 2014,
PI V. Watson, Co-PIs M. Garner and B. Rogers.

Four days in summer and four days during the year.

Participants received $500 for participation, paid substitutes for follow-up session, and Tinkerplots, *Conned Again Watson*, *How to Lie with Statistics* and probability kit.

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Setting

Our problem on the afternoon of November 12, 2013 in a follow-up session.

14 middle grades teachers, teaching:

- 5 sixth grade,
- 5 seventh grade,
- 2 eighth grade,
- 2 sixth through eighth.

Two men, 12 women.

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The Problem

A confused secretary stuffs twelve checks into twelve envelopes and seals them. Then he realizes that he paid absolutely no attention to which check went into which envelope. Nevertheless, he goes ahead and mails them because he was a mathematics major; so he knows that the chances are good that no one will ever find out about his foolish lapse.

The Questions

1. What is the probability that no check is in its correct envelope?
2. What is the probability that –
   a. Exactly one check is in its correct envelope?
   b. Exactly two checks are placed correctly?
   c. Exactly three checks are placed correctly?
3. On average, how many checks are in correct envelopes?
4. During the stuffing process, what percentage of the checks , on average, is wrongly paced before a correct placement is made?

The Source

"It was not until my fourteenth year of mathematics courses that I was introduced to open-ended problems. During my second year in graduate school at the University of Colorado, Wolfgang Thron taught a topology course that had accessible problems without accessible solutions. It was the first time in my life that I was challenged to express myself mathematically. It was so exciting …

Yet when I began teaching, I forgot entirely about open-ended problems. I taught in the same style that I had been taught – "For homework, do the first ten problems; the answers to the odd-numbered ones are in the back of the book." Ten years later when I spend a sabbatical semester in Israel, I became reacquainted with open-ended problems. Shmuel Avital of the Technion University was creating these kinds of problems for precollege students."

The Source

"The word creative in mathematics is, more often than not, a synonym for wrong. After all, the student believes, the world of mathematics has already been created. It already has its algorithms and formulas. . .

Fortunately, this is not a fair description of the world of mathematics. Mathematics is as creative as the arts. Furthermore this creative aspect can be shared by grade school, junior high school, and high school students. It is not the private domain of geniuses or of Ph.D.'s This book is an attempt to inspire such creativity."

Reactions

Overwhelmed at first.

Confusion/discussion about questions 3 and 4.

Focus on questions 1 and 2:

1. What is the probability that no check is in its correct envelope?
2. What is the probability that –
   a. Exactly one check is in its correct envelope?
   b. Exactly two checks are placed correctly?
   c. Exactly three checks are placed correctly?

Progress

Try a simpler problem – 3 envelopes, 3 checks.

For n=3

aA bB cC	All match
aA bC cB	One match
aB bA cC	One match
aC bB cA	One match
aB bA cB	No match
aC bB cA	No match

For n=3

1. What is the probability that no check is in its correct envelope? **2/6**
2. What is the probability that –
   a. Exactly one check is in its correct envelope? **3/6**
   b. Exactly two checks are placed correctly?
   c. Exactly three checks are placed correctly?
3. On average, how many checks are in correct envelopes?
4. During the stuffing process, what percentage of the checks , on average, is wrongly paced before a correct placement is made?

n=4

aA	bB	cC	dD	All match
aA	bB	Cd	dB	One match
aA	bD	cB	dC	
aC	bB	cD	dA	
aD	bB	cA	dC	
aB	bD	cC	dA	
aD	bA	cC	dB	
aB	bC	cA	dD	
aC	bA	cB	dD	
aB	bA	cC	dD	Two matches
aC	bB	cA	dD	
aA	bC	cB	dD	
aD	bB	cC	dA	
aA	bD	cC	dB	
aA	bB	cD	dC	

No matches:

24 – 8 – 6 – 1 = 9

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A Table

	n=3	n=4	n=5	n=6
All match	1	1	1	1
One match	3	4C1 x 2 = 8	5C1 x 9 = 45	6C1 x 44 = 264
	None 6-4=2	Two match 4C2 x 1 = 6	Two match 5C2 x 2 = 20	Two match 6C2 x 9= 135
		None 24 – 15 = 9	Three match 5C3 x 1 = 10	Three match 6C3 x 2 = 40
			None 120 – 76 = 44	Four match 6C4 x 1 = 15
				None 720-455=265

And then what happened?

The teachers who had actually worked out all the possibilities for n=4, confirmed the results the rest of the group had obtained.

The teachers who were simulating the results for n=3, 4, 5 shared their observations and a discussion ensued about the differences and how we would calculate the answers from question 3 and question 4.

That ended the day and everyone was tired of the problem!

Observations – Common Core

- Derangements are nowhere in the middle school curriculum, but not one teacher commented that this problem was not relevant to what they teach.
- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and express regularity in repeated reasoning.

Other Problems Same Book

The Josephus Problem in disguise:

Ten thousand sailors are arranged around the edge of their ship. They hold, in order, the numbers 1, 2, 3, 4, … 10,000. Starting the count with number 1, every other sailor is pushed overboard until they are all gone. Where should you be standing to be the last survivor? What if every third sailor were pushed overboard? Every Fourth?

Other Math Circle Sources

Derangements by Brian Conrey and Tom Davis
http://www.geometer.org/mathcircles/derange.pdf

Emory Math Circle
http://www.mathcs.emory.edu/mathcircle/fa15/E100315

North Carolina State University Math Circle
https://www.math.ncsu.edu/MathCircles/Coat_Problem.pdf

BMTC
http://boisemathcircles.org/wp-content/uploads/2015/03/BMTC-Derangements-handout-Nov-7-2015-revised.pdf

James Tanton
https://www.youtube.com/watch?v=6XeCheL9XnI

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Thank you!

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