1 “I”-Trominos. One square from a $32 \times 32$ chessboard is removed at random. What is the probability that this 1023-square board can be tiled by $3 \times 1$ “I”-trominos?

2 Missing digit. The following is the product of two twin primes, where “X” indicates a digit. What are the possible value(s) for X?

$85070591X30234644163149060928396584963$

3 Analog Clock. Between the start and end of most hours on a regular clock there is a time when the hour and minute hands coincide. When this happens we are interested in the small angle formed by the second hand with these other two. (Since on a circle we can go two directions from one place to another every angle between hands has a small version and a large version.) At which time(s) (to the nearest second) is this angle a maximum?

4 A Slightly Weird Function. Specify a function $f(n)$ from the positive integers to the positive integers satisfying the following two conditions for all positive integers $n$:

(a) $f(n+1) > f(n)$,

(b) $f(f(n)) = 3n$.

Find $f(2019)$.

5 Billiard ball. A mathematical billiard ball (i.e., a point with zero radius) is shot from corner $A$ of the square below at an angle $\theta = \angle BAC$ where $\tan \theta = \frac{1000}{2019}$. How many times will it bounce off a wall of the square before it returns to a corner, and which corner will it return to?

6 Cosines and Squares. Let $\theta = \frac{2\pi}{17}$. Compute

$$\sum_{k=1}^{16} \cos (k^2\theta) = \cos \theta + \cos 4\theta + \cos 9\theta + \cos 16\theta + \cdots + \cos 16^2\theta.$$ 

7 A 9th-degree equation. Find all solutions, real and complex, to

$$(2000x^3 - 17)^3 + (19x^3 - 12)^3 = (2019x^3 - 29)^3$$

8 A Boring Polynomial. Let $P(x)$ be a 2018th-degree polynomial with real coefficients such that $P(n) = n$, for $n = 1, 2, 3, \ldots, 2018$. Find all possible values for $P(2019)$. 