MATH WRANGLE JMM 2019 QUESTIONS

1 *"I"-Trominos*. One square from a 32×32 chessboard is removed at random. What it the probability that this 1023-square board can be tiled by 3×1 "I"-trominos?

2 *Missing digit*. The following is the product of two twin primes, where "X" indicates a digit. What are the possible value(s) for X?

$$85070591X30234644163149060928396584963$$

3 *Analog Clock*. Between the start and end of most hours on a regular clock there is a time when the hour and minute hands coincide. When this happens we are interested in the small angle formed by the second hand with these other two. (Since on a circle we can go two directions from one place to another every angle between hands has a small version and a large version.) At which time(s) (to the nearest second) is this angle a maximum?

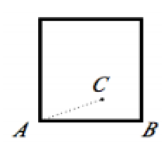
4 *A Slightly Weird Function*. Specify a function $f(n)$ from the positive integers to the positive integers satisfying the following two conditions for all positive integers n:

(a) $f(n+1) > f(n)$,

(b) $f(f(n)) = 3n$.

Find $f(2019)$.

5 *Billiard ball*. A mathematical billiard ball (i.e., a point with zero radius) is shot from corner A of the square below at an angle $\theta = \angle BAC$ where $\tan\theta = 1000/2019$. How many times will it bounce off a wall of the square before it returns to a corner, and which corner will it return to?



6 *Cosines and Squares*. Let $\theta = 2\pi/17$. Compute

$$\sum_{k=1}^{16} \cos\left(k^2\theta\right) = \cos\theta + \cos 4\theta + \cos 9\theta + \cos 16\theta + \cdots + \cos 16^2\theta.$$

7 *A 9th-degree equation*. Find all solutions, real and complex, to

$$(2000x^3 - 17)^3 + (19x^3 - 12)^3 = (2019x^3 - 29)^3$$

8 *A Boring Polynomial*. Let $P(x)$ be a 2018th-degree polynomial with real coefficients such that $P(n) = n$, for $n = 1, 2, 3, \ldots, 2018$. Find all possible values for $P(2019)$.