

Math Wrangle Problems

Joint Math Meetings 2015

American Mathematics Competitions

January 11, 2015

1. Find the sum of all positive two-digit integers that are divisible by each of their digits.
2. An equilateral triangle is inscribed in the ellipse whose equation is $x^2 + 4y^2 = 4$. One vertex of the triangle is $(0, 1)$, one altitude is contained in the y -axis. Find the length of each side of this equilateral triangle.
3. A fair die is rolled four times. Find the probability that each of the final three rolls is at least as large as the roll preceding it.
4. Three of the vertices of a cube are $P = (7, 12, 10)$, $Q = (8, 8, 1)$, and $R = (11, 3, 9)$. What is the surface area of the cube?
5. Find the integer that is closest to $1000 \sum_{n=3}^{10000} \frac{1}{n^2 - 4}$.
6. Let \mathcal{S} be the set $\{1, 2, 3, \dots, 10\}$. Let n be the number of sets of two non-empty disjoint subsets of \mathcal{S} . (*Disjoint sets* are defined as sets that have no common elements.) What is n ?