Stations Around the Room, Advertisements, and Other Activities

Jim Tanton brought the mathematical salute and related arm twisting puzzles, visual problems (Given an $n \times n$ grid, when does there exist a path of horizontal and vertical lines that starts in a given square and visits every square exactly once), felt braiding topology puzzles and a freeform braiding challenge using three ribbons – braid them how you like (but return the middle strand to the middle) – tie them to the wooden stick – untie the braid without untying any ribbons, lots of editions of his newsletter, Instant Insanity puzzle, Square with wonder puzzle, a trio of problems.

BAMA has a series of newsletters and brought a wooden puzzle

Math Teacher Circle Network has a "How to Run a Math Teachers' Circle Workshop" this summer Brought geometric puzzles

Fliers for *A Decade of the Berkeley Math Circle: The American Experience, Volume I* by Zvezdelina Stankova

Fliers for *Circle in a Box* by Sam Vandervelde

Julia Robinson Mathematics Festival (Josh Zucker)

San Francisco Math Circle

San Jose Math Circle's current and former students participated in the Sample Math Circle and the Math Wrangle

MSRI's Emissary Newsletter has an article on the Great Circles Conference last April.

National Association of Math Circles has event in March –Circle on the Road at Arizona State University, brought bookmarks and information.

Kaplans are holding a Math Circle Institute this summer

Amanda Serenevy brought One-Cut Theorem and Sona Drawing activities

Joshua Zucker – Sample Math Circle Session

Josh started by giving students a handout.

The Mad Veterinarian

The Mad Veterinarian builds machines that turn combinations of animals into other animals. The first set of machines he built turned combinations of cats and dogs into cats and dogs according to the following rules.

$$CC \to D$$
$$CD \to C$$
$$DD \to D$$

The veterinarian likes cats more than dogs and wants to alleviate animal overpopulation.

Questions:

- Suppose the mad veterinarian wants to turn 3 cats and a dog into just one cat. Can this be done? How? Is there more than one way? Is there a way to get a different result?
- Is there a combination of starting dogs and cats that could not be reduced to just one cat?
- Suppose there was a large number of starting cats and dogs. Is there a way to tell which combinations can lead to just one cat and which cannot?

One student noted that this is similar to what would happen if c=-1 and d=1 and the operation was multiplication. If a starting sequence is worth -1, does that mean that we can reduce to one cat necessarily?

The mad veterinarian constructed another set of machines that works with cats, dogs, and mice.

$$CD \to M$$
$$DM \to C$$
$$CM \to D$$

Questions:

- Suppose we want to get one cat in the end, and we start with 3 cats and a dog. Is there a way to get just one cat in the end?
- Can we figure out what starting configurations make it possible to obtain a cat and what machines we need to use to obtain this result?

One student noted that it can help to work backwards. The group introduced reverse machines that do the opposite of each operation.

Another student wanted to try base 3.

One student wanted to try setting this up as multiplication of three real numbers, and then solve the system of three equations in three unknowns. The group discovered that c=1, d=1, m=1 is one solution. Another solution is c=1, d=-1, m=-1. Another solution is c=-1, d=1, m=-1. OR c=-1, d=-1, m=1.

- Following up with this, we have a question. If the starting sequence is not equal to the value of a cat, does that guarantee that there is no way to reduce down to one cat?
- If the starting sequence is equal to the value of a cat, does that guarantee that there is a way to reduce down to one cat?
- What if we use both the reverse machines and the original machines?
- Can we turn four cats into one cat? (The students showed we can turn four cats into two cats if we can use the reverse machines also.)
- What about CCDM? What can that end up with?
- Using different solutions found earlier leads to different conclusions for whether this ends up as a cat, ends up as a cat or a mouse, or ends up as a cat or a dog. But all of those are solutions, so it seems like that must end up as a cat no matter what we do.
- If I end up with two matching animals, is it possible to reduce that to a single animal of any kind?

Suppose the mad veterinarian uses these machines

$$CD \to MM$$
$$DM \to CC$$
$$CM \to DD$$

Suppose we start with CCCD. Questions:

- Can we get down to one cat? (No, because we will have four animals forever.)
- Can we get down to four cats?

One student suggested assigning numbers and using multiplication again.

Another student suggested working backwards.

After getting stuck, the group decided to go back to the first case and try asigning D=0 and C=1 and adding modulo 2. In the next case, they had trouble assigning different numbers and using addition modulo something. Is there a way to do this for the second case? Is there a way to do this in the third case? The students found C=1, D=2, M=3 working modulo 3 in the third case.

Final machine.

$$C \to 2D$$
$$D \to CM$$
$$M \to CDDD$$

Machines are also reversible

Start: 1 million cats

Goal: End up with the fewest possible number of cats and no dogs and no mice.

How could we tackle this? Should we start by writing 1 million C's?

One student suggested reducing the problem to 100 cats and try that.

Josh suggested starting with one cat, and seeing what we could build that up to.

What about making an organized list of combinations? Here are some combinations that are equivalent using the machines.

C	D	M
1	0	0
0	2	1
1	5	0

So we have learned that five dogs can be created or destroyed at will as long as we have a cat as a catalyst!

Can anyone find another cool relationship like that?

The students found more things that are zero.

Let's go back to equations

$$
\begin{aligned}
C = 2D + C + 3D \\
5D = 0 \\
-C = 3M
\end{aligned}
$$

What does $-C = 3M$ mean?

Maybe it would be better if we write all of our equations with a zero so we know things that can be cancelled out.

$$
\begin{aligned}
0 = 3M + C \\
0 = D + 2M \\
0 = 2C + 2D
\end{aligned}
$$

One student thought of a way to reduce cats. You can combine $5D = 0$ with $0 = D + 2M$ to conclude that any multiple of ten mice can be created or destroyed. So we can make thirty mice and then use $0 = 3M + C$ to destroy ten cats. We should be able to reduce the million cats down to ten cats. Could we get ten cats down to something smaller?

Can we assign some numbers to dogs, cats, and mice in this case? There seem to be a lot of multples of 5 and 10 running around.

$$
\begin{aligned}
C = 2D + M \\
D = C + M \\
M = C + 3D
\end{aligned}
$$

A dog needs to be an even number. The mouse needs to be six more than the cat.

Let's try D=2, C=3, M=9. What does that prove? Starting from 1,000,000 cats, if we want to end up with pure cats, the number of cats needs to be a multiple of 10. So that proves that 10 cats is the smallest number of pure cats we can make.

These problems have lots of connections with deep mathematical ideas (see the handout for links).

Math Wrangle

2 Handouts – Mathematical Wrangle Problem Set and Mathematical Wrangle Rules.

End of Day Announcements

Brandy Wiegers announced that Math Circles.org is growing http://mathcircles.org

New features include a wiki page about Math Circle related events and booths at the Joint Meetings and Math Fest.

Math Circle on the Road information is posted there.

You can add YOUR information to this listing very easily on the site. Add information about YOUR Math Circle or YOUR summer program on the site.