SIGMAA Thursday special session

Jim Tanton: Welcome and *On Finding Successful Math Circle Questions and Topics*

A math circle is a formal or semiformal experience that unites kids with mathematics professionals to engage in a joyful way. True mathematics.

“Research class” - 8-10 weeks of an hour a week on one problem with MS and HS students.

See handout.

Need to choose something that is engaging and easy to enter and leads to deep, rich, interesting mathematics. No lesson plans, just following tangents and going where the kids take the problem.

A seed of an idea: a 5x5 grid of squares. Engage in the power of sitting with a mathematical thought and playing with it.

Count! How many squares. Already the mystery of why the number of each size is a square number as well.

What else can we count? Paths. Rectangles. There are 5triangle * 5triangle rectangles. Interesting. Why? Also why is this equal to the sum of the first 5 cubes? Is there a nice 1-1 correspondence? Can we make a proof without words for this?

What about tilted squares? Also notice their areas. Numbers that can be expressed as the sum of two squares.

Path walking: which spots can be starting points for an open path visiting every square? Why can the (3,2) point in the 5x5 grid not be the starting point? Other size grids? Third dimension?

Different ways of counting: 5^2 = 1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1. Also shading to show sum of odd numbers equals the square.

The shading leads to a proof that the sum of the first k odds to the next k odds is 1/3. (Galilean ratios)

Take shaded square to flip a half of it and get (3+7) / (1 + 5 + 9) = 2/3 and so on, (3+7+11)/(1+5+9+13) = 3/4.

Also alternating version if one shading is +1 and the other is -1 per square. And more!

Let the classroom remain silent - for 2 minutes - for 20 minutes!

Good topics: Number theory, figurate numbers, combinatorics, graph theory, games, integer sequences, lattice polygons, intermediate-value theorem.

New things: Pick's theorem proof, sequence of mountain and valley folds from a piece of paper.

Pay attention and you'll notice them when they come around. They come all the time but if you're not sensitive to the ideas they will pass you by.

Where to run into them: Read! Journal of recreational math, back issues of SciAm, Focus, College math journal, and so on. Attend talks! Books, web resources, SIGMAA, NAMC, AoPS.

Develop them: Try, try, try and do, do, do. Set aside a bit of time and flail and have fun flailing. Work with kids in your area, your kids, your friends' kids. Organize your own more structured group. Try different things to develop your own sense of what works. Take topics from other people but not lessons or presentations - they need to fit your style. Math is a human experience.

Tatiana: If kids do all their math on graph paper forever, then just like this topic you will discover a lot of mathematics.

??: How do you break through the "answer in the back of the book" syndrome with a new batch of kids? Ans: math salute!

??: How do you present the problem? Do you just put the grid up or do you pose a problem? Ans: Just put up the grid with no words and "is there anything interesting about that?" They start commenting and you build questions from there. Be a listener and a nudger. If things are really stuck maybe give a bigger shove. Sometimes start with a serious question like "Can an irregular pancake be cut in half by a single line?"

?Zeitz: How do you deal with personalities? There's chaos, some people try to dominate, some people are uncomfortable in that environment. Ans: Let it be the first few sessions. Some kids will sort themselves out. Sometimes need to talk to kids and tell them to let other people have a turn. Strategies such as talking quietly until people quiet down to listen to you, or having a private marker to remind them unobtrusively such as moving pieces of chalk from one spot on the board. "That's a great idea, why don't you write that up for us?"

Steve Dunbar: *Lincoln Area Math Teachers' Circle: 3 Years and Growing*
Summer '07 core group of 5 attended AIM teachers' circle workshop.
Now summer planning sessions to set up about 6 monthly meetings through the school year. Team includes middle school teachers and math coaches as well as university faculty.

Financial support from UN-L center for math science education

Coalition: attendees include faculty in math and math-ed, grad students, teachers mostly from middle school, and math ed students/student teachers/preservice teachers. About 25 people attending each session. Some of the grad students have started their own circles when they went on to their own institutions.

Would like to grow to HS teachers as well.

Meeting place: Now are meeting at a middle school, cafeteria. A comfortable and familiar place for the teachers and the parking is easy. Also sometimes community college with nice meeting rooms, but meeting in the schools is probably best.

Format: "Dinner and a math problem", dinner first and then a math session for 90 minutes or so. A night out with their friends, social and collegial with professional benefits and some serious good mathematics.

"Wow" factors: Conway tangle, jug problems and euclidean algorithm (scraps of wood in measured lengths to do it physically, plus film clip from Die Hard with a vengeance), place value by Tanton's base machines, hat trick (red hat/blue hat problem, with real hats), mad veterinarian, game of SET. Manipulatives plus some whimsy or goofiness.

Conway tangle lesson developed.

Multiple levels: Connection to higher-level mathematics, along with school math.

Sources of ideas:Visit mathteacherscircle.org and mathcircles.org. Canned materials there but better to learn them and then make your own instead of just directly using them.
And www.nebrwesleyan.edu/people/cminer/teachercircle0708.html
Also lots of good ideas to find here at the joint meetings!

Teachers Circles are extensions of and support for math coaching and math coaches as well as math teachers.

Joint leadership: a teacher keeps the grounding in school curriculum topics and a university faculty member provides connections to higher-level topics.

Effect: Connections between university and school, enrichment for teachers, indirectly for students. Quotes from teachers!

Jamylle Carter: *What I Learned Running the Oakland Math Circle*

How started: SF math circle designed to get teachers to bring kids and serve the unenriched kids who wouldn't have parents bringing them to the circles. Racial diversity not so great at that circle, mostly white and asian.

Why Oakland: SF pop is 50% white 30% asian, vs Oakland is 30% white 35% black 15% asian. Also Oakland median income is about 3/4 of SFs. Seek out middle school students, primarily African-American and Latino, low SES. Ended up all African-American to address the struggle between racial/cultural identity and academic interest.

Social safety net: Uri Treisman: Even when Black students are motivated, prepared, and encouraged by mentors, they tend to work alone. Isolation leads ultimately to their decision to leave the sciences. Oakland math circle designed to prevent this. "It's OK to be black and like math"

Community partnerships: Northern California Council of Black Professional Engineers, Museum of African American Technology Science Village (a source of meeting location and advice and science activities). Also Institute for the Advanced Study of Black Family Life and Culture (501c3 to help organize and take the checks and provide some inspiration and ideas as well). Exploratorium (Jamylle post-doc Exploratorium Math/Science Internship) teacher institute provided activities and motivation and support from veteran high school teachers.

Who: 7 girls, 16 boys, private and public schools. Avg about 4 students per week for 20 weeks. Reached by word of mouth, schools, churches, fliers, newspapers, ...

Topics: Isometries (Escher drawings etc). Pre-trig (exploratorium activity) with inclinometer to find height of tall objects and rocket flights. Music with slinkies and dominoes and amplitude/frequency analysis with Audacity - also leads to LCM. Statistics (M&Ms - population characteristics vs sample, mean/median/mode on measurements of their bodies).

Also guest teachers (NASA, biostatistics) and assistant too.

Funding: MAA Tensor-SUMMA grant, California Math Council Student Activities Trust, and MSRI minigrant.

Quotes from kids, also from parent: "You can look like this and be a mathematician."

Lessons learned: Pilot ended after one year. Had five months mostly in the summer to get the word out to the schools and develop the activities during intense Exploratorium professional development work. Communication between partners with new relationships was difficult. Dennis Bartels "You've gotta work with people you already know. Form the relationship and then the funding opportunity naturally arises." Don't try to do too much at once. Have a relationship with schools before you start.

Tom Kilkelly: *ARML Power Contest from Math Circles to Classrooms 1994-2010*

Teams wished they had practice with proofs to prepare for the power question at the national ARML competition. “A cooperative effort in exploring a problem situation through the solution of chains of related problems. Meaningful problem situations for novice and veteran. Attract schools with strong competition teams and offer experiences for students new to such events. Thus they must build mathematically significant results out of trivial material.”

Lots of these topics could be used in math circles.

Topics: Color transformations (rows/cols of black and white squares, row flips and column flips, is it possible to transform position A to position B?). Lots of polynomial topics not from regular classroom math. Integer geometry: integer sides and diagonals. Cevians (median reflected in angle bisector) Right triangles with inscribed circles, election math, billiards, yahtzee, basimals (decimals in other bases), slitherlink.

25-point affine geometry.

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ABCDE     AILTW     AXQOH
FGHIJ     SVEHK     RKIBY
KLMNO     GORUD     JCUSL
PQRST     YCFNQ     VTMFD
UVWXY     MPXBJ     NGEWP
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Lines are rows/cols in the above. Two lines meet in a point or are parallel, two points determine a line, unique perpendiculars, unique parallel through a given point. Distance cyclically around the line etc. Define triangle (3 noncollinear points) - how many are there? How many are isosceles/equilateral? Is right = scalene? How about triangle centers (circum, in, ortho)? Euler line! 6500 quadrilaterals classified into 11 types. Some familiar theorems fail: “If opposite sides of a quad are congruent and parallel”

Insane tic-tac-toe (Russia): 3x4 board, players can make either mark, if you get 3 in a row of one mark you win. Is going first a disadvantage? X... .OX. ...O forces win. How about two moves for first player? How about playing on a torus?

Errors in math reasoning: a + bc = (a+b)(a+c) works when? For instance 0.5 and 0.2 and 0.3.

Factoring: 6x^2 -7x - 3 -> “multiply by 6” x^2 - 7x - 18 = (x-9)(x+2) so “divide by 6” to (x-9/6)(x+2/6) to (x-3/2)(x+1/3) to (2x-3)(3x+1) always works!

Pythagorean triples ...

See book at ARML website.

Brandy Wiegers *National Association of Math Circles*

See http://mathcircles.org and see upcoming events, problem database, and so on.

Circle on the road conference Mar 13-15, like last year's Great Circles conference.
Going to Tempe and helping them start a math circle.
Both for helping people start circles and networking among established circles.

Ralf Youtz and Jon Yaggie: *Student-to-student connections: Graduate students in math circles*

(CM)^2 (Creating Momentum through Communicating Mathematics) project at SFSU: Mathematicians who can explain their work to the general public. Seminars focused on communicating mathematics. Grad students in classrooms and math circles 10 hrs/week.

SF Math Circle founded in 2005 at SFSU. Satellite programs at high schools. CM^2 people run the satellite programs now.

Grad students: Role models for K12, plus deeper understanding of math, developing student ideas at higher levels. Age closer to students. Also benefit to grad students: teaching more open-ended instead of teaching remedial algebra at SFSU.

The future: We have tested material, written up papers on math circle lessons, to share with people in the future. Also can bring in grad students in smaller ways who are not funded through CM^2.

Grad students learn a lot: How to teach, more flexible lesson plans.

Goals: Improve fellows' communication, teaching, teamwork, outreach.

Experience: Training (previous summer, workshops, demo lessons), seminar (debrief what happened, planning for future, twice weekly), preparation, circles!

Communication with circle students was easy. With teachers, to give continuity to previous year. Seminar for communication with each other.

Teamwork: Planning, circle roles. "Steal more, be more creative, or just abandon ideas that don't gel into full activities."

Teaching: Design and lead lesson. "I didn't have the experience to plan and run the circle." Adaptability and thinking on your feet.

Students: "Seemed intrigued, asking questions, had ideas, insights." "Weird walking around because these kids want to figure problems out on their own and don't want help."

Outreach: Students (K12) interact with students (grad) who understand math and can relate to them as individuals. Value of math circles and GK12 programs. "Wherever we go we want to do a math circle"

Evaluation: Difficult time during the semester but more enthusiastic at end than beginning. Retention numbers at SFMC are better than ever. Took 2-3 circles to be prepared for all the challenges and flexibility.

Lee Windsperger: *The LSU Math Circle*

Circle run completely by grad students as part of assistantships.

Summer enrichment program: talented students come for 3 weeks. Local day campers and statewide residential camp.

Started GK12 in 2006. Math Saturdays for local students. Teachers offering extra credit, food, maybe attracting students more than the mathematics.

Summer 06 - 9 students - get committed students. Recruit through professional development programs at LSU. By '09 accepting 30 students of 120 applicants. Up until now free: stipends for commuter students and free dorm for residential. Time to start charging and grow? Parallel program for students with less elite test scores now?

Grad student run: instructional and administrative duties (meal forms, field trip driving authorization, and so on).

Two weeks of morning presentations from grad students/professors, afternoon problem sets with undergrad mentors from REU and SMILE(?). Field trips. Intro to MATLAB.

Last week students choose project topic and work with grad student advisor. Ends with presentation. Projects often continue during the school year (publication, talk at JMM).

Topics: Ptolemy's theorem, control theory, sudoku solver via integer programming, cryptography (RSA, Fibonacci?), industrial applications, knot theory (invariants like Jones polynomial, surfaces associated with knots).

HS students presentation (Hira Khan and Joyce Ward):
Infinite chain of beads connected by weightless string. Oscillating vertically (orthogonal to string) on frictionless poles. What patterns can occur?

All beads oscillating at same frequency. So can make the vertical position a projection of circular motion and get more geometry instead of calculus.

Restoring force proportional to difference in position to adjacent beads.

Two cases: beads each on same amplitude but constant phase difference. Or beads each on same phase difference 180 but constant amplitude ratio.

Depends on dimensionless parameter "b" (related to angular frequency and restoring force constant of proportionality?)

Second case isn't physically possible except for when amplitude ratio is 1 (which intersects the two cases). But if you have one defective bead you can get exponential decay on both sides of it instead of exponential growth on one side.

Tatiana Shubin and Mary Fay-Zenk: *Bringing Math Teachers' Circle Lessons to a Middle School Classroom*

Circles for teachers running since summer 06 - but we have never until now made lessons directly for their classrooms, instead showing them the beautiful mathematics and letting them figure out how to use it.

"Promoting a culture of problem-solving through creating a community of math teachers and mathematicians"

Teachers' Circle Network: Establish foundation for a culture of problem-solving by fostering the enjoyment of mathematics among MS math teachers. (Map of programs) http://mathteacherscircle.org Materials & Resources: for Circle Leaders, and now materials for teachers as well: lesson plans from master teachers focusing on problem solving, classroom ready, with links to NCTM and CA standards.

Hard problems, real problem solving, and the joy of problem solving.

Three problems all relating to triangular numbers, letting kids discover that connection on their own. Can be used separately. Full lesson plans including instructional plan, questions for students, assessment, extensions, teacher reflection, and solutions.

Difference from normal curriculum: math as a coherent whole, recognizing underlying themes and parallels within various contexts.

Teacher reflection: patience and perseverance, allowing students time (we often rush to cover material rather than uncover it, or are too eager to show them something beautiful), ..., analysis of different approaches.

Goal: problem solving as a way of thinking, habit of mind, willingness to take on a challenge.

Alex McFerron: *A Great Math Circle - an Exploration of Deltahedra with Origami*

Origami script for math circle available if you want an activity for your circle! Much more detailed discussion because of collaboration with people new to circles.

Wanting to do too much (powerpoint etc) instead of letting participants do it. Script was a way to get people to see how to run the event.

Make the two triangles from a business card. Two cards -> tetrahedron. Four cards, octahedron. Then icosahedron - tape helps.

Make a table of the properties (vertices, edges, degree of vertices, and so on). Convex, only 8 are possible - why? See them all and find patterns.

Don't have to make the complicated ones, just try to make them so they can see what makes 18 or 22 faces impossible. That's what you discover in doing the circle over the course of an hour or so.

alexmcferron.com has origami script.

Can get to make all three and find the question