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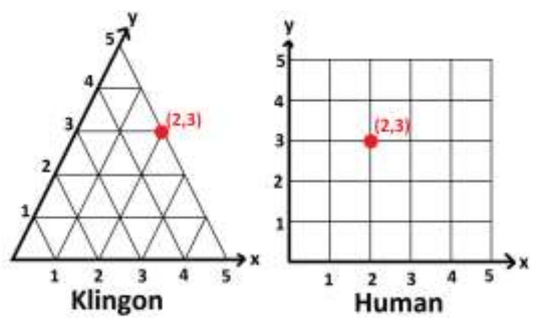
TANTON'S TAKE ON …

THE QUADRATIC FORMULA IN KLINGON

APRIL 2016

Have you noticed in the series *Star Trek* that Klingon computer graphics, as displayed on view-screens throughout Klingon ships at least, seems to be based on equilateral triangular grids rather than square grids?

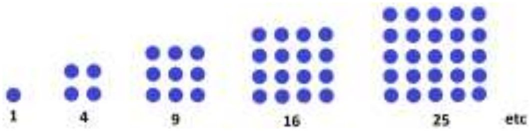
Is Klingon mathematics based on triangles?



Rather than naturally think of geometric squares and the square numbers, do Klingons naturally first think of equilateral triangles and the triangle numbers?

If so, there would be consequences.

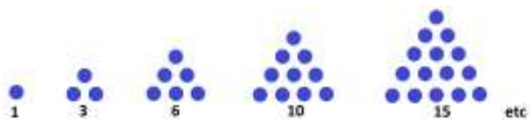
Humans
Humans think square numbers.



The N th square number is $S(N)=N^2$.

Humans extend this formula to non-integer values too: $S(x)=x^2$.

Klingons
Klingons think triangular numbers.



The N th triangular number is

$$T(N)=1+2+3+\cdots+N=\frac{N^2+N}{2}.$$

Klingons extend this formula to non-integer values too: $T(x)=\frac{x^2+x}{2}=\frac{x(x+1)}{2}$.

Humans
Humans define the *square root* of a number x to be a value a such that $S(a)=x$, that is, so that $a^2=x$.

Humans recognize that there are two square roots of a positive value x and denote them with one symbol $\pm\sqrt{x}$.

Humans notice that if a is the positive square root of x, then $-a$ is the negative square root of x. For example, $S(5)=25$ and $S(-5)=25$.

And Human students develop fluency with basic square roots.

$$\pm\sqrt{1}=1 \;\; or \;\; -1$$
$$\pm\sqrt{4}=2 \;\; or \;\; -2$$
$$\pm\sqrt{9}=3 \;\; or \;\; -3$$
$$\pm\sqrt{16}=4 \;\; or \;\; -4$$

etc.

Klingons
Klingons define the *triangular root* of a number x to be a value a so that $T(a)=x$.

(To humans, this equation is $\frac{a^2+a}{2}=x$.)

Klingons recognize that positive numbers (and some negative numbers even) have two triangular roots. They denote the two triangular roots of a number x as $\sqrt[T]{x}$.

(Humans would translate $\sqrt[T]{x}$ as $\frac{-1\pm\sqrt{8x+1}}{2}$.)

Klingons notice that if a is the positive triangular root of x, then $-(a+1)$ is the negative triangular root of x. For example, $T(5)=15$ and $T(-6)=15$.

(Humans would explain this by noting that if $\frac{a(a+1)}{2}=x$, then $\frac{(-(a+1))\times(-a)}{2}=x$ as well.)

And Klingon students develop fluency with basic triangular roots.

$$\sqrt[T]{1}=1 \;\; or \;\; -2$$
$$\sqrt[T]{3}=2 \;\; or \;\; -3$$
$$\sqrt[T]{6}=3 \;\; or \;\; -4$$
$$\sqrt[T]{10}=4 \;\; or \;\; -5$$
$$\sqrt[T]{15}=5 \;\; or \;\; -6$$
$$\sqrt[T]{21}=6 \;\; or \;\; -7$$

etc.

EASY QUADRATIC EQUATIONS
EASY TRIANGULIC EQUATIONS

Humans
Some human quadratic equations are easy to solve. For example,

$S(x)=25$ has solutions
$x=5$ or -5.

$S(x-3)=64$ yields
$x-3=8$ or -8
$x=11$ or -5.

In fact, any equation of the form $S(x+k)=d$ has solution

$$x+k=\pm\sqrt{d}$$
$$x=-k\pm\sqrt{d}$$

(or no real solution if the square root of d does not exist).

Klingons
Some Klingon triangulic equations are easy to solve. For example,

$T(x)=15$ has solutions
$x=5$ or -6.

$T(x-2)=10$ has yields
$x-2=4$ or -5
$x=6$ or -3.

In fact, any equation of the form $T(x+k)=d$ has solution

$$x+k=\sqrt[T]{d}$$
$$x=-k+\sqrt[T]{d}\ .$$

(or no real solution if the triangle root of d does not exist).

GENERAL QUADRATIC EQUATIONS
GENERAL TRIANGULIC EQUATIONS

Humans
Humans notice that

$$S(x+k)=S(x)+2xk+S(k).$$

So given an equation of the form

$$aS(x)+bx+c=0$$

they rewrite it as

$$S(x)+\frac{b}{a}x+\frac{c}{a}=0$$
$$S(x)+2\cdot x\cdot\frac{b}{2a}+\frac{c}{a}=0$$

which starts to look like $S(x)+2xk+S(k)$. Continuing the process, they get

$$S(x)+2\cdot x\cdot\frac{b}{2a}+S\left(\frac{b}{2a}\right)=S\left(\frac{b}{2a}\right)-\frac{c}{a}.$$

This is

$$S\left(x+\frac{b}{2a}\right)=S\left(\frac{b}{2a}\right)-\frac{c}{a}$$

which has solutions

$$x+\frac{b}{2a}=\pm\sqrt{S\left(\frac{b}{2a}\right)-\frac{c}{a}}$$
$$x=-\frac{b}{2a}\pm\sqrt{S\left(\frac{b}{2a}\right)-\frac{c}{a}}\ .$$

Humans call this the quadratic formula.

The solutions to $aS(x)+bx+c=0$ **are**
$$x=-\frac{b}{2a}\pm\sqrt{S\left(\frac{b}{2a}\right)-\frac{c}{a}}.$$

If we write this with our usual squaring notation we are saying here that the solutions to $ax^2+bx+c=0$ are given by

$$\begin{aligned} x &= -\frac{b}{2a}\pm\sqrt{\frac{b^2}{4a^2}-\frac{c}{a}} \\ &= -\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{4a^2}} \\ &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}. \end{aligned}$$

Klingons
Klingons notice that

$$T(x+k)=T(x)+xk+T(k).$$

So given an equation of the form

$$aT(x)+bx+c=0$$

they rewrite it as

$$\begin{aligned} &T(x)+\frac{b}{a}x+\frac{c}{a}=0 \\ &T(x)+x\cdot\frac{b}{a}+\frac{c}{a}=0 \end{aligned}$$

which starts to look like $T(x)+xk+T(k)$. Continuing the process, they get

$$T(x)+x\cdot\frac{b}{a}+T\left(\frac{b}{a}\right)=T\left(\frac{b}{a}\right)-\frac{c}{a}.$$

This is

$$T\left(x+\frac{b}{a}\right)=T\left(\frac{b}{a}\right)-\frac{c}{a}$$

which has solutions

$$x+\frac{b}{a}=\sqrt[T]{T\left(\frac{b}{a}\right)-\frac{c}{a}}$$

$$x=-\frac{b}{a}+\sqrt[T]{T\left(\frac{b}{a}\right)-\frac{c}{a}}.$$

Klingons call this the triangulic formula.

The solutions to $aT(x)+bx+c=0$ **are**
$$x=-\frac{b}{a}+\sqrt[T]{T\left(\frac{b}{a}\right)-\frac{c}{a}}.$$

Example: Solve $T(x)+4x=35$.

Answer: We have $T(x)+4x-35=0$.
Using the triangulic formula with $a=1$, $b=4$, $c=-35$ we get

$$\begin{aligned} x &= -\frac{4}{1}+\sqrt[T]{T(4)+35} \\ &= -4+\sqrt[T]{10+35} \\ &= -4+\sqrt[T]{45} \\ &= -4+9 \quad or \quad -4+(-10) \\ &= \quad 5 \quad\; or \quad -14. \end{aligned}$$

A HUMAN ADVANTAGE?

The formula $S(x)=x^2$ matches the area formula for a square of side length x. The quadratic formula can be derived geometrically by literally completing the picture of a square. (See the quadratics course at www.gdaymath.com.)

Aside: What is the word for square in Latin?

Is there a geometric "completing the triangle" approach for deriving the triangulic formula?

CREATIVE ADVENTURES

We see now that triangular thinking not only changes the look of coordinate geometry in Klingon, but also the look of high school algebra. The fun need not stop with the quadratic formula.

What is the distance formula in Klingon?

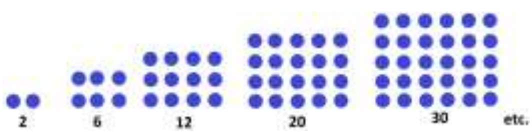
What is the equation of a line in Klingon?
What is the equation of a circle in Klingon?

Humans call x^3 "x cubed." What is the Klingon equivalent?

What other human formulas and ideas have Klingon counterparts?

We can go to other alien races too.

Oblongians always think in terms of oblongs one unit wider than they are tall.



They set $L(x) = x(x+1)$ *and are looking to solve equations of the form* $aL(x) + bx + c = 0$.

What is the general oblongic formula that solves these equations?

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