Good Problems: Planning in Context JMM 2015

January 13, 2015

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a Math Circle for middle school boys.

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Additional goal for teachers:

- Introduce them to good problems they can use with students.

Practice Standards are meant to be the hallmarks of effective use and understanding of mathematics.
They are more conceptual than the Content Standards.

There are eight Practice Standards

http://www.corestandards.org/

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We start by having participants pair off and play the game to get familiar with it.

Then we ask them to determine if there is a good strategy for winning.

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How can one nudge people in a productive direction?

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For middle schoolers, set the task of them advising Alice (first player) and Bob how to play.
Pairs that were competing must now work cooperatively to discover a strategy.

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Conjecture: Alice has a winning strategy if n is not a multiple of 3 and Bob has a winning strategy otherwise.
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How to get there if participants don't see a pattern:
Pairs are assigned specific values of n, 1,2,3,. . . ,8,. . . and asked who wins and how?

Collectively build chart:

n	win
1	A
2	A
3	B
.	.
.	.
.	.
8	A

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An acceptable proof will depend on the background of group.
Teachers should be able to handle the concept that once A picks, A becomes B.
Middle schoolers will be able to handle “They take 1 you take 2, they take 2 you take 1.”

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Change picking rule:
Pick 1,2,3.
Pick 1,2,4.
Pick any positive number.
Classic Nim.
Misère versions.

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Math Circles developed to engage students interested in math and accelerate and deepen their understanding.
If you have a circle of this type solving Nim might be a goal. Even bright participants are unlikely to discover “Nim sum” as a solution.

Summary:
What is the goal of the Circle?
Who is the target audience?
What are good stopping and branch points?

Where can I get good "low floor high ceiling" problems?

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AMS Math Circle Library,
http://www.ams.org/bookstore/mclseries
Math Teachers' Circle Netwok.
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http://www.mathteacherscircle.org/resources/mathematical-materials/
Julia Robinson Math Festival
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