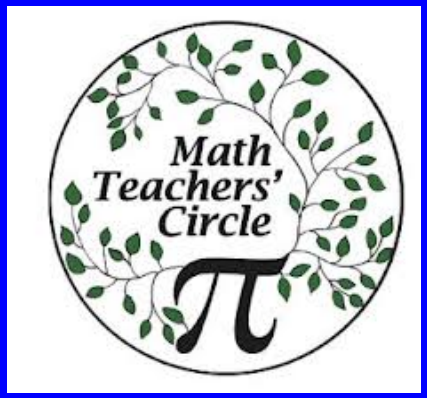


Bard Math Circle and
Mid-Hudson Math Teachers Circle

Developing Collaborative Lesson Plans for Math Enrichment

Lauren Rose, Bard College Math Professor

Beth Goldberg, Linden Avenue Middle School Teacher

Joy Sebesta, Bard MAT Student

Developing Collaborative Lessons for Math Enrichment: A Four Step Process

1. Select an interesting problem
   - Can be readily simplified and expanded
   - Employs manipulatives
   - Facilitates a ‘hands on’ approach
   - Connects to a rigorous mathematical theory
   - Has real world applications
2. Brainstorm with others
   - Math professors
   - Middle school math teachers
   - Students
3. Develop a lesson plan for the problem
   - Try to link to the common core
4. Utilize feedback, refine, customize and reuse!

Developing Collaborative Lessons for Math Enrichment: Success Stories

1. Take Away Games and NIM
   - Group exercise in a 200 level Proofs class
   - Used in a local MathCounts club session
   - Customized for 8th grade Math/Science Day at Bard
   - Reused for a Elementary Math Circle Session
   - Presentation to Senior Citizens in Bard's Lifetime Learning Institute

Game: 12 chips, on your turn take 1, 2, or 3, winner is the one who takes the last chip.

Analysis: Figure out a winning strategy, describe it in mathematical terms.

Expansion: Change the number of chips, the rules for taking chips, etc.

Developing Collaborative Lessons for Math Enrichment Success Stories

2. Simpson's Paradox
   - Initial presentation developed during AIM Math Teachers Circle workshop.
   - NYC Teacher's Math Circle Summer Workshop.
   - Ten County Math Teachers' Conference
   - Senior Citizens in Bard's Lifetime Learning Institute

Game Plan:

- Started with a Math Olympiad problem
- Simplified with hands on computations
- Lots of examples and links to middle school math
- Real world applications

Developing Collaborative Lessons for Math Enrichment Success Stories

3. Dominoes and Graph Theory

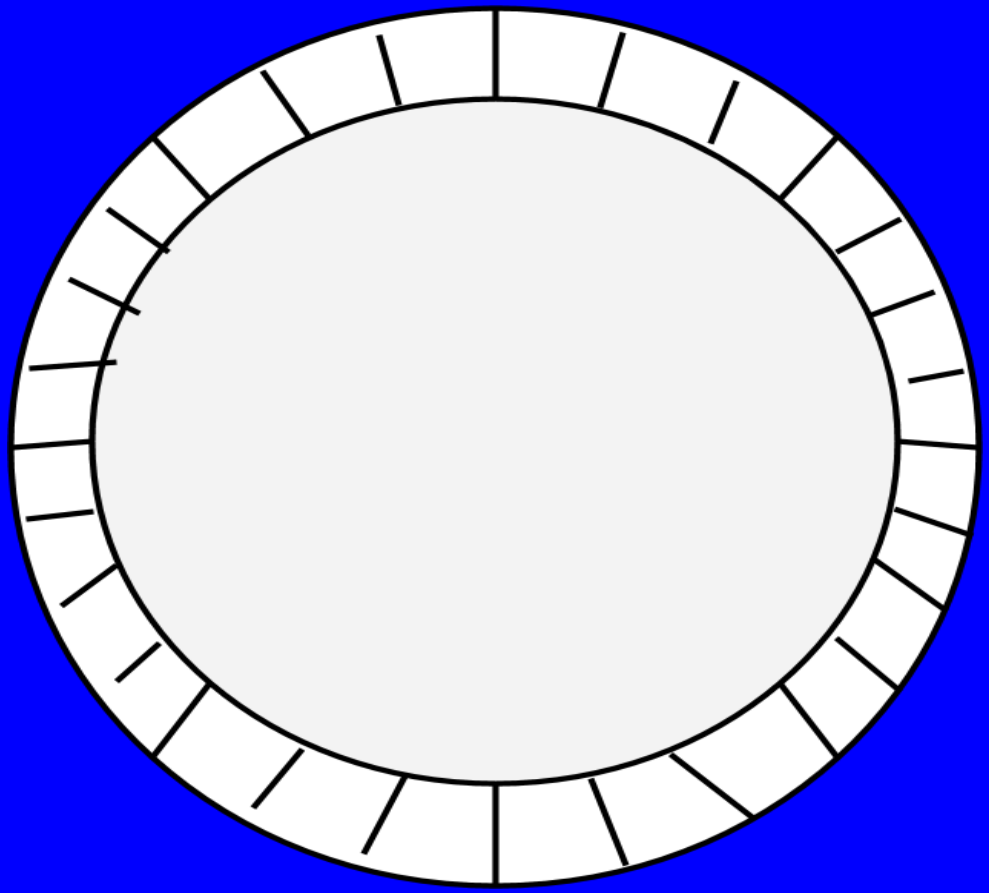
- Problem from Paul Zeitz' book about Domino circles.
- Assigned in Problem solving class at Bard
- Mid-Hudson Math Teachers Circle Presentation
- Presented to 7th graders in SPMPS at Bard
- (8th grade Math/Science Day at Bard)
- (Senior Citizens in Bard's Lifetime Learning Institute)

Developing Collaborative Lessons for Math Enrichment Success Stories

Dominoes and Graph Theory Lesson Plan

1. Using all 28 dominoes, can you put them in a single circle, using the usual domino rules?

If so, write in the dominoes in the circle below:

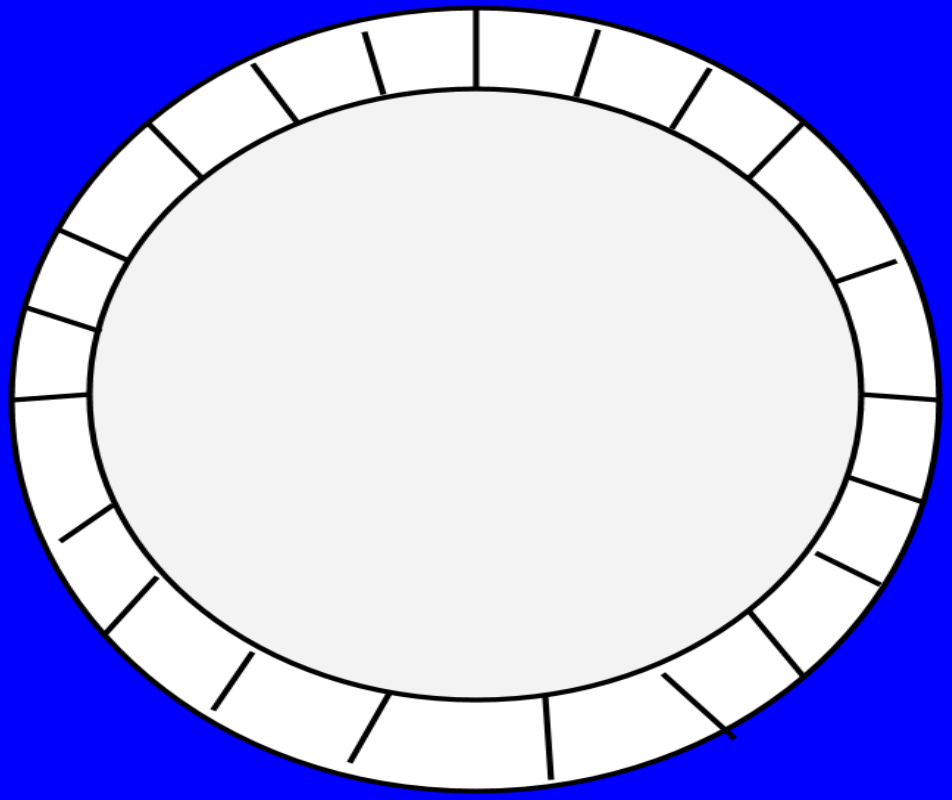


Developing Collaborative Lessons for Math Enrichment Success Stories

Dominoes and Graph Theory Lesson Plan

2. Now, remove all the dominoes with 6 pips (dots). Can you put the remaining 21 dominoes in a circle?

If so, write them in the circle below:



Developing Collaborative Lessons for Math Enrichment Success Stories

Dominoes and Graph Theory Lesson Plan

3. Try to make smaller domino circles and complete the chart below:

n = highest # of pips	Dominoes	Can you make a domino circle? (Yes or No)
0	00	
1	00, 01, 11	
2	00, 01, 02, 11, 12, 22	
3	00, 01, 02, 03, 11, 12, 13, 22, 23, 33	
4	00, 01, 02, 03, 04, 11, 12, 13, 14, 22, 23, 24, 33, 34, 44	
5	00, 01, 02, 03, 04, 05, etc.	
6	00, 01, 02, 03, 04, 05, 06, etc.	

Developing Collaborative Lessons for Math Enrichment Success Stories

Dominoes and Graph Theory Lesson Plan

4. Discuss with your group and write down your observations:

   1. Do you notice a pattern?

   2. Can you make a "conjecture" that works for any "n"?

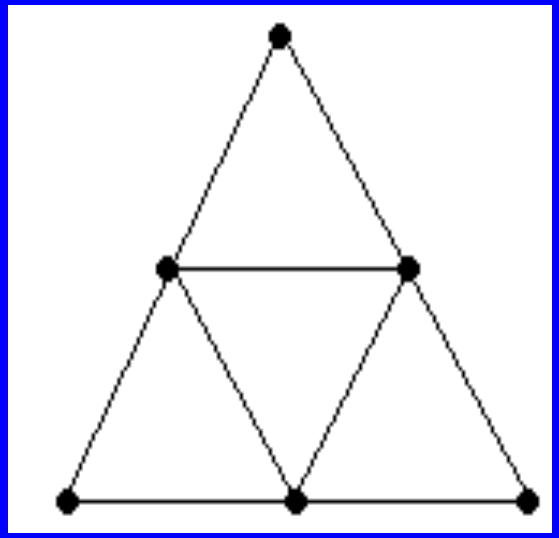
   3. Why do you think this pattern holds?

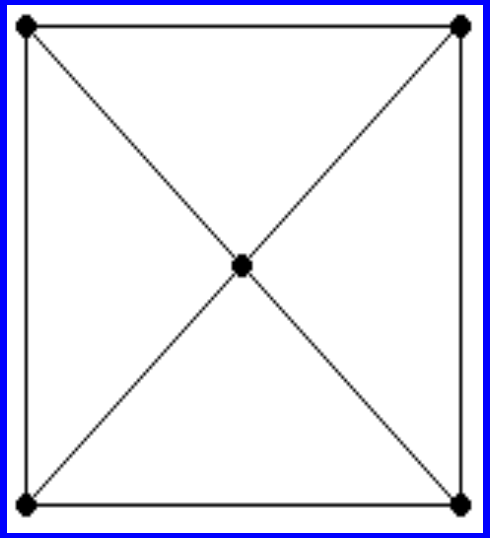
Developing Collaborative Lessons for Math Enrichment Success Stories

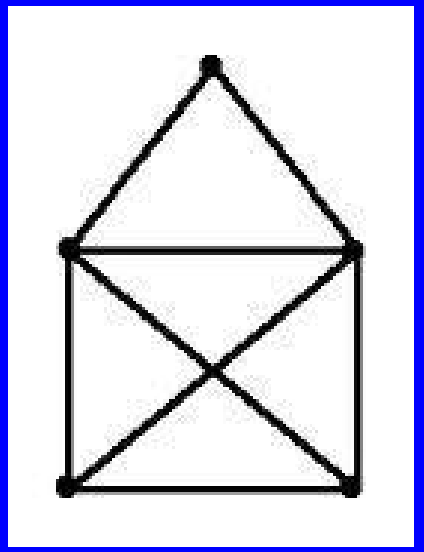
Dominoes and Graph Theory Lesson Plan

5. Graph Theory

- In which figures can you retrace the edges exactly once without lifting your pencil? In which figures can you also start and end in the same place?







Developing Collaborative Lessons for Math Enrichment Success Stories

Dominoes and Graph Theory Lesson Plan

6. Further Analysis and Examples, depending on age level
   - Get them to discover Eulerian Circuits and Paths
   - Königsburg Bridge Problem
   - Connect graph theory back to the domino problem
   - Applications of Graph Theory that involve Eulerian or Hamilton paths

Developing Collaborative Lessons for Math Enrichment

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