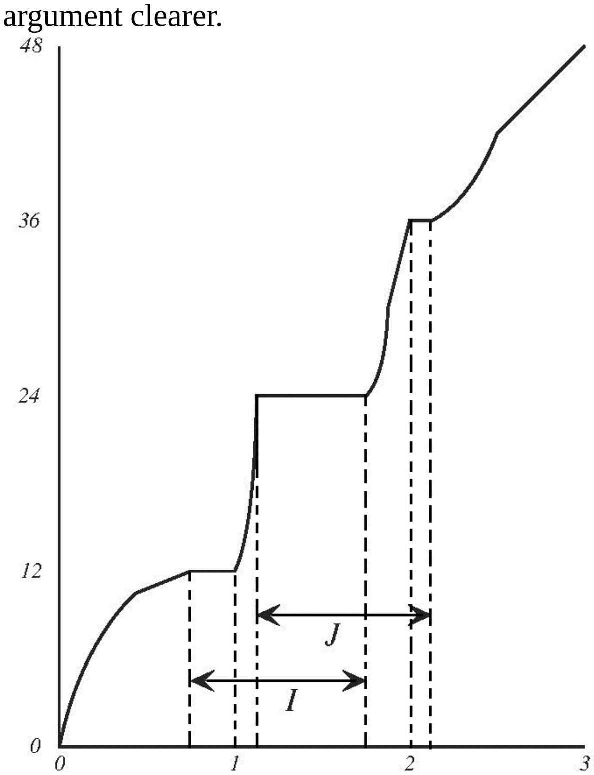
1 **The perceived speed is 4 revolutions per second, clockwise**. In $1/48$ seconds, the wheel spins $100/48 = 4\frac{1}{12}$ turns, so we perceive $1/12$ of a complete clockwise turn. Hence in one second, we perceive $48/12 = 4$ complete clockwise turns.

2 **The answer is 48**. Let $[a,b]$ denote the time interval from a to b minutes. Someone had to observe the worm during $[0,1]$; the worm thus traveled 12 inches during this time. Likewise, someone observed the worm during $[2,3]$, and the worm again traveled 12 inches during this final minute. There is a gap of one minute (the interval $[1,2]$) during which we claim the worm could travel at most 24 inches from two different observers. Consider the two overlapping observation intervals $I = [1-\varepsilon, 2-\varepsilon]$ and $J = [1+\varepsilon, 2+\varepsilon]$, where ε is a small (positive) number. If the worm had already traveled its first 12 inches by time $1-\varepsilon$, then it could travel another 12 inches during the portion of time interval within I that does not belong to $[0,1]$ and which is not a part of J. Similarly, the worm could travel another 12 inches during the portion of time interval J before 2 minutes that does not belong to I. Notice that this is the maximum amount possible, for we can always isolate at most two overlapping intervals of length 1 minute to cover the time in $[1,2]$ during which the worm travels exactly 12 inches for each. Here is a plausible distance-time graph, which should help make the above argument clearer.



3 Since $5 \cdot 2 = 10$ and there are plenty of 2's to go around, we can count factors of 5 from

$$z(2015) = \lfloor 2015/5 \rfloor + \lfloor 2015/5^2 \rfloor + \lfloor 2015/5^3 \rfloor + \lfloor 2015/5^4 \rfloor = 502.$$

This method first counts the multiples of 5, then adds in the multiples of 25, etc. Each time n reaches a multiple of 5, $z(n)$ will increase, but it will increase by 2 when n hits a multiple of 25, and by 3 when n hits a multiple of 5^3, etc. In other words, starting from the second term in the above sum, we will have an exact count of the number of "misses" as $z(n)$ increments. So our answer is just $502 - \lfloor 2015/5 \rfloor = 99$. The general formula for the number of non-wranglish numbers between 1 and $z(n)$ is thus $z(\lfloor n/5 \rfloor)$.

4 **It will take 28 moves.** The little cubies that make up the object fall into three categories: those that are never altered by the moves, those that are altered by just one move, and those that are altered by both moves A and B. For the cubies in the second category, four repeats of M will bring these back to the start. For cubies in the third category, there will be a total of 7 (the first three where only A moves the original and then on the 4th M, we see that A returns the color but then B sends it off and thus the final 3 moves of B from M restore the original). The answer follows since $28 = \text{LCM}(4,7)$.

5 The minimal time is $\sqrt{\mathbf{10}} + \sqrt{\mathbf{20}}$ **seconds**, which is accomplished by traveling on two straight line paths from $(1,3)$ to the origin and then the origin to $(-4,-2)$. The only question is why it is never optimal to visit the interior of quadrant II:

Any such trip would involve entering quadrant II at $(0,a)$ and leaving it at $(-b,0)$ for some positive a,b. Thus we need to compare the time traveled on the hypotenuse versus just staying on the legs. The hypotenuse time is $H = \frac{10}{7}\sqrt{a^2+b^2}$ and the leg time is $L = a+b$. Since

$$H^2 - L^2 = \frac{51}{49}a^2 + \frac{51}{49}b^2 - 2ab = \frac{2}{49}(a^2+b^2) + a^2 - 2ab + b^2 = \frac{2}{49}(a^2+b^2) + (a-b)^2 > 0,$$

it never makes sense to travel along the hypotenuse. Clearly this holds for any quadrant II speed that is less than $\sqrt{1/2}$.

6 **The answer is 499950**. Any palindrome will have the form *abba* where a and b are (possibly the same) digits. Thus there is one such Palindrome for each choice of $00, 01, \ldots, 99$. The sum of these numbers is 4950 and hence when we add this (for the *ba* in *abba*) to 100 times this (for the *ab* in *abba*) we will get $495000 + 4950 = 499950$.

7 The answer is 17. Clearly
$$\frac{1}{\sqrt{n}} = \frac{2}{\sqrt{n}+\sqrt{n}}.$$
Note that
$$\frac{2}{\sqrt{n}+\sqrt{n}} < \frac{2}{\sqrt{n}+\sqrt{n-1}} = 2(\sqrt{n}-\sqrt{n-1}).$$
Likewise,
$$\frac{2}{\sqrt{n}+\sqrt{n}} > \frac{2}{\sqrt{n+1}+\sqrt{n}} = 2(\sqrt{n+1}-\sqrt{n}).$$
Telescoping yields
$$2(\sqrt{L+1}-\sqrt{2}) < \sum_{n=2}^{L} \frac{1}{\sqrt{n}} < 2(\sqrt{L}-1).$$
Thus $2(\sqrt{101}-\sqrt{2}) < S < 18$. The lower bound is clearly greater than 17, so we conclude that $\lfloor S \rfloor = 17$.

8 The max distance is $60\pi + 5$ **with string** 1111100000. Given a 10 digit binary number consisting of k 0s and $10-k$ 1s means the bug will travel around k circles and then in addition go a length of $10-k$. Clearly to maximize distance those circles should be as big as possible. Thus all the 1s in the string should appear before any zeroes. It remains to find the value of k. The table below shows why $k = 5$ is the best:

STRING	**DISTANCE**	**COMMENTS**
0000000000	20π	
1000000000	$36\pi + 1$	
1100000000	$48\pi + 2$	
1110000000	$56\pi + 3$	
1111000000	$60\pi + 4$	
1111100000	$60\pi + 5$	**MAXIMUM**
1111110000	$56\pi + 6$	
1111111000	$48\pi + 7$	
1111111100	$36\pi + 8$	
1111111110	$20\pi + 9$	
1111111111	10	**MINIMUM**