

- 1 Let $P(x)$ be a 2015th-degree polynomial with real coefficients, satisfying

$$P(n) = n, \quad \text{for } n = 1, 2, 3, \dots, 2015.$$

What values are possible for $P(2016)$?

- 2 Find a positive integer N and a_1, a_2, \dots, a_N with $a_k = \pm 1$ for $k = 1, 2, \dots, N$, such that

$$\sum_{k=1}^N a_k k^3 = 2016,$$

or show that this is impossible.

- 3 Emilee and Zach play the following game: They flip a fair coin, alternating turns, and Emilee always goes first. The first person to flip heads wins a dollar from the other. Since Emilee flips first, she has an advantage, so they decide to start the game with Emilee having 2 dollars and Zach having 4 dollars. What is the probability that Emilee wins (i.e., takes all of Zach's money)?

- 4 A *tromino* tile consists of three squares glued end-to-end (i.e., like a domino, only the length is 3 times the width). Obviously, a standard 8×8 chessboard cannot be tiled by trominos that are made out of chessboard squares, since the chessboard has 64 squares, which is not a multiple of 3. ("Tiling with trominos" means covering with horizontal and vertical tiles creating no overlaps or gaps.)

However, if you remove one square from a chessboard, you are left with 63, which is a multiple of 3. So perhaps, if one removes a square from a chessboard, it will be possible to tile the 63-square shape remaining with trominos? Is this true for any choice of square to remove? Or for just some of the choices? Explain!

- 5 Consider the data set $\{33, 17, 21, 94, 46, 50, 66, 12, 75\}$. By ordering the data from smallest to largest we define Q_1, Q_2, Q_3 as follows:

$$12 \quad 17 \quad Q_1 = \left(\frac{17+21}{2}\right) = 19 \quad 21 \quad 33 \quad Q_2 = 46 \quad 50 \quad 66 \quad Q_3 = \left(\frac{66+75}{2}\right) = 70.5 \quad 75 \quad 94$$

In other words, Q_2 is the median and Q_1 and Q_3 are the lower half and upper half medians (with Q_2 removed). The *inter-quartile range* is $IQR = Q_3 - Q_1 = 70.5 - 19 = 51.5$. By changing 4 values but keeping the median and mean unchanged, make the inter-quartile range as small as possible. What is this minimum IQR?

- 6** Anna walks up an escalator which is going up. When she walks at one step per second, it takes her 20 steps to get to the top. If she walks at two steps per second, it takes her 32 steps to get to the top. She never skips over any steps. How many steps (i.e. those showing if we turn the escalator off) does the escalator have?
- 7** For a fixed real number u , define the sequence

$$a_2 = \frac{\sqrt{-u\sqrt{u}+u}}{u^2}, \quad a_3 = \frac{\sqrt{-u\sqrt{u\sqrt{u}+u}+u}}{u^3}, \quad a_4 = \frac{\sqrt{-u\sqrt{u\sqrt{u\sqrt{u}+u}+u}+u}}{u^4}, \dots$$

In other words, for each $n \geq 2$, the numerator of the expression for a_n contains n nested square roots, and the denominator is u^n .

Let $u = 0.5$. Compute the value of a_{100} , accurate to two decimal places.

- 8** Sonia is driving along a busy highway which has the same constant rate of traffic flow in each direction. Among all the drivers that day, everyone is traveling at exactly the legal limit except for Sonia, who is speeding. Sonia passes one car on her side of the highway for every 26 which she meets in the opposite direction. By what percentage is Sonia exceeding the speed limit?