The Missing "M" in STEM

(It's not just STE!)

A Math Circles and Modeling Approach

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What do we mean by “missing”?

- In STEM education, whether in the curriculum, extra-curricular, or external programs, mathematics is usually presented with
  - “Trust us!”
  - Hand-waving
  - Little (if any) context or background

Or perhaps "missing" ≈ in service of?

"Some people put the M under the STE, showing that math is actually the basis of it all. But then that falls into/promotes the idea that math is a utilitarian subject, a tool for solving complicated other problems."

--James Tanton

Where is it missing?

- Science fairs: The former director of the Santa Fe Alliance for Science, which runs or supports science fairs and their judging in New Mexico observed that very few science fair projects are driven by mathematics
- External or extracurricular programs
  - New Mexico's Supercomputing Challenge

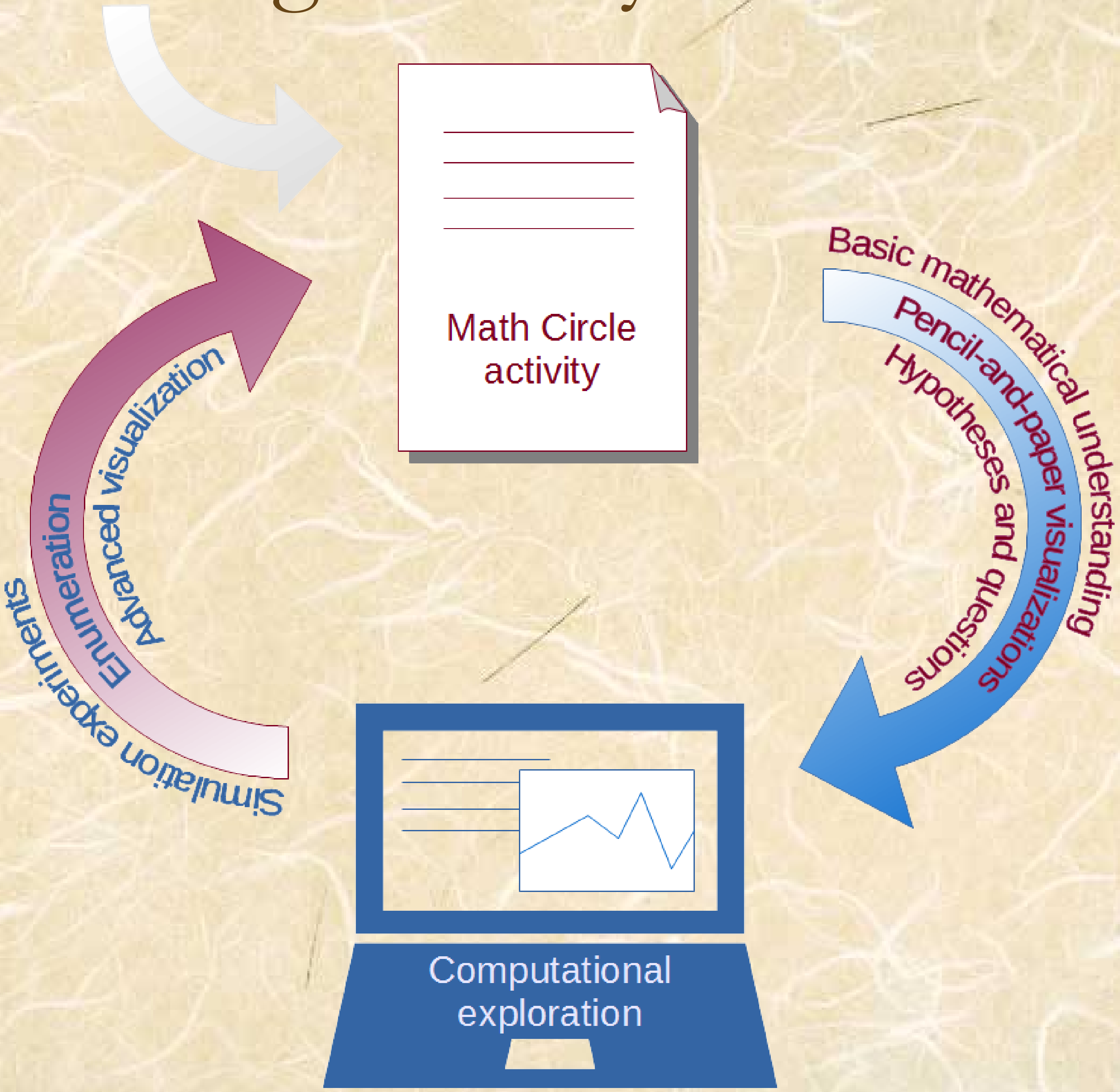
    (a three-decade old computational science research project for secondary students. Students are largely told what mathematics to use by their scientist-mentors)
- Science and computer science courses

An alternative approach

- Mathematics as the driver
  - It is then the hard-won and newly gained understanding of the mathematics involved that spurs students to use the tools of computational and computer science to extend and enrich their understanding of the math
- Not limited to computational approaches
  - **Starting with the math circle investigation of the mathematics** could be used for leading into investigations in any of the sciences

The Investigation Cycle

- Take standard K-12 math circles activities and extend or integrate them with computational science and visualization
- The mathematics drive the exploration and the students are grounded in the nascent mathematical understanding



Computational Approach Elements

- Visualization of data and dynamics
- Exhaustive data explorations
- Parameter space sweeps
- Constructing models
  - Conceptual
  - Agent-based
  - Mathematical

The Activity Setting Matters

- What sort of knowledge and skills with computational tools can we assume the students bring to the table?
- This depends in part on the setting:
  - Computer science class
  - Math circle
  - Mathematics course
  - Extra-curricular program
  - External STEM program
- We have chosen one of these settings for each of our examples

Student background and depth of approach

- No background in modeling or programming
  - Using the model or creating a basic spreadsheet following some introductory lessons
  - Focus on visualization and varied representations of the system
- Modest knowledge of modeling or programming tools
  - Adding to or modifying code
  - Parameter space exploration
  - More advanced spreadsheet use
- Advanced modeling or programming experience
  - Writing code
  - Designing and programming models
  - Creating parameter space sweep, consolidating data (e.g. w/pivot tables)

Current Implementation

- Use **widely available, free, cross-platform tools**
  - Visualization of data and dynamics using NetLogo and spreadsheet plots
  - Exhaustive data exploration and enumeration using spreadsheets and Python
  - Parameter space sweeps using NetLogo BehaviorSpace
  - Agent-based and systems dynamics model building using NetLogo or Vensim/STELLA

Our approach in early use

- Tsehootsooi PLC
  - Grades 5 & 6 math circle
  - Folding Fractals activity
  - Computer lab using NetLogo model and beginning programming in NetLogo
- Los Lunas Middle School
  - New Mexico Supercomputing Challenge students
  - Boomerang Fractions activity
  - Excel spreadsheet investigation

Our three pilot projects

Folding Fractals

1. Take a long piece of paper and tape one of the short edges of it on your left. Fold it in half, right over left, so that the free end lands on the tape. Unfold it. We'll call this V to represent the direction of the fold.

2. Fold the strip back in half, like it just was, and fold it in half again, right over left again. Unfold it. It should now look like V V Λ, by which we mean that the first two folds are "up" and the last fold is "down", sometimes called "valley" and "mountain".

3. Make an organized table of the folding patterns with three, four, five, and six folds, along with the one and two fold patterns you already have. Can you find a rule that helps you predict what the seven fold pattern will look like? Perhaps it would be easier to keep track of if you used some other symbols for the up and down folds – ones that don't look so similar, like 1 and 0 perhaps.

4. Try writing one sequence of folds at half size unde
suggest a different rule than the one you just foun

5. Explain why each of the two rules you found in th

6. Arrange your folded strip carefully so that each fo
fold, when viewed on edge, it looks like |__. Wha
folds? Three folds? And so on. Eventually it sho
itself. Can you give a recipe for the pattern of left
explain why it will never cross itself? It might hel
or to draw your shape on graph paper.

7. Starting with the digit 1 written down, and your fi
following rules, each time writing some digits at th
moving your finger one digit over:
If your finger is on a 1, write a 32 on the end.
If your finger is on a 2, write a 42 on the end.
If your finger is on a 3, write a 31 on the end.
If your finger is on a 4, write a 41 on the end.

After you've written for a while, replace each odd
digit with a Λ. What do you observe? Why does t

Boomerang Fractions

David Wilson suggested this question, which was explored by Gordon Hamilton, Joshua Zucker, Richard Guy, and other participants at the BIRS Conference on Integer Sequences February 27 – March 1, 2015. Bob Klein contributed to this summary.

Choose a fraction m/n. Begin a sequence of numbers with 1. To get the next number of the sequence, add m/n. On subsequent steps either add m/n or take the reciprocal. We say that the *longevity* of the fraction is the minimum number of steps needed to return to 1.

Consider the fraction 1/2. The sequence below shows the quickest way to return to 1 using this fraction, and so the *longevity* of 1/2 is 4.

$$1 \to \frac{3}{2} \to 2 \rightsquigarrow \frac{1}{2} \to 1.$$

Investigate some of the questions below or create your own questions about boomerang fractions.

back to 1 using the minimum

turn to 1?

ny branches will the tree have

Candy Sharing Game Rules

Here are the rules for the Candy Sharing game.

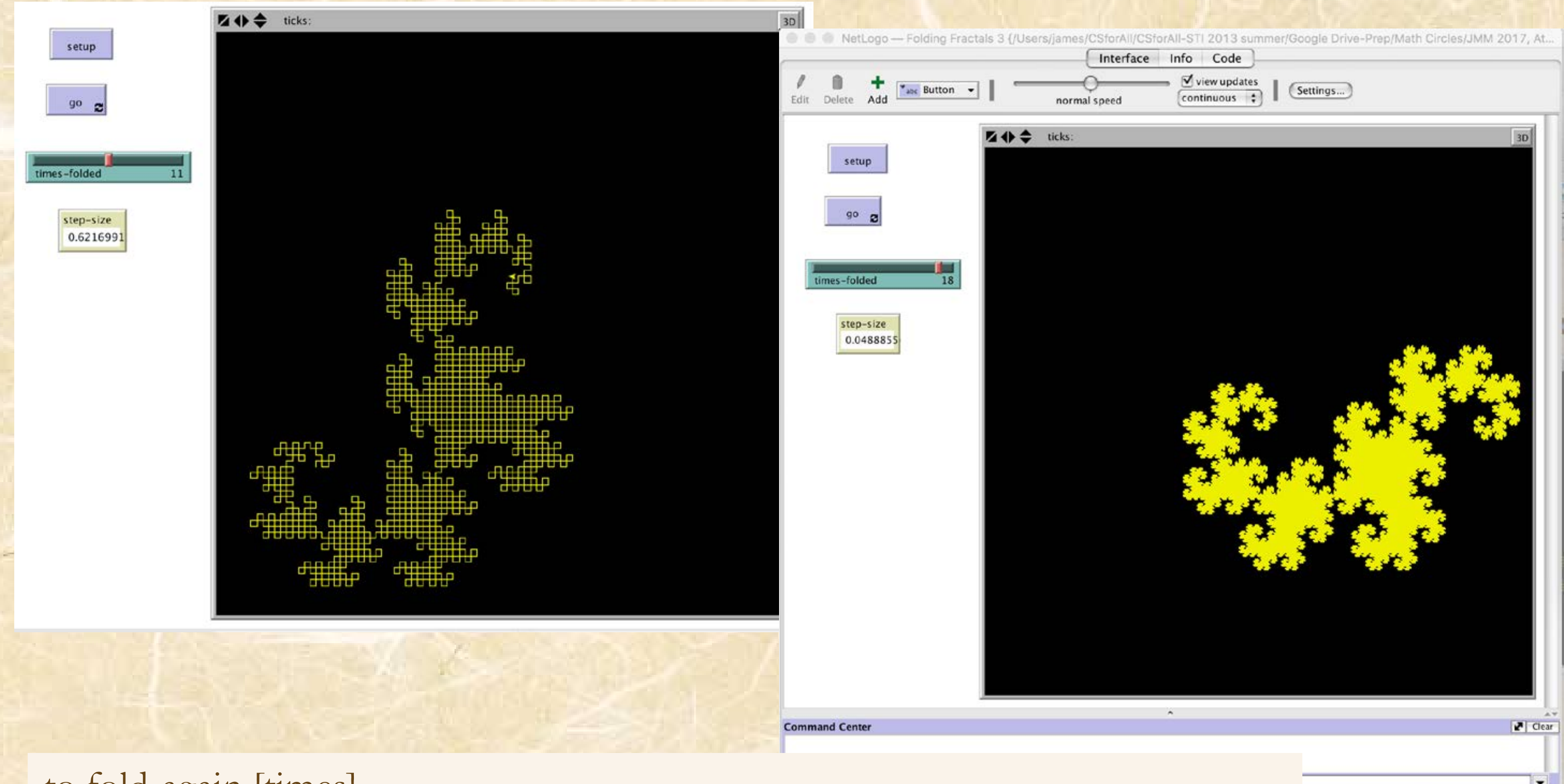
- The players should sit in a circle.
- Each circle will have a leader who starts the game by distributing wrapped candies among participants. The distribution of candy will not necessarily be even – some playerswill start out with more and some players may start out with none.
- During the game, players should keep their candy on the ground in front of them. They should make sure that the candy does not get lost under their legs and they should not hold it in their hands.
- When the leader says "Share!", everyone who has two or more pieces of candy in front of them gives one piece to the person on their right and one piece to the person on their left. They should use both arms to do this at the same time. Players with one or zero pieces of candy do nothing.
- After the appropriate players have shared candy, the leader will say "Share!" again. This process repeats until the group sees a pattern emerge in the game.
- Several things might happen with the game.
  1. The game might stop because no one is passing candy any more.
  2. The game might settle down so that even though everyone passes candy every time, the amount of candy that each player has is always the same. This is called a *fixed point*.
  3. A repeating pattern might emerge in the way that the candy is shared. This is called a *cycle*.

After you have tried the challenges on the next page, you might want to try changing the rules of the game.

Project Summary: Folding Fractals

- The math circle activity (Author: J. Zucker)
  - Begin with folding some adding machine paper in half, then again and again, taking note of the sequence of the two fold types "\/" and "/\" created in each successive set of folds. "Can you discover a pattern to the development of the fold lists? Can you find a rule that helps you predict what the *n*-fold pattern will look like? Arrange your folded strip carefully so that each fold is a right angle." Topics: Fractals, L-Systems, List Manipulation
- The computational extension
  - **Activity class: Visualization, computation & algorithms**
  - **Setting: Math circle**
  - Use or modify an agent-based **NetLogo** model to extend and visualize the paper folding process beyond what is physically possible
  - Alternatively, the model can be introduced without the essential algorithm for the fold list construction, and students follow a circle-type activity sheet for expressing "walking a list", then writing pseudocode and NetLogo code
  - Add box-counting fractal dimension calculation

Folding Fractals




```
to fold-again [times]
  repeat (times - 2) [set folds (sentence folds R reverse map not folds)]
end
```

Project Summary: Boomerang Fractions

- The math circle activity (Authors: Serenevy, Klein, Zucker, et al)
  - “Choose a fraction m/n. Begin a sequence of numbers with 1. To get the next number of the sequence, add m/n. On subsequent steps either add m/n or take the reciprocal. We say that the *longevity* of the fraction is the minimum number of steps needed to return to 1.” Topics: Sequences, Number Theory

$$1 \to \frac{3}{2} \to 2 \rightsquigarrow \frac{1}{2} \to 1.$$

- The computational extension
  - **Activity class: Computation, enumeration, & algorithms**
  - **Setting: Math circle, computer science course, math class, external STEM program**
  - Students create a **spreadsheet** using randomly-generated sequences , then enumeration. If a CS class then more sophisticated, generalized, and versatile solutions written in Python, etc.
  - Particularly easy here to teach students how to use the initial tool—the ubiquitous spreadsheet.
  - Students in computer science courses can explore solution algorithms

Boomerang Fractions: Random Choice

F	G	H	I	J	K	L	M	N
Boomerang Fractions: Random sequence generator								
Start	1				7	= Total count of 1s		
Fraction	2/3	1 2/3						
	Step numbe	Count of 1s by row						
	Starter valu	0	5/3	5/3	5/3	5/3	5/3	5/3
	1	0	7/3	7/3	3/5	3/5	7/3	7/3
	2	0	3/1	3/1	5/3	19/15	3/7	3/7
	3	0	11/3	11/3	7/3	29/15	7/3	7/3
	4	0	13/3	13/3	3/7	15/29	3/1	3/7
	5	0	5/1	3/13	7/3	29/15	1/3	7/3
	6	1	17/3	35/39	3/7	15/29	3/1	3/7
	7	1	19/3	61/39	23/21	103/87	1/3	7/3
	8	1	7/1	39/61	37/21	87/103	3/1	3/7
	9	[illegible]	[illegible]	[illegible]	21/37	467/309	1/3	23/21
	10	[illegible]	[illegible]	[illegible]	37/21	309/467	1/1	37/21
	11	[illegible]	[illegible]	[illegible]	17/7	1861/1401	5/3	21/37
	12	[illegible]	[illegible]	[illegible]	65/21	1401/1861	7/3	37/21
	13	[illegible]	[illegible]	[illegible]	79/21	7925/5583	3/1	21/37
	14	0	11/1	605/183	31/7	11647/5583	1/3	137/111
	15	1	35/3	727/183	107/21	5583/11647	1/1	211/111
	16	1	37/3	283/61	121/21	40043/34941	1/1	111/211
	17	0	13/1	61/283	45/7	63337/34941	5/3	211/111
	18	0	41/3	749/849	7/45	28877/11647	3/5	95/37
	19	0	43/3	1315/849	45/7	109925/34941	19/15	37/95
	20	0	15/1	849/1315	149/21	133219/34941	29/15	95/37
	21	0	47/3	1315/849	21/149	52171/11647	15/29	37/95
	22	0	49/3	627/283	149/21	11647/52171	29/15	301/285
	23	0	17/1	2447/849	163/21	139283/156513	13/5	285/301

Sheet16 Sheet17 Sheet18 Sheet19 Sheet20 Sheet21 Sheet22 Sheet23 Sheet24 Sheet25 Sheet26 Sheet27 Sheet28 Sheet29 Sheet30 Sheet31 Sheet32 Sheet33

Here we see a 1
is reached in
column M

Boomerang Fractions: Full Search

Boomerang Fractions: Exhaustive sequence search, depth 9

A	B	C	D	E	F	G	H	I	J	K
	Depth	9		Start	1					
	Count	511		Fraction	2/3	**1 2/3**				
				Add	0					
				Reciprocal	1					
Decimal	Binary									
1	000000001	0	0	0	0	0	0	0	0	1
2	000000010	0	0	0	0	0	0	0	1	0
3	000000011	0	0	0	0	0	0	0	1	1
4	000000100	0	0	0	0	0	0	1	0	0
5	000000101	0	0	0	0	0	0	1	0	1
6	000000110	0	0	0	0	0	0	1	1	0
7	000000111	0	0	0	0	0	0	1	1	1
8	000001000	0	0	0	0	0	1	0	0	0
9	000001001	0	0							
10	000001010	0	0							
11	000001011	0	0							
12	000001100	0	0							
13	000001101	0	0							
14	000001110	0	0							
15	000001111	0	0							
16	000010000	0	0							
17	000010001	0	0							
18	000010010	0	0							
19	000010011	0	0							
20	000010100	0	0							
21	000010101	0	0							
22	000010110	0	0							
23	000010111	0	0							
24	000011000	0	0							
25	000011001	0	0							
26	000011010	0	0							
27	000011011	0	0							
28	000011100	0	0							
29	000011101	0	0							
30	000011110	0	0							
31	000011111	0	0							
32	000100000	0	0							
33	000100001	0	0							
34	000100010	0	0							
35	000100011	0	0							
36	000100100	0	0							

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	Returns to 1	127										
	Depth	1	2	3	4	5	6	7	8	9		
	Column min	3/5	3/7	1/3	3/11	3/13	1/5	3/17	3/19	1/7		**Operation Sequence**
Where 1s occur-->>		0	0	0	32	16	32	16	20	11		
0	5/3	7/3	3/1	11/3	13/3	5/1	17/3	19/3	7/1	1/7		AAAAAAAAR
0	5/3	7/3	3/1	11/3	13/3	5/1	17/3	19/3	3/19	47/57		AAAAAAARA
0	5/3	7/3	3/1	11/3	13/3	5/1	17/3	19/3	3/19	19/3		AAAAAAARR
0	5/3	7/3	3/1	11/3	13/3	5/1	17/3	3/17	43/51	77/51		AAAAAARAA
0	5/3	7/3	3/1	11/3	13/3	5/1	17/3	3/17	43/51	51/43		AAAAAARAR
0	5/3	7/3	3/1	11/3	13/3	5/1	17/3	3/17	17/3	19/3		AAAAAARRA
0	5/3	7/3	3/1	11/3	13/3	5/1	17/3	3/17	17/3	3/17		AAAAAARRR
0	5/3	7/3	3/1	11/3	13/3	5/1	1/5	13/15	23/15	11/5		AAAAARAAA
0	5/3	7/3	3/1	11/3	13/3	5/1	1/5	13/15	23/15	15/23		AAAAARAAR
0	5/3	7/3	3/1	11/3	13/3	5/1	1/5	13/15	15/13	71/39		AAAAARARA
0	5/3	7/3	3/1	11/3	13/3	5/1	1/5	13/15	15/13	13/15		AAAAARARR
0	5/3	7/3	3/1	11/3	13/3	5/1	1/5	5/1	17/3	19/3		AAAAARRAA
0	5/3	7/3	3/1	11/3	13/3	5/1	1/5	5/1	17/3	3/17		AAAAARRAR
0	5/3	7/3	3/1	11/3	13/3	5/1	1/5	5/1	1/5	13/15		AAAAARRRA
0	5/3	7/3	3/1	11/3	13/3	5/1	1/5	5/1	1/5	5/1		AAAAARRRR
0	5/3	7/3	3/1	11/3	13/3	3/13	35/39	61/39	29/13	113/39		AAAARAAAA
0	5/3	7/3	3/1	11/3	13/3	3/13	35/39	61/39	29/13	13/29		AAAARAAAR
0	5/3	7/3	3/1	11/3	13/3	3/13	35/39	61/39	39/61	239/183		AAAARAARA
0	5/3	7/3	3/1	11/3	13/3	3/13	35/39	61/39	39/61	61/39		AAAARAARR
0	5/3	7/3	3/1	11/3	13/3	3/13	35/39	39/35	187/105	257/105		AAAARARAA
0	5/3	7/3	3/1	11/3	13/3	3/13	35/39	39/35	187/105	105/187		AAAARARAR
0	5/3	7/3	3/1	11/3	13/3	3/13	35/39	39/35	35/39	61/39		AAAARARRA
0	5/3	7/3	3/1	11/3	13/3	3/13	35/39	39/35	35/39	39/35		AAAARARRR
0	5/3	7/3	3/1	11/3	13/3	3/13	13/3	5/1	17/3	19/3		AAAARRAAA
0	5/3	7/3	3/1	11/3	13/3	3/13	13/3	5/1	17/3	3/17		AAAARRAAR
0	5/3	7/3	3/1	11/3	13/3	3/13	13/3	5/1	1/5	13/15		AAAARRARA
0	5/3	7/3	3/1	11/3	13/3	3/13	13/3	5/1	1/5	5/1		AAAARRARR
0	5/3	7/3	3/1	11/3	13/3	3/13	13/3	3/13	35/39	61/39		AAAARRRAA
0	5/3	7/3	3/1	11/3	13/3	3/13	13/3	3/13	35/39	39/35		AAAARRRAR
0	5/3	7/3	3/1	11/3	13/3	3/13	13/3	3/13	13/3	5/1		AAAARRRRA
0	5/3	7/3	3/1	11/3	13/3	3/13	13/3	3/13	13/3	3/13		AAAARRRRR
0	5/3	7/3	3/1	11/3	3/11	31/33	53/33	25/11	97/33	119/33		AAARAAAAA
0	5/3	7/3	3/1	11/3	3/11	31/33	53/33	25/11	97/33	33/97		AAARAAAAR
0	5/3	7/3	3/1	11/3	3/11	31/33	53/33	25/11	11/25	83/75		AAARAAARA

Z	AA	AB	AC	AD	AE	AF	AG	AH	AI
AAAAAAAAR	A	A	A	A	A	A	A	A	R
AAAAAAARA	A	A	A	A	A	A	A	R	A
AAAAAAARR	A	A	A	A	A	A	A	R	R
AAAAAARAA	A	A	A	A	A	A	R	A	A
AAAAAARAR	A	A	A	A	A	A	R	A	R
AAAAAARRA	A	A	A	A	A	A	R	R	A
AAAAAARRR	A	A	A	A	A	A	R	R	R
AAAAARAAA	A	A	A	A	A	R	A	A	A
AAAAARAAR	A	A	A	A	A	R	A	A	R
AAAAARARA	A	A	A	A	A	R	A	R	A
AAAAARARR	A	A	A	A	A	R	A	R	R
AAAAARRAA	A	A	A	A	A	R	R	A	A
AAAAARRAR	A	A	A	A	A	R	R	A	R
AAAAARRRA	A	A	A	A	A	R	R	R	A
AAAAARRRR	A	A	A	A	A	R	R	R	R
AAAARAAAA	A	A	A	A	R	A	A	A	A
AAAARAAAR	A	A	A	A	R	A	A	A	R
AAAARAARA	A	A	A	A	R	A	A	R	A
AAAARAARR	A	A	A	A	R	A	A	R	R
AAAARARAA	A	A	A	A	R	A	R	A	A
AAAARARAR	A	A	A	A	R	A	R	A	R
AAAARARRA	A	A	A	A	R	A	R	R	A
AAAARARRR	A	A	A	A	R	A	R	R	R
AAAARRAAA	A	A	A	A	R	R	A	A	A
AAAARRAAR	A	A	A	A	R	R	A	A	R
AAAARRARA	A	A	A	A	R	R	A	R	A
AAAARRARR	A	A	A	A	R	R	A	R	R
AAAARRRAA	A	A	A	A	R	R	R	A	A
AAAARRRAR	A	A	A	A	R	R	R	A	R
AAAARRRRA	A	A	A	A	R	R	R	R	A
AAAARRRRR	A	A	A	A	R	R	R	R	R
AAARAAAAA	A	A	A	R	A	A	A	A	A
AAARAAAAR	A	A	A	R	A	A	A	A	R
AAARAAARA	A	A	A	R	A	A	A	R	A

Boomerang Fractions: Solution

Spyder (Python 3.5)

File Edit Search Source Run Debug Consoles Projects Tools View Help

C:\Users\Nicholas Bennett

Editor - C:\Users\Nicholas Bennett\Documents\Math Circles\boomerang_3.py

boomerang_1.py | boomerang_2.py | boomerang_3.py | boomerang_4.py

```python
def boomerang(fraction, limit=100):
    """Construct and return a boomerang fraction sequence.

    Arguments:
        fraction -- Additive term in boomerang fraction sequence
        limit    -- Maximum number of inversions that will be pe
                    before the function terminates unsuccessfull
                    value of 0 or None disables the limit check.
    Return value:
        List of interleaved values and operations.  If the first
        in the list is not 1, then the function has terminated w
        successfully completing a boomerang fraction sequence.
    """
    value = Fraction(1)
    sequence = [value]
    inversion_count = 0
    while True:
        value -= fraction
        sequence = [value, ADDITION_OPERATION] + sequence
        if value == 1:
            break
        if (
            value <= fraction  # Subtraction not possible; inver
            or (
                value < 1
                and (1 / value - 1) % fraction == 0
            )  # Shortcut available; invert.
        ):
            if limit and inversion_count >= limit:  # No more ir
                break
            value = 1 / value
            inversion_count += 1
            sequence = [value, INVERSION_OPERATION] + sequence
    return sequence
```

IPython console — Console 1/A

```
object?   -> Details about 'object', use 'object??' for extra details.

In [1]: runfile('C:/Users/Nicholas Bennett/Documents/Math Circles/
boomerang_3.py', wdir='C:/Users/Nicholas Bennett/Documents/Math
Circles')
1/2
1 [add] 3/2 [add] 2 [invert] 1/2 [add] 1

2/3
1 [add] 5/3 [add] 7/3 [add] 3 [invert] 1/3 [add] 1

1/4
1 [add] 5/4 [add] 3/2 [add] 7/4 [add] 2 [invert] 1/2 [add] 3/4 [add] 1

4/5
1 [add] 9/5 [add] 13/5 [add] 17/5 [add] 21/5 [add] 5 [invert] 1/5 [add]
1

2/5
1 [add] 7/5 [add] 9/5 [add] 11/5 [add] 13/5 [add] 3 [add] 17/5 [add]
19/5 [add] 21/5 [add] 23/5 [add] 5 [invert] 1/5 [add] 3/5 [add] 1

7/9
Failed

2/7
1 [add] 9/7 [add] 11/7 [add] 13/7 [add] 15/7 [add] 17/7 [add] 19/7
[add] 3 [add] 23/7 [add] 25/7 [add] 27/7 [add] 29/7 [add] 31/7 [add]
33/7 [add] 5 [add] 37/7 [add] 39/7 [add] 41/7 [add] 43/7 [add] 45/7
[add] 47/7 [add] 7 [invert] 1/7 [add] 3/7 [add] 5/7 [add] 1


In [2]:
```

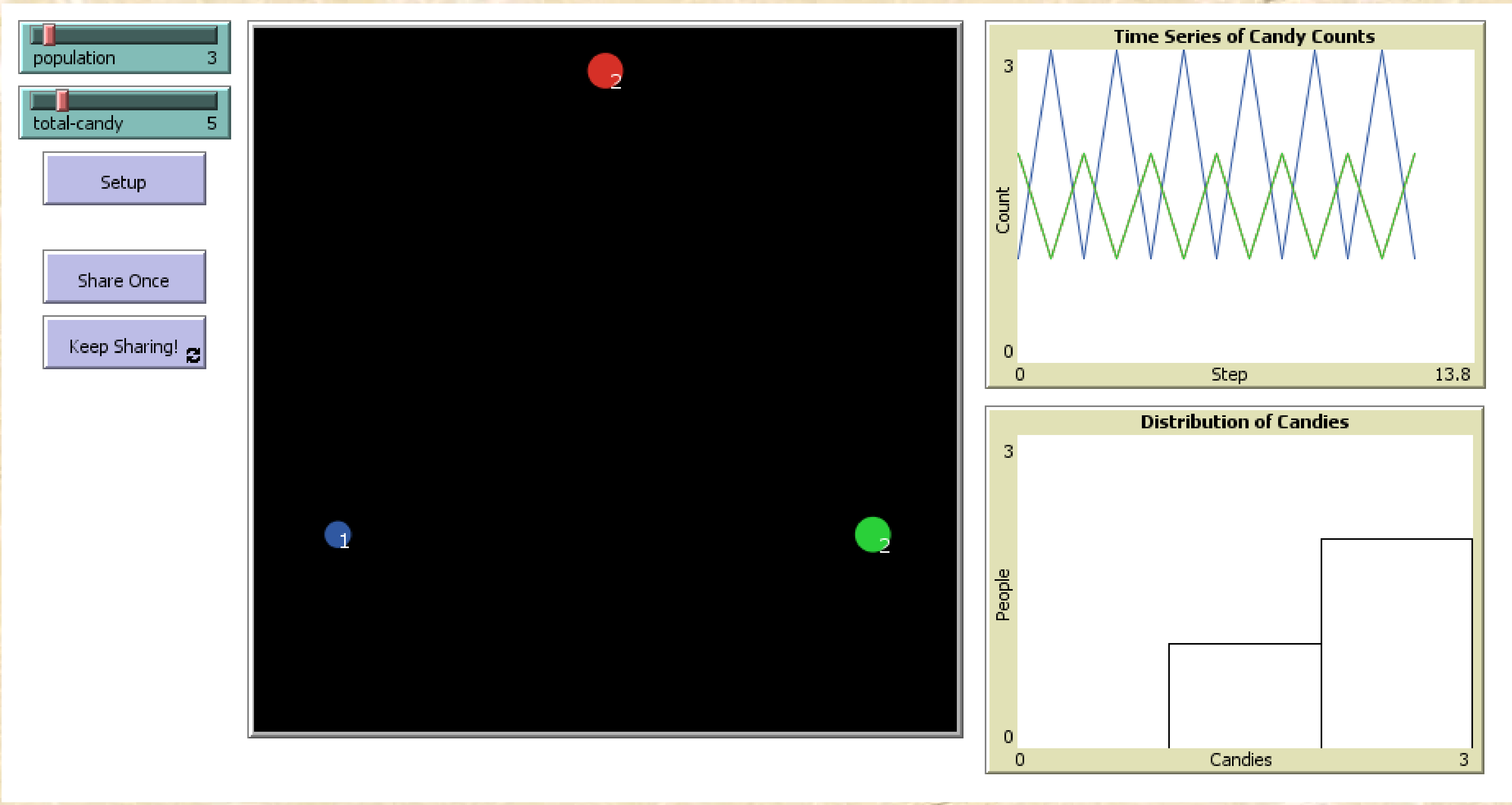
Python console | History log | IPython console

Permissions: RW | End-of-lines: CRLF | Encoding: UTF-8 | Line: 22 | Column: 32 | Memory: 51 %

Project Summary: Candy Sharing

- The math circle activity (Author: A. Serenevy)
  - "Students learn a candy sharing game with a simple rule for passing candy around the circle. Students will experiment to figure out how to make the game stop, end in a fixed point, or end in a cycle depending on the amount of starting candy, the number of people playing, and the initial distribution of candy." Topics: Dynamical Systems, Number Patterns
- The computational extension
  - **Activity class: Algorithm variations and parameter space investigation**
  - **Setting: Computer science course**
  - Students build or use a dynamic agent-based **NetLogo** model to rapidly explore different conditions, varying number of players, total amount and distribution of candy
  - Using NetLogo's Behavior Space facility, students can automate large-scale exploration of parameter spaces over these variables
  - Students can explore alternative sharing algorithms by coding procedures that sequence the candy sharing steps differently

Candy Sharing: Agent-based Model



Candy Sharing: Parameter Sweep

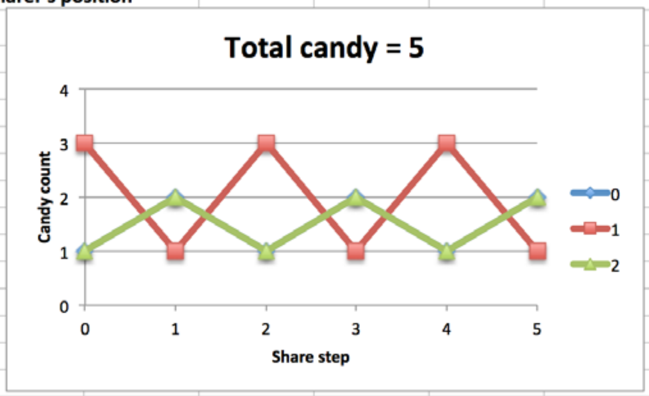
BehaviorSpace results (NetLogo 5.3.1)
Candy Sharing 11.nlogo
Vary total-candy 1
11/23/2016 09:15:11:036 -0700

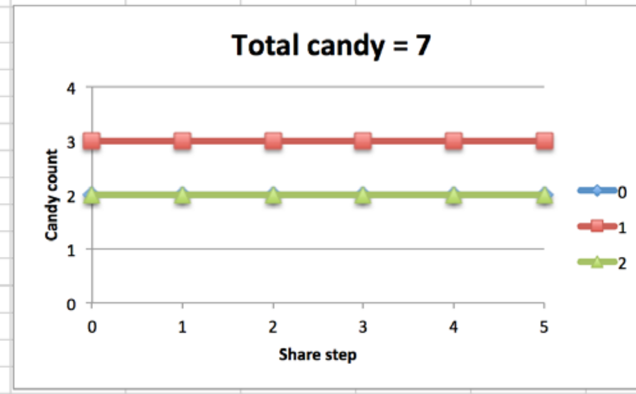
min-pxcor	max-pxc	min-pycor	max-py
-16	16	-16	16

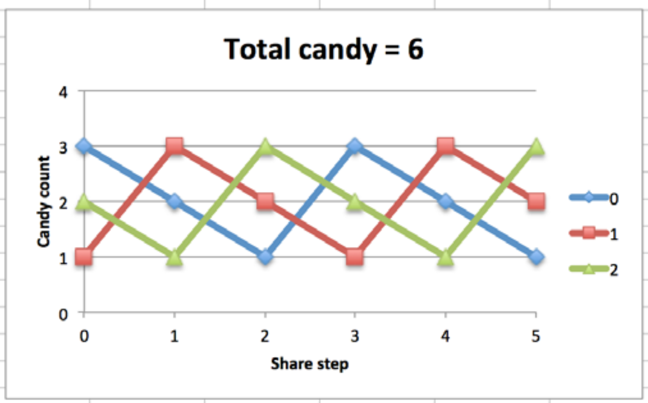
Sort by run-number, then step.

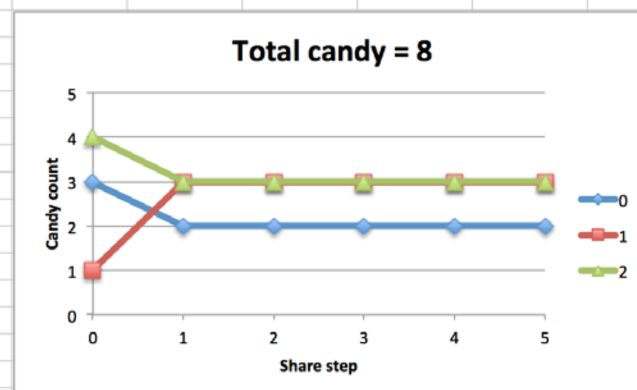
Candy-count by sharer's position

[run number	people	total-candy	[step]	0	1	2
1	3	5	0	1	3	1
1	3	5	1	2	1	2
1	3	5	2	1	3	1
1	3	5	3	2	1	2
1	3	5	4	1	3	1
1	3	5	5	2	1	2
2	3	6	0	3	1	2
2	3	6	1	2	3	1
2	3	6	2	1	2	3
2	3	6	3	3	1	2
2	3	6	4	2	3	1
2	3	6	5	1	2	3
3	3	7	0	2	3	2
3	3	7	1	2	3	2
3	3	7	2	2	3	2
3	3	7	3	2	3	2
3	3	7	4	2	3	2
3	3	7	5	2	3	2
4	3	8	0	3	1	4
4	3	8	1	2	3	3
4	3	8	2	2	3	3
4	3	8	3	2	3	3
4	3	8	4	2	3	3
4	3	8	5	2	3	3

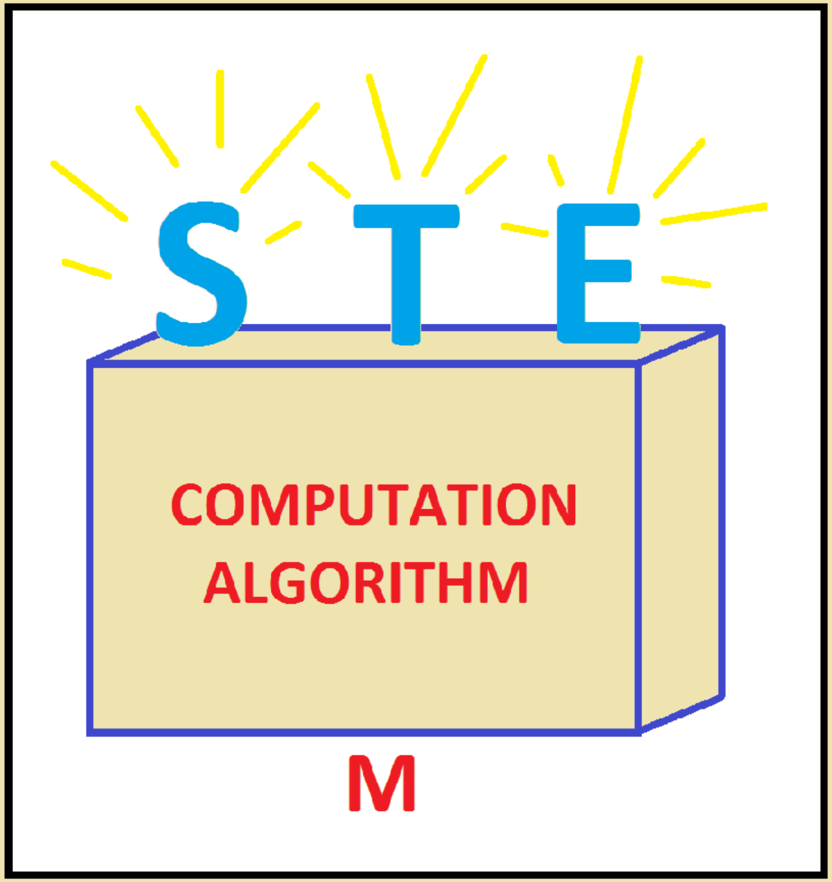




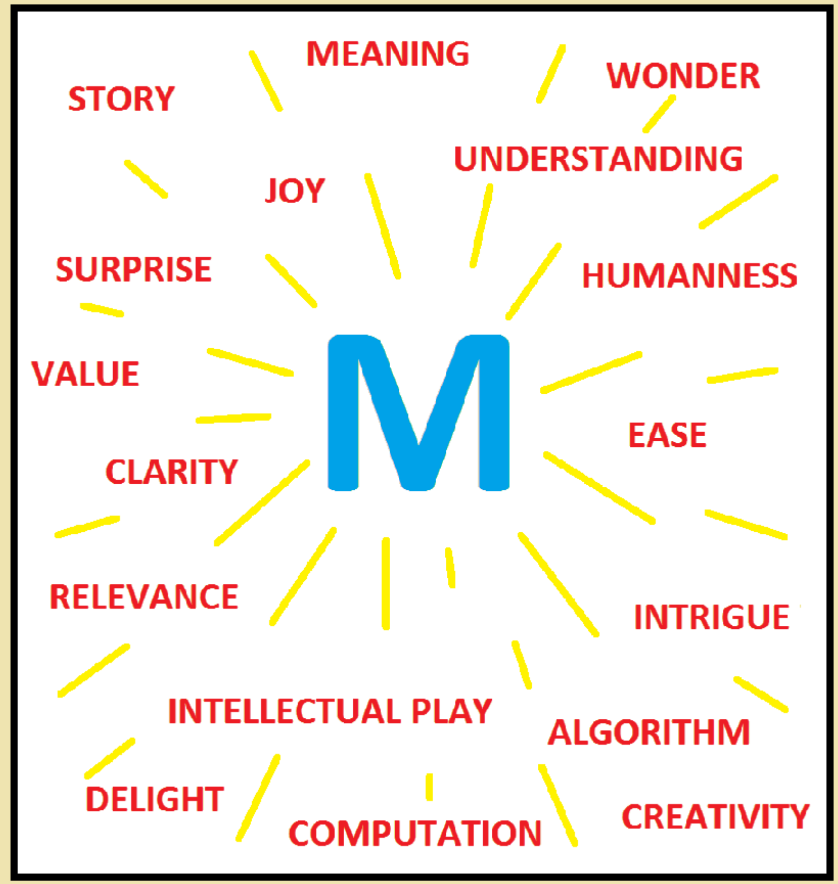




Seeing M just as a fourth letter



How the Global Math Project sees M



The Global Math Project

www.theglobalmathproject.org

Resources

Sample Missing "M" activity sheet packets up front!

Slideshow, spreadsheets, models, applications & handouts:

- http://www.mathcirclesnm.org/missing-m/jmm2017
- http://www.mathcirclesnm.org/

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