Möbius Band Bouquets

Background

During this activity, you will encounter an area of mathematics called *topology*. Topology is sometimes called “rubber sheet geometry” because it studies properties that do not change when an object is deformed or rearranged. Loops, knots, braids, Möbius bands, tori (donut shapes), and Klein bottles are just a few examples of objects that topologists study.

Joan Birman is a topologist who has made important contributions in knot and braid theory. Biologists are using her ideas to investigate how DNA strands in living cells get tangled and untangled.

Materials Needed

- 1 Pencil
- Tape
- 1 Marker
- Scissors
- About 30 Paper Strips

Tips to Keep in Mind During This Activity

- TWISTING: Practice keeping track of how many twists you make in a paper strip. Start by using two hands to bring the ends together as if you were making an untwisted loop. Put in each twist by turning ONE of your hands over once.
- TAPING: Make all pieces of tape go all the way across the strip.
- CUTTING: When cutting loops down the middle, start by pinching the loop. Make a small snip in the middle of the paper, open the paper, and insert the scissors to cut around the loop.

The Basics

- The Untwisted Loop
  - Make a normal loop with no twists.
  - Choose one of the edges of the untwisted loop and color a continuous border along that edge until you return to where you began.
  - What will happen if you cut your untwisted loop around the middle? Try it!
- The Möbius Band
  - Form a loop with a single twist in it. This shape is called a Möbius Band.
  - Choose one of the edges of the Möbius Band and color a continuous border along that edge until you return to where you began.
  - What happened when you colored the border? How is this result different from what happened when you colored the untwisted loop. The untwisted loop had an inside surface and an outside surface. How many separate surfaces does the Möbius Band have? The untwisted loop had two edges: an edge touched by the colored border and one that was not touched by the border. What happened with the Möbius Band? How many edges does the Möbius Band have?
  - What do you think will happen if you cut your Möbius Band down the middle? Try it! Why is the result different from the untwisted loop?

Twisting Time

Try cutting loops with 2, 3, 4, 5, 6, 7, or more twists down the middle.

How many components did the shape have after you cut it – two like the untwisted loop or one like the Möbius Band? When will there be two components and when will there be only one component? Why?

Multi-Loop Bouquets

- A Bouquet With Two Untwisted Loops
  - Begin by making a paper chain out of two untwisted loops.
  - Hold the two loops together so that they cross in a little square. Tape around the square in four places – two on one side and two on the other. (See the sample.)
  - What do you think will happen when you cut both loops down the middle? Try it! Don't stop cutting until both paper strips are cut completely in half.
- A Bouquet With Two Möbius Bands Twisted in Different Directions
  - Make a paper chain out of two Möbius Bands that are twisted in different directions. One way to do this is to form the first Möbius Band by twisting your writing hand towards you, and form the other Möbius Band by twisting your writing hand away from you.
  - Tape the two loops together so that they cross in a little square and tape around the square in four places like the sample.
  - Cut this bouquet down the middle!
- Try the following loop bouquets or make up your own – remember to tape loops together in all the places where the loops cross before cutting down the middle.
  - Two Möbius Bands Twisted in the Same Direction
  - One Möbius Band One Untwisted Loop
  - One Untwisted Loop and a Loop with Two Twists
  - Two Untwisted Loops Crossing at an Angle
  - Three Untwisted Loops

Joan Birman and Topology

Joan Birman (1927 –) is an American mathematician who has made fundamental contributions to topology. This field of mathematics concerns the properties of shapes that do not change when they are deformed. In 1975, Joan laid the foundations for several new subfields in topology by writing the book *Braids, Links, and Mapping Class Groups*. During her thirty-five year career as a research mathematician, Joan has gone on to write over ninety articles. Her ideas have been applied to tasks ranging from the control of robotic arms in factories to understanding the tangled strands of DNA in living creatures.

As a young girl, Joan loved mathematics. She recalls, "My love of patterns — the swirling patterns of marbles as I spilled them out on the floor, and the matching of plaids as my mother was sewing skirts for me — merged right into formal math as I got older. I remember very clearly when an elementary school teacher asked about whether the sum and products of two odd numbers was even or odd, and how excited I was to understand it, and how beautiful it seemed that such patterns existed if you only knew enough to ask the right questions." In high school, Joan had excellent math teachers. Her fellow students loved to have friendly competitions to see who could solve the challenge problems their teachers gave them.

When Joan took her first math class in college, she was bored and confused and felt like she was wasting her time. At first, she thought that that she was just not cut out to study math. When she took another math class later on, she realized that the first class was just taught poorly. Joan ended up earning her bachelor's degree in mathematics in 1948, and received a master's degree in physics in 1950.

Joan married and found a job as a systems analyst in the aircraft industry. This job allowed her to use math and physics to tackle interesting real-world problems. When Joan's three children were born, she stopped working for a while to be at home with them. In 1961, she took a few graduate math courses. She became excited about what she was learning and decided to become a research mathematician. She set her own pace and did much of her work at home so that she could spend time with her family.

In 1968, Joan received her Ph. D. from the Courant Institute of Mathematical Sciences at the age of forty-one. Joan became a professor in a university and began to collaborate with other mathematicians. This work led her to think of exciting research questions to explore on her own, resulting in numerous publications and her seminal book.

Joan is an outstanding mentor and has supervised the Ph. D. dissertations of fifteen students. She has received many awards in recognition of her innovative research and well-written expository articles. Although she is now 79 years old, Joan continues to teach courses, give invited talks at conferences around the world, and publish papers with new ideas.

Joan's work has led to the development of applications by biologists studying knotted configurations of DNA. "Biologists really had a problem knowing if the coiled strands of DNA they observed were knotted or not, or if two knotted pieces of DNA were the same, so they came to mathematicians and we said, 'Oh, we've been working on that for years.' I'm pleased when my work is useful, but I'm more pleased by the beauty of it. It's a bit like art, and art is not necessarily useful."

The information on this page comes from

- *In Her Own Words: Joan S. Birman, Columbia University*
  http://www.awm-math.org/articles/notices/199107/six/node1.html
- *The Poster Project Biographies: Joan Birman*
  http://www.math.sunysb.edu/posterproject/www/biographies/birman.html
- *Profiles of Women in Mathematics: Joan S. Birman – Studying Links via Braids*
  http://www.awm-math.org/noetherbrochure/Birman87.html

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