Interior Angles In Regular Polygons

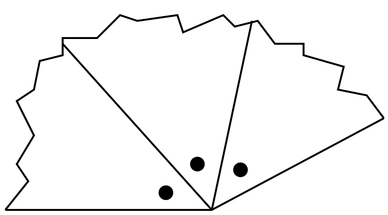
A *regular* polygon is a polygon which is as symmetrical as possible – having equal lengths and equal angles. What is the measure of each angle of a regular octagon, nonagon, or dodecagon? Let's find out!

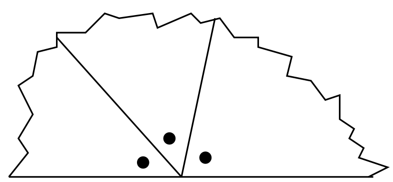
Kinds of Triangles

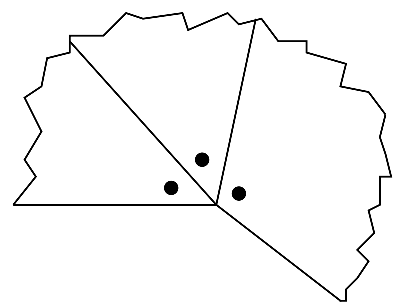
There are many terms that we use to describe triangles. Sometimes we classify triangles based on the size of the largest angle. So we have obtuse, right, and acute triangles. Sometimes we classify triangles based on the number of matching side lengths. So we have equilateral, isosceles, and scalene triangles. A given triangle can be described using one of each of these words. Is every combination possible? Can you create a triangle with every combination? Can you create a right isosceles triangle? What about an obtuse equilateral triangle? Is it possible to build a right scalene triangle?

Angles in Triangles

Use paper to create many different types of triangles. Be sure to draw and cut them very carefully so that the edges are straight. Put a dot near each vertex (corner). If you tear off the three angles and place them together, the angles might make one of the three pictures below.







What kind of triangles do you think will have angles that fit together like the first picture? Which triangles might look more like the middle picture? Which triangles will look more like the last picture?

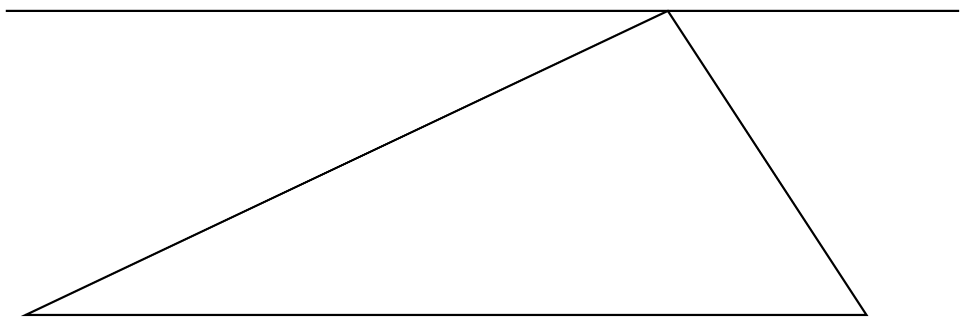
Try it and find out! Be very careful to use only triangles with very straight edges and place the angles together so that the vertices all meet at one point and the edges meet.

What did you find?

A complete circle around a point is an angle of 360°. How many degrees are in half a circle? Did the total degrees in the triangles you tested add up to more than half a circle, less than half a circle, or exactly equal to half a circle? Based on these examples what do you conjecture is true about the sum of the angles in any triangle?

The Sum of Angles in a Triangle

Consider the triangle below. The line on top is parallel to the base of the triangle. Can you see any angles that are congruent to one another? How do you know for sure that these angles must be congruent?



Can you use this diagram to prove your conjecture about the sum of angles in the triangle? Will this proof work for any triangle? What if the picture was drawn differently?

From Triangles to Polygons

Draw any quadrilateral. Use one line to divide it into triangles. Based on what you know about the sum of the angles of each triangle, what can you say about the sum of the angles in any quadrilateral? A *regular* quadrilateral (the quadrilateral with greatest symmetry) is also known by what name? How many degrees are in each angle of this shape?

Draw any pentagon. Draw just enough lines to divide it into the smallest number of triangles possible. Based on your drawing, what can you say about the sum of the angles in any pentagon? What is the measure of each angle of a regular pentagon?

Use the chart on the next page to find the minimal number of triangles, the sum of the angles, and the measure of each angle of different polygons. The last line will allow you to generalize to find the formula for the internal angles in any regular polygon.

Folding Parallelograms

Follow the first line of instructions for making Robert Neale's Magic Pinwheel. When you open the paper, use a pencil to trace all of the folded lines. Label every angle that you see with the correct number of degrees. Robert Neale designed his Magic Pinwheel so that the largest angle of the parallelogram forms the internal angle of a regular polygon. Based on the chart you completed above, do you see why the pinwheel requires eight units?

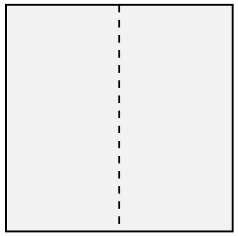
Use the directions to fold eight parallelogram units and assemble them to create a magic pinwheel.

If you wanted to design a magic pinwheel made from six units, how many degrees would the largest angle need to have? What if you wanted to design a magic pinwheel made from twelve units?

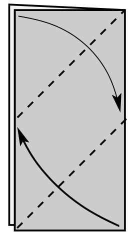
Degrees in Polygons

Angles in polygon	**Triangles in polygon**	**Sum of polygon angles**	**Each angle of regular polygon**
3			
4			
5			
6			
7			
8			
9			
10			
n			

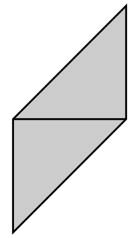
Robert Neale's Magic Pinwheel



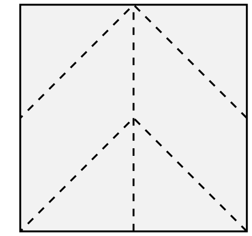
Fold paper in half



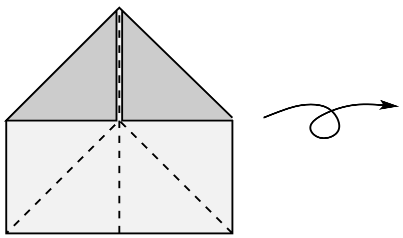
Bring one corner to opposite edge to form a triangle and repeat with opposite corner



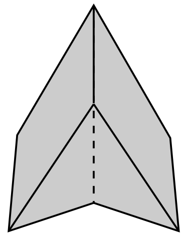
You should see a parallelogram



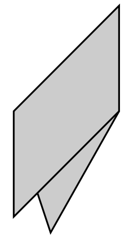
Open the paper



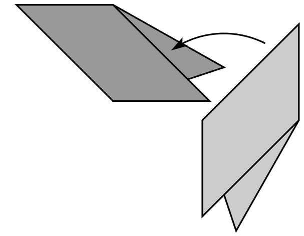
Fold top two corners to the center and flip the paper over



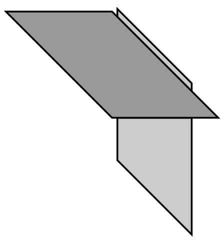
Push the large triangular region inside



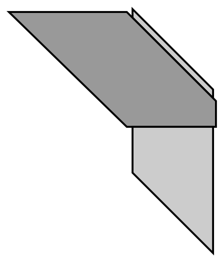
Flatten the parallelogram to complete the module.
Make seven more modules.



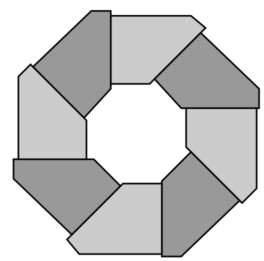
Insert one piece into another so that the folded edge of each module is on the outside



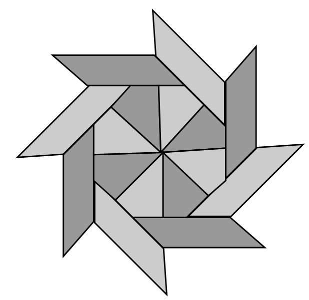
Be sure to tuck the inside piece as far into the fold as possible



Tuck the corners of the outside module into the groove of the inside module as snuggly as possible



Continue to add modules around the octagon. Tuck the corners of the last module on either side of the parallelogram inside the first module.



Push gently to transform into a pinwheel. If necessary, sharpen the creases and slide it in and out a few times.