Testing Your Marbles

You are in charge of quality control for the Magnificent Marble Menagerie. The MMM produces the finest marbles in the world. Not only are they visually stunning, but they are also break resistant. They are also very expensive! On each package is a certificate stating how break resistant each package of marbles is. You are in charge of determining the breaking point for each marble. To do this you randomly select a marble from a batch, take it to a multistory to, climb to a floor and drop the marble out of a window. You do this repeatedly until the marble breaks. If it survives on being dropped from floor F but breaks on being dropped from any floor above F, the you declare the marble to be of "Strength F." As everyone knows, marbles are very consistent. This means

a. If a marble survives a drop from floor F, then it will survive any subsequent drop from floor F or from any floor below F.

b. If a marble breaks on being dropped from a floor, then it will break if dropped from any higher floor.

You have an 18 story building for your tests. (Later we will consider higher buildings. If a marble survives a drop from floor 18, then the marbles in that batch are certified to be of Strength 18 (which for now is the highest rating.)

1. Marbles are expensive, so you want to use only one marble to determine the strength of your batch. Describe in detail the process you would use to determine the strength of the batch. With your procedure how long might it take to determine the strength of the batch of marbles?
2. Making a lot of drops is expensive too. You have to repeatedly climb stairs, readjust equipment, etc. Suppose you are willing to sacrifice two marbles to determine the strength of your batch. What is a good strategy? With your strategy what is most number of drops you may have to do? Is your strategy the best, that is, is there another strategy that will determine the strength of any marble in a smaller number of drops?

A problem of the type described in Problem 2 is called a "mini-max" problem. In the marble scenario it means that for a given strategy, you are concerned with the maximum number of drops you might have to make to determine the strength of a marble. *Then* you are concerned with finding a drop strategy that makes this maximum as small as possible.

3. You would like to certify your marbles relative to a taller building. Suppose that you decide you can sacrifice two marbles and are willing to make up to six drops (but no more) to determine the strength of the marble. What is the tallest building (e.g. most floors) that you can handle with these restrictions?
4. Suppose you are willing to sacrifice two marbles and make d drops. What is the tallest building that you can handle?

Testing Your Marbles—Notes.

This project is based on the article *The Tower and Glass Marbles Problem* by Richard Denman, David Halley, and Michael Rothenberg that appeared in the November 2010 issue of *The College Mathematics Journal.* To date I have only used this activity in one circle with a group of sixth through twelfth graders.

Problem 1. was the warm-up problem, thought it is admittedly trivial. I gave this after describing the marble testing scenario but did not give them assumptions a. and b. After this activity I asked for possible further questions or for things we should be concerned about with this problem. I got responses like"air resistance," "do previous drops weaken a marble before it breaks?" and "are all marbles the same?" These were not the types of questions I was expecting, but it gave me a chance to talk about mathematical modeling and the need to make decisions about what to assume and what to ignore. Further discussions with students led to assumptions a. and b.

I gave students a lot of time in working on Problem 2. Progress was slow at first but I kept at them to think broadly and creatively. Finally one group hit on the idea of dropping the first marble from floors 2, 4, 6,... until it broke, then quickly considered the more general idea of dropping the first marble from floors d, $2d$, $3d\ldots$ until it broke. We spent some time discussing the optimal d for this approach. For some buildings the situation can get more interesting with the optimal drop strategy resulting from varying the distance to the next level for the first ball. You might have students work on the 15 floor case as an example, though questions 3. and 4. get at this from another direction.

We did some work on the three marble case but had time for very little discussion. I think some students understood the iterative nature of moving from the two to the three marble case.