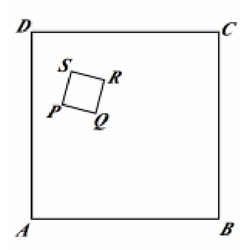
1 Find values of a, b, c, and d so that for the following functions there is a number y such that $f(y) = g(y) = h(y)$:

$$f(x) = a^x \quad g(x) = bx + c \quad h(x) = \log_d(x)$$

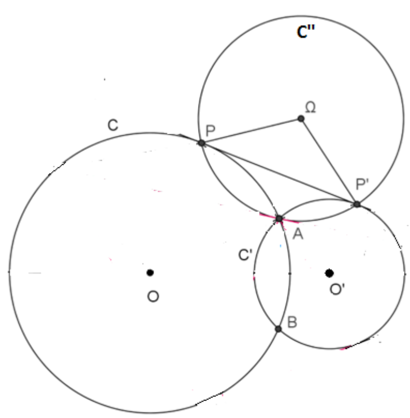
2 Square $PQRS$ lies inside square $ABCD$ as shown. Points S, P, and A are collinear, with $m\angle SAD = 15^\circ$ and $m\angle ASD = 120^\circ$. Let points X, Y, and Z be, respectively, the midpoints of segments $\overline{DS}, \overline{CR}, \overline{BQ}$. If $XY = 2016$, find YZ.



3 The 13-digit base-10 number $16926Z8244483$ is the product of a pair of twin primes. The sixth digit of this number is represented by the letter Z. Find Z.

4 Determine, without a calculator, the rightmost four digits of 7^{2016}.

5 In the diagram below the circles C and C' with respective centers O and O' intersect at the point A. The line segment PP' is part of the common tangent to C and C'. The circle C'' with center Ω circumscribes $\triangle APP'$. Show that $\angle P\Omega P'$ equals the angle between the two circles C and C'. *Here we define the angle between the two circles at a point of intersection as the angle between their respective tangents at this same point.*



6 Take a strip of paper and fold the right end so that it lies on the left end. Then when the paper is unfolded, there will be a single "downward" facing crease which we will indicate with the symbol "V." Next, imagine performing two right-over-left folds, starting with a fresh strip of paper: first fold the right end over the left, and then, without unfolding, again fold the right end over the left. When the strip is unfolded, reading from left to right, you will see, in order, two "V" folds and then one "upward" folding crease, denoted by "A."

Now imagine performing this left-over-right folding 2016 times, and then unfolding the strip of paper. Reading from left to right, what is the 2016th symbol? Explain your answer since there is no credit for a lucky guess!

7 Let $N = 2016^{2016}$, and let S be the set of consecutive integers from 1 to N. In how many different ways can three consecutive integers be removed from S so that the average of the remaining numbers is an integer?

8 3 bugs named Ark, Bark, and Spark (these rhyme) are located on 3 disjoint circles of respective radii 1,2, and 3. The circles are centered on the x-axis and at $t = 0$ each bug is on the rightmost point of the x-axis on their circle. The bugs all travel counter-clockwise according to the following rules:

- When traveling above the x-axis the bugs move at π units per second.
- When traveling below the x-axis the bugs move at 2π units per second.
- The changes in speed are instantaneous when each bug crosses the x-axis.

Besides at $t = 0$, what is the smallest t when all the bugs are again at their same respective locations on these circles? At this time how many complete laps has each bug made on their circle?