Modular Arithmetic

Background

In this activity, you will explore a strange type of arithmetic that results from arranging numbers in a circle instead of on a number line. On some of our circles, $4 + 5$ is not equal to 9!

Modular arithmetic is useful in situations where something repeats in a cyclic fashion. Time on a clock is one example of this. You use modular arithmetic when you try to figure out what time is 4 hours after 9 AM. In computer programming the "mod" operation is used for programs that need to assign values cyclicly.

Modular arithmetic is one of the first example systems that math majors encounter in abstract algebra. Algebraists study underlying patterns to determine when two systems that look different on the surface really have the same structure. Examples of systems that algebraists study are arithmetic of real numbers, modular arithmetic, arithmetic of string braids, motions, and operations based on symmetries of physical objects.

Emmy Noether was a mathematician who made important contributions to abstract algebra and modern physics. Instead of thinking in terms of long and complicated computations, Emmy looked for simple underlying patterns. This enabled her to create new and more powerful ways of thinking about difficult problems in mathematics and physics.

Moduli and Stepping Numbers

- Choose a circle with the numbers zero up to some number.
- How many numbers are in your circle? (Remember to count the zero.) This number is called the "modulus". (The plural of "modulus" is "moduli".)
- Choose a magic stepping number. This is the number of steps you will take each time.
- Find the space in the chart on the back of this page corresponding to the modulus and the stepping number you chose. Put a big circle in that box so you remember which spot you are working on.
- Start on the zero.
- Start stepping in the direction of the 1 on the circle. Go as many steps as are in your magic stepping number. Call out the final number you land on (but not the numbers in between).
- Keep going like this until you see a pattern in the numbers you are calling out.
- Did you call out all the numbers? If the answer is "yes" then write that word in the chart where your circle is. If you did not land on all the numbers, then write the word "no".

Modulus

Stepping Number	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

Modular Counting

Choose a circle with 5 numbers (0 through 4).

- Start on the 0. If you take 5 steps, where do you land?
- Start on the 0. If you take 6 steps, where do you land?
- Start on the 0. If you take 9 steps, where do you land?
- Start on the 0. If you take 13 steps, where do you land?
- Start on the 0. If you take 15 steps, where will you land?
- Start on the 0. If you take 100 steps, where will you land? Why?
- Start on the 0. If you take 147 steps, where will you land? Why?
- Start on the 0. If you take 284 steps, where will you land? Why?

Now choose a circle with 7 numbers (0 through 6).

- Start on the 0. If you take 15 steps, where will you land?
- Start on the 0. If you take 482 steps, where will you land? Why?

Suppose that you are working on a circle with modulus m (m is the number of numbers in the circle) and you take k steps starting from zero. What procedure can you use to figure out where you will land?

Modular Arithmetic

Addition

Find the circle with 5 numbers (0 through 4).

$2 + 2 \equiv_5$ _____

$3 + 4 \equiv_5$ _____

Subtraction

Find the circle with 7 numbers (0 through 6).

$6 - 4 \equiv_7$ _____

$3 - 6 \equiv_7$ _____

Multiplication

Find the circle with 10 numbers (0 through 9).

$2 \times 4 \equiv_{10}$ _____

$5 \times 3 \equiv_{10}$ _____

Division

Find the circle with 8 numbers (0 through 7).

$6 \div 2 \equiv_8$ _____

$5 \div 3 \equiv_8$ _____

Modular division is not as straightforward as other arithmetic operations. Sometimes there is no answer to a modular division problem. Find an example of a modular division problem with no solution.

More Practice With Modular Counting

Use a circle with 3 numbers (0 through 2).

- Start on the 0. If you take 5 steps, where do you land?
- Start on the 0. If you take 6 steps, where do you land?
- Start on the 0. If you take 9 steps, where do you land?
- Start on the 0. If you take 13 steps, where do you land?
- Start on the 0. If you take 15 steps, where will you land?
- Start on the 0. If you take 100 steps, where will you land? Why?
- Start on the 0. If you take 147 steps, where will you land? Why?
- Start on the 0. If you take 284 steps, where will you land? Why?

Now use a circle with 9 numbers (0 through 8).

- Start on the 0. If you take 15 steps, where will you land?
- Start on the 0. If you take 482 steps, where will you land? Why?

Suppose that you are working on a circle with modulus m (m is the number of numbers in the circle) and you take k steps starting from zero. What procedure can you use to figure out where you will land?

Modular Arithmetic

Addition

Use a circle with 6 numbers (0 through 5).

$3 + 2 \equiv_6$ _____

$5 + 12 \equiv_6$ _____

Subtraction

Use a circle with 9 numbers (0 through 8).

$8 - 10 \equiv_9$ _____

$1 - 6 \equiv_9$ _____

Multiplication

Use a circle with 4 numbers (0 through 3).

$4 \times 5 \equiv_4$ _____

$7 \times 3 \equiv_4$ _____

Division

Use a circle with 7 numbers (0 through 6).

$6 \div 4 \equiv_7$ _____

$2 \div 5 \equiv_7$ _____

Modular division is not as straightforward as other arithmetic operations. Sometimes there is no answer to a modular division problem. Find an example of a modular division problem with no solution.