Mathematical Explorations of Musical Scales

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Background on the IEMTC

- K - 12 teachers with varying backgrounds but have an interest in doing mathematics
- Average of 16 participants per session
- Teachers come from many different districts (as far as 60 miles)
- Sessions are primarily driven by teachers with minimal instruction from facilitators
- Goal is to introduce problems with accessible entry point and room to expand to a deeper level of thinking

What do you notice about a piano keyboard?

Model of Piano Keyboard

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Model of Piano Keyboard
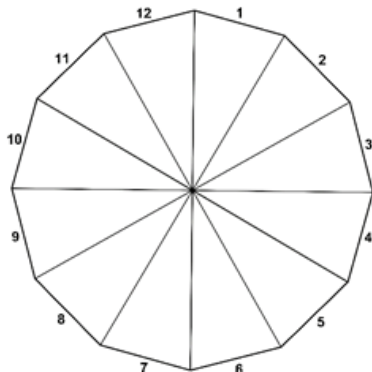
How many keys are in a single period?

Model of Piano Keyboard

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Model of Piano Keyboard

To form the model, bend a single period into 12-gon where adjacent wedges represent one half-step

Mathematical Definition of Musical Scale

Participants constructed a mathematical definition of musical scale using the model:

A musical scale is a shading of wedges in the model so that given any two neighboring wedges, at least one wedge must be shaded
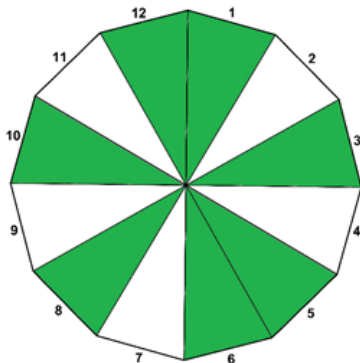

Figure: Major Scale: WWHWWWH

When are two musical scales equivalent and when might they be considered different?

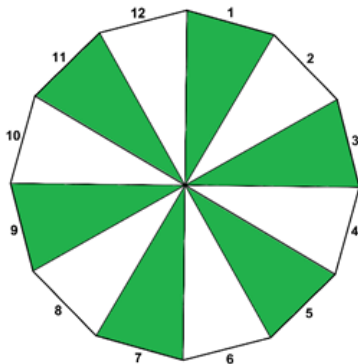Figure: Whole -Tone Scale: WWWWWW

When are two musical scales equivalent and when might they be considered different?
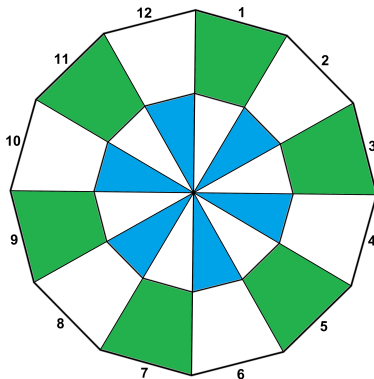

Figure: Whole -Tone Scale: WWWWWW

Creating Musical Scales

Use the definition to create your own musical scale. How many different types of scales exist?
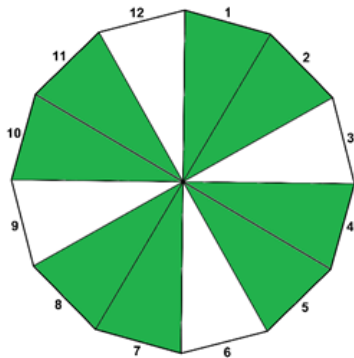

Figure: Diminished Scale: HWHWHWHW

- Is it possible to create a musical scale having exactly 4, 5, 6, 7, or 8 different types?

- How many different musical scales can our model produce?

- Explore symmetries when altering definition of musical scale to include three-halves step. In other words, shade the wedges so that given any three consecutive wedges, at least one wedge must be shaded.

Thank You
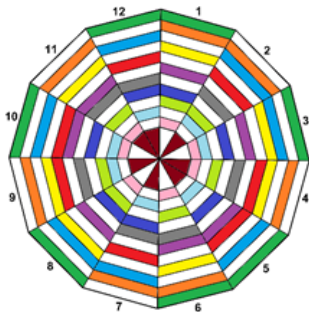
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