Foldable Numbers

led by Thomas Clark

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What Numbers can be Folded?

1. You have a strip of paper and you are allowed two "moves:" fold from the left side leaving a crease and fold from the right side leaving a crease. The left side of the paper is 0 and the right side is 1. Since you don't have a crease to begin with, fold the paper in half, leaving a crease at 1/2.

2. Since we can fold the paper to get "1/2" lets call it a *foldable number*.

3. What other numbers can you fold? What set of folds gets you to that number?

4. Do you notice any patterns? If you had a certain number, could you figure out how to fold it?

Function, Function, What's Your Function?

1. Suppose your crease is currently at a number x between 0 and 1. If you do a left fold, what number is your new crease at? Call it $l(x)$ and write a formula for it.

2. Suppose your crease is currently at a number x between 0 and 1. If you do a right fold, what number is your new crease at? Call it $r(x)$ and write a formula for it.

3. If you start with a "crease" at 0 and do a right fold or a "crease" at 1 and do a left fold, do you get to 1/2 with your formulas?

4. Use function composition and a calculator or Desmos to check your answers on the front page.

5. Find the connection between questions 1. and 2. How does this fit with what you did on the first page?

Binary Decimals or "Binicimals?" or "Binimals?"

A decimal like 0.123 means $\dfrac{1}{10} + \dfrac{2}{100} + \dfrac{3}{1000} = \dfrac{1}{10^1} + \dfrac{2}{10^2} + \dfrac{3}{10^3}$.

1. What would a binimal look like? What would it mean?

2. What is the value of the binimal 0.101?

3. Write the binimal for $5/16$.

4. What is the binimal for $4/7$ or $2/3$ or $\pi/6$?

5. What does this have to do with folding?

6. What does this have to do with functions on the last page. What would l and r look like if you wrote them as functions of binimals?

Fixed Points

A *fixed point* for a function f is a number x such that $f(x) = x$.

1. Are there any fixed points for $l(x)$? (What does this mean "on the paper"?)

2. Are there any fixed points for $r(x)$? (What does this mean "on the paper"?)

3. Are there any fixed points for $l(r(x))$? (What does this mean "on the paper"?)

4. Are there any fixed points for $r(l(x))$? (What does this mean "on the paper"?)

5. Suppose $f(x)$ is some composition of some number of applications of l and r. What fixed points are possible?

Fixed Points and Binimal Representations

1. Use the fixed points/patterns/binary versions of r and l to find the binimal representation of 1/3 and 2/3 and 4/7 and 11/15.

2. Let's call a number *infinitely foldable* if we can describe a set of folds that if repeated infinitely would fold the number. What numbers are infinitely foldable? What does this have to do with the fixed points of the function compositions?

3. What about all the other numbers? Can we get close to them? How close? What does that even mean?

Classroom Connections

1. Are there any connections between the math we've done tonight and what you teach in your classroom?

2. Is there something we did tonight you would incorporate into one of your lessons?

3. How would you have improved the activity we did tonight?

4. What standards from the CCSS connect to what we did tonight? (I thought about making a list but so many standards talk about rational numbers and decimals, they wouldn't all fit!) Which ones from your grade level apply?