PRIMARY SOURCE PROJECT IN A MATH TEACHER CIRCLE: PYTHAGOREAN TRIPLES: CONNECTIONS BETWEEN ALGEBRA AND GEOMETRY

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MATH**FEST** - DENVER

GOALS FOR MATH TEACHER CIRCLE

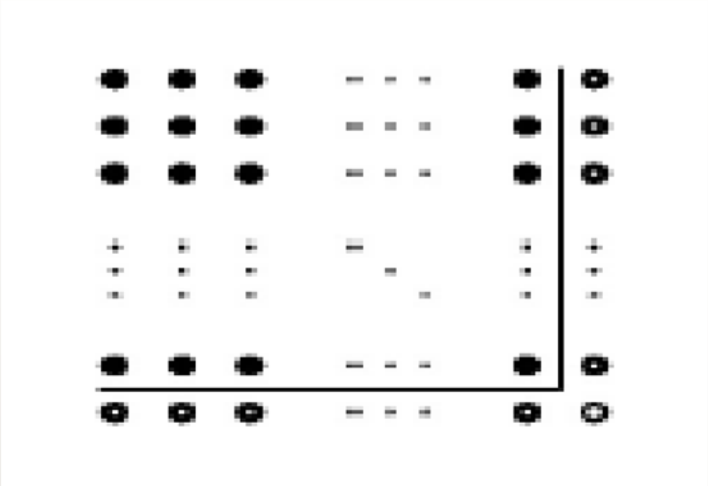
- Inquiry-based, collaborative activity
- Mathematics as sensemaking
- Modeling of Effective Teaching Practices
- Connect the teachers' learning experience to teaching

HISTORICAL PROJECT - (TRIUMPHS)

- Pythagorean Theorem – The relationship was known before Pythagoras (~500 BCE)
- Pythagorean Triples – Three whole numbers x,y,z that satisfy $x^2+y^2=z^2$
- Explore two methods to generate Pythagorean Triples geometrically; one from Pythagoras and one from Plato and then analyze the methods algebraically

PLIMPTON 322 (~1900 BCE)

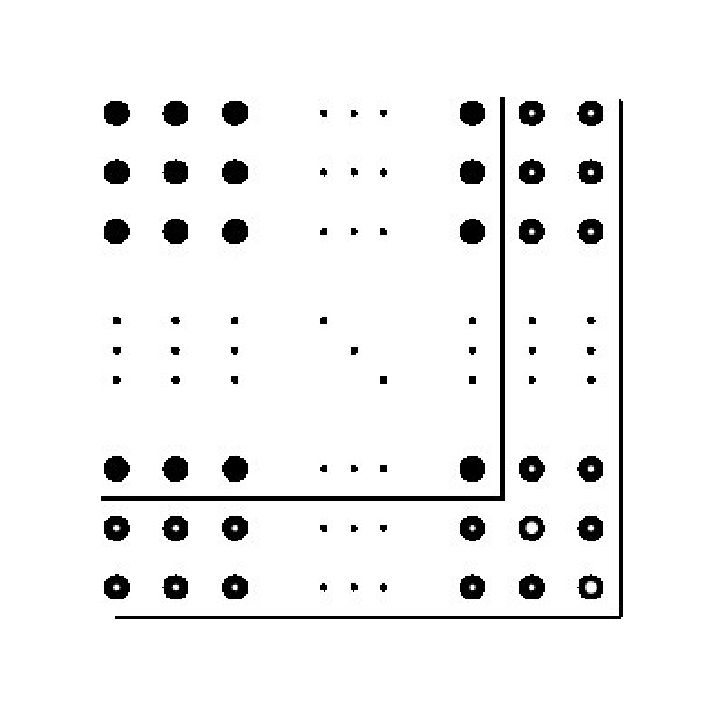
PYTHAGORAS' METHOD – SINGLE GNOMOM



PROCLUS' *COMMENTARY*

- The method of Pythagoras begins with odd numbers, positing a given odd number as being the lesser of the two sides containing the angle, taking its square, subtracting one from it, and positing half of the remainder as the greater of the sides about the right angle; then adding one to this, it gets the remaining side, the one subtending the angle. For example, it takes three, squares it, subtracts one from nine, takes the half of eight, namely, four, then adds one to this and gets five; and thus is found the right-angled triangle with sides of three, four, and five.

PLATO'S METHOD – DOUBLE GNOMON



PROCLUS' *COMMENTARY*

- The Platonic method proceeds from even numbers. It takes a given even number as one of the sides about the right angle, divides it in two and squares the half, then by adding one to the square gets the subtending side, and by subtracting one from the square gets the other side about the right angle. For example, it takes four, halves it and squares the half, namely, two, getting four; then subtracting one it gets three and adding one gets five, and thus it has constructed the same triangle that was reached by the other method. For the square of this number is equal to the square of three and the square of four taken together.

EXTENSION QUESTIONS

- What are primitive Pythagorean Triples?
- Do these two methods generate all Pythagorean Triples?
- If not, what are other Pythagorean Triples and how could they be generated?
- How many Pythagorean Triples are there?
- Which numbers are in more than one Pythagorean Triple and what is the greatest number of Pythagorean triples a number can be a part of?

VALUE OF THE HISTORICAL PROJECT

- High-level task
- Narrative
- Human-based
- Algebra-geometry connected

STANDARDS OF MATH PRACTICE

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics**

STANDARDS OF MATH PRACTICE

- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning

RESULTS

- Increased Mathematical Knowledge of Teaching (MKT) through connections between algebra and geometry
- Connections to the Standards of Mathematical Practice
- Connections to the Teaching Practices in *Principles to Action*

2.1 Shifts in Classroom Practice Self-Assessment

Instructions: Place an *X* along each continuum that best represents your classroom practice.

Shift 1: From *stating-a-standard* toward *communicating expectations for learning*

Teacher shares broad performance goals and/or those provided in standards or curriculum documents.	→	Teacher creates lesson-specific learning goals and communicates these goals at critical times within the lesson to ensure students understand the lesson's purpose and what is expected of them.

Shift 2: From *routine tasks* toward *reasoning tasks*

Teacher uses tasks involving recall of previously learned facts, rules, or definitions and provides students with specific strategies to follow.	→	Teacher uses tasks that lend themselves to multiple representations, strategies, or pathways encouraging student explanation (how) and justification (why/when) of solution strategies.

Shift 3: From *teaching about representations* toward *teaching through representations*

Teacher shows students how to create a representation (e.g., a graph or picture).	→	Teacher uses lesson goals to determine whether to highlight particular representations or to have students select a representation; in both cases, teacher provides opportunities for students to compare different representations and how they connect to key mathematical concepts.

Shift 4: From *show-and-tell* toward *share-and-compare*

Teacher has students share their answers.	→	Teacher creates a dynamic forum where students share, listen, honor, and critique each other's ideas to clarify and deepen mathematical understandings and language; teacher strategically invites participation in ways that facilitate mathematical connections.

Shift 5: From *questions that seek expected answers* toward *questions that illuminate and deepen student understanding*

Teacher poses closed and/or low-level questions, confirms correctness of responses, and provides little or no opportunity for students to explain their thinking.	→	Teacher poses questions that advance student thinking, deepen students' understanding, make the mathematics more visible, provide insights into student reasoning, and promote meaningful reflection.

Shift 6: From *teaching so that students replicate procedures* toward *teaching so that students select efficient strategies*

Teacher approaches facts and procedures with the goal of speed and accuracy.	→	Teacher provides time for students to engage with mathematical problems, developing flexibility by encouraging student selection and use of efficient strategies; teacher provides opportunities for students to evaluate when a strategy is best suited for the problem at hand.

Shift 7: From *mathematics-made-easy* toward *mathematics-takes-time*

Teacher presents mathematics in small chunks so that students reach solutions quickly.	→	Teacher questions, encourages, provides time, and explicitly states the value of grappling with mathematical tasks, making multiple attempts, and learning from mistakes.

Shift 8: From *looking at correct answers* toward *looking for students' thinking*

Teacher attends to whether an answer or procedure is (or is not) correct.	→	Teacher identifies specific strategies or representations that are important to notice; strategically uses observations, student responses to questions, and written work to determine what students understand; and uses these data to inform in-the-moment discourse and future lessons.

SHIFTS IN CLASSROOM PRACTICE