Path Counting

for Math Circles

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Math Circle Styles

Modes of student involvement

- **Good:** Interested (math talk format)

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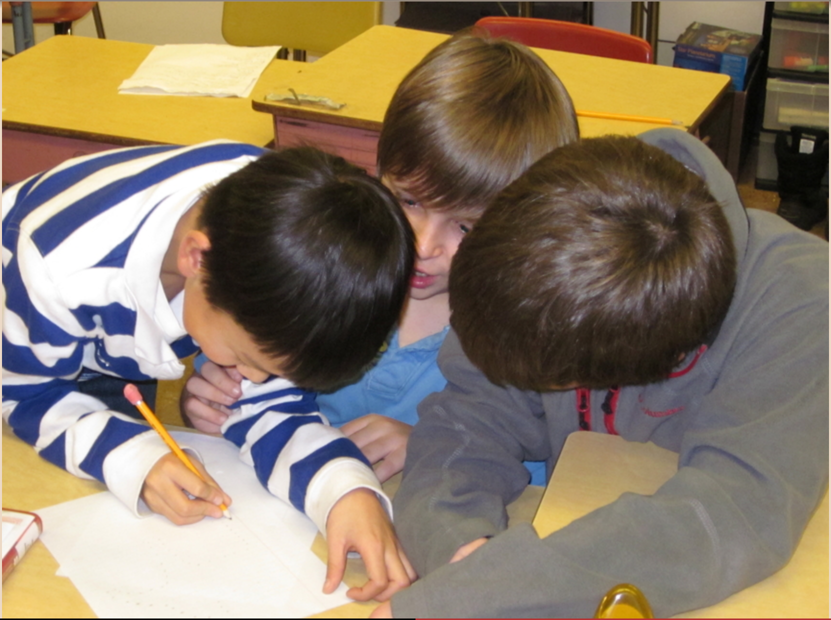

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Modes of student involvement

- **Good:** Interested (math talk format)
- **Better:** Engaged (math class format)
- **Best:** Engrossed (math circle format)
- **Not good:** Out of control

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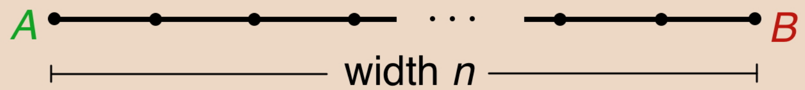
Modes of student involvement

- **Good:** Interested (math talk format)
- **Better:** Engaged (math class format)
- **Best:** Engrossed (math circle format)
- **Not good:** Out of control
- **Very bad:** Bored stiff

Math Circle Styles

The Prototype

How many paths are there from A to B?



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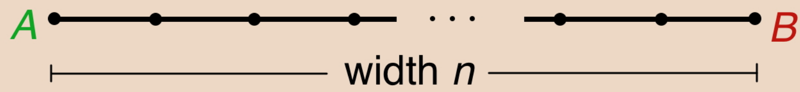
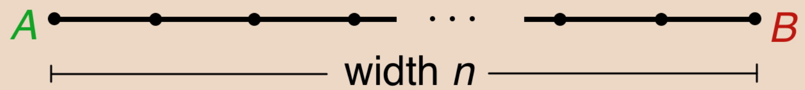


To lay some ground rules, we'll say that a valid route must move from one point to another through the network along the edges pictured, beginning at A and ending at B.

Everything else is up for grabs.

The Prototype

So, how many paths are there from A to B?

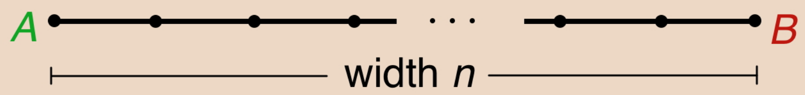


Possible answers:

- 1 (move from left to right along edges)
- 2^{n-1} (use each edge exactly three times)
- F_{2n+1} (use each edge up to three times)

Embellishment

For the time being let's agree to always move from left to right. How might we add more edges to create new counting problems?



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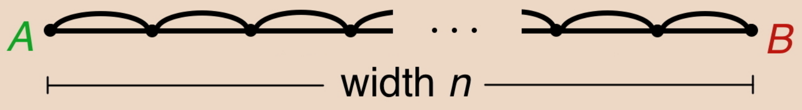

One simple modification is to double each edge. Now how many paths from A to B?

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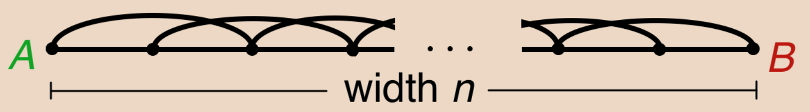
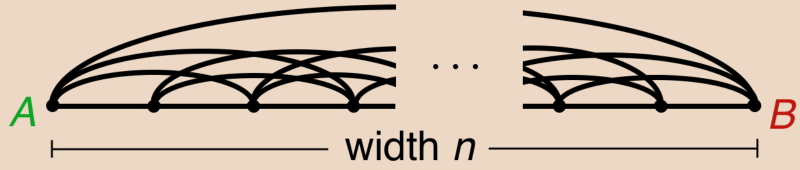


Or we could make the extra edges longer. Now how many paths from A to B?

Embellishment

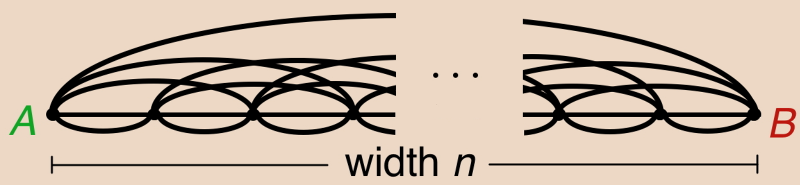
One could easily get carried away, including more and more edges.



Now how many paths from A to B?

Embellishment

For our final trick of the afternoon, we'll incorporate a previous idea by doubling all of the shortest edges as well.

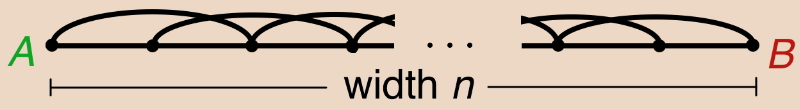


Now how many paths from A to B?

Embellishment

There is a beautiful explanation of this fact.

Recall the network giving Fibonacci numbers:



Embellishment

There is a beautiful explanation of this fact. Recall the network giving Fibonacci numbers:

Focus on every other vertex. (Assuming that n is even.) How could we create a network equivalent to this one, using only the circled points as vertices in our network?

Your Turn

If all has gone well, kids will be ready to create their own path counting problems.

- Encourage creativity, not complexity.
- Work individually, then in small groups.
- Create a gallery of favorite networks.
- Issue a challenge with prizes for solutions.
- Provide a concluding activity as a group.

Wrapping Up

Create a network (rules can vary) having the following number of paths through them. The network with the fewest edges wins.

$90 \qquad 120 \qquad 129$

Query: can one prove an upper bound for N if all edges are traversed left to right?

Thanks for listening!