Path Counting
for Math Circles

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Math Circle Styles

Modes of student involvement

- **Good:** Interested (math talk format)
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Path Counting
Modes of student involvement

- **Good:** Interested (math talk format)
- **Better:** Engaged (math class format)
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Modes of student involvement

- **Good:** Interested (math talk format)
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- **Best:** Engrossed (math circle format)
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- **Good:** Interested (math talk format)
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- **Not good:** Out of control
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Modes of student involvement

- **Good:** Interested (math talk format)
- **Better:** Engaged (math class format)
- **Best:** Engrossed (math circle format)
- **Not good:** Out of control
- **Very bad:** Bored stiff
How many paths are there from $A$ to $B$?

To lay some ground rules, we'll say that a valid route must move from one point to another through the network along the edges pictured, beginning at $A$ and ending at $B$. Everything else is up for grabs.
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To lay some ground rules, we’ll say that a valid route must move from one point to another through the network along the edges pictured, beginning at $A$ and ending at $B$.

*Everything else is up for grabs.*
So, how many paths are there from \( A \) to \( B \)?

Possible answers:
- \( 1 \) (move from left to right along edges)
- \( 2^{n-1} \) (use each edge exactly three times)
- \( F_{2n+1} \) (use each edge up to three times)
Embellishment

For the time being let’s agree to always move from left to right. How might we add more edges to create new counting problems?

![Diagram showing a path from A to B with width n](image-url)

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Embellishment

For the time being let’s agree to always move from left to right. How might we add more edges to create new counting problems?

One simple modification is to double each edge. Now how many paths from $A$ to $B$?
Embellishment

For the time being let’s agree to always move from left to right. How might we add more edges to create new counting problems?

Or we could make the extra edges longer. Now how many paths from $A$ to $B$?
One could easily get carried away, including more and more edges.

Now how many paths from $A$ to $B$?
For our final trick of the afternoon, we’ll incorporate a previous idea by **doubling** all of the shortest edges as well.

Now how many paths from $A$ to $B$?
There is a beautiful explanation of this fact. Recall the network giving Fibonacci numbers:

![Network Diagram]
There is a beautiful explanation of this fact. Recall the network giving Fibonacci numbers:

Focus on every other vertex. (Assuming that $n$ is even.) How could we create a network equivalent to this one, using only the circled points as vertices in our network?
Your Turn

If all has gone well, kids will be ready to create their own path counting problems.

- Encourage **creativity**, not **complexity**.
- Work **individually**, then in **small groups**.
- Create a **gallery** of favorite networks.
- Issue a challenge with **prizes** for solutions.
- Provide a **concluding activity** as a group.
Create a network (rules can vary) having the following number of paths through them. The network with the fewest edges wins.

90  120  129

Query: can one prove an upper bound for \( N \) if all edges are traversed left to right?

Thanks for listening!