From 5th to 12th: Discoveries and Challenges of Multi-leveled Math Circles

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Goals of Talk

• Prepare organizers for challenges with expanding their Math Circle to other age groups

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- Ideas to resolve existing issues with Math Circles
- Provide an idea for a new Math Circle activity

History of the TAMU Math Circle

- Started in 2011 as an after-school club
- Moved to Texas A&M in 2012 and became the TAMU Math Circle
- Open to all local schools
- Initially for grades 5-8, was expanded to grades 9-12 in Fall 2014



Structure of TAMU Math Circle

- Runs Saturdays, 3-5pm
- Starts with 30 minute unstructured activity, followed by a 90-minute structured activity
- Kids are divided into 3 groups:
 - Beginner: In Pre-algebra or below
 - Intermediate: In Algebra I or above
 - Advanced: In Algebra II or above



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- Optional 30 minute competition

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- Coming up with activities for 3 groups
 - Following a curriculum: Anna Burago, "Mathematical Circle Diaries, Year 1: Complete Curriculum for Grades 5 to 7"
 - Creating activities that can be modified for all three groups

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Art Gallery Problem: For any art gallery with *n* corners, how many guards should you hire to ensure that it is guarded?

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Art Gallery Problem: For any art gallery with *n* corners, how many guards should you hire to ensure that it is guarded? Part of the solution involves vertex-coloring for graphs

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 - Limit these tasks
 - Ensure enough time/volunteers to help students

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- Behavior issues
 - Volunteers hover near unruly kids
 - Disruptive kids go last for snack time

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• Stick with concrete problem: How many guards should you hire for a 20-cornered gallery to ensure that it is guarded?

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Interesting Discovery: The youngest group came up with the worst-case scenario galleries the fastest.



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Intermediate Group Challenges and Solutions

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Intermediate Group Challenges and Solutions

- Many of them have been to math camps/attended Math Circle before, so they may have seen activities before
 - Send a volunteer to quietly discuss the problem with the student
 - Plan challenge problems
 - Task them with helping other students
 - When discussing well-known topics (Pi, Fibonacci numbers), start activity with invitation to share facts

Keep it similar to the beginner group, with the following additions:

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Keep it similar to the beginner group, with the following additions:

- Discuss the concepts of necessary and sufficient
- How many guards should you hire for a 200-cornered gallery to ensure that it is guarded?

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• Would 200 guards work? 100 guards? 66 guards?

Advanced Group Challenges and Solutions

Advanced Group Challenges and Solutions

- High-school aged students have extremely busy schedules
 - Be aware of common time conflicts: Concerts, sports events, academic competitions
 - Create stand-alone activities
 - Ask them for feedback on activities and be willing to adapt to their preferences

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- Why can you 3-color the graph made from the triangulated polygon?
- What goes wrong when you add holes to the polygon?

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• Why is the brute force method for placing guards not feasible?

- Why can you 3-color the graph made from the triangulated polygon?
- What goes wrong when you add holes to the polygon?
- How does the problem change when you restrict to right-angled walls?

Other activities suitable to all three groups

- Catalan Numbers
- Hyperbolic Soccerball
- Euler Characteristic: Polyhedron, Sphere, Torus



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Acknowledgements

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The TAMU Math Circle is partially funded by the Dolciani Mathematics Enrichment Grant.

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If you'd like more information about the Art Gallery Problem activity, check out my website:

http://www.math.tamu.edu/~kaitlyn/Materials/ArtGalleryProblem/

Thank you for listening!