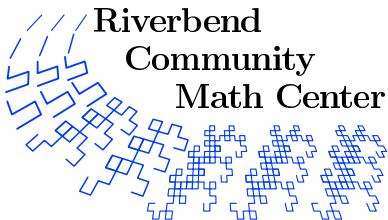


Queen Dido Problems

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Acknowledgements

- Hung Tran. “Math Circle Isoperimetric Problems”.
<https://www.math.uci.edu/mathcircle/materials/IsoperimetricL2.pdf>
- Alan Siegel. “A Historical Review of the Isoperimetric Theorem in 2-D, and its place in Elementary Plane Geometry.”
Based on Jakob Steiner’s proof of the isoperimetric problem.

Queen Dido

- Queen Dido was born in Tyre in 839 B.C.
- She was supposed to rule jointly with her brother Pygmalion.
- Dido was forced to flee from Tyre with her followers when her brother had her husband killed.
- They fled to northern Africa (modern-day Tunisia).
- Queen Dido persuaded King Iarbas to give her as much land as she could encompass within an ox's hide.
- She cut the hide into thin strips, stitched them together, and made a large loop.

Queen Dido's Problem

- Now Queen Dido wants to enclose as much land for her followers as possible using the loop of ox hide.

- How should she do this?

Queen Dido Triangle Problems

- Let's start with some simpler problems.
- First consider a triangle with given sides a and b . Queen Dido is free to choose the third side to be anything she likes to maximize the area. What length should she choose?
- For example, suppose that one side is length 3 and the other is length 4. Would it be best to have the third side be 1? 2? 6? 8? Some other value?
- Is it best to choose as large a number as possible for the third side?

Queen Dido Triangle Problems

- Suppose that Queen Dido uses her fixed length loop and the length of one side of the triangle she is making has been pre-determined.
- For example, suppose that the loop of string is 16 units long and one edge of the triangle must be 3 units long.
- Which triangle with these conditions would have the largest area?

Queen Dido Triangle Problems

- Suppose that Queen Dido may make any triangle out of her loop of fixed length. Which one would have maximal area?
- Suppose that Queen Dido has a string (not a loop) of length 1 and that there is a straight cliff or sea coast forming a natural boundary on one side. What should she do with the rest of the string to form a triangle with maximal area using the cliff as one edge?

Queen Dido Quadrilateral Problems

- Suppose that Queen Dido wants to make a quadrilateral with her fixed loop of string. Suppose that two edges of the quadrilateral and the angle between them have already been dictated, but she is free to decide what to do with the other two edges. Which choice will maximize the area?
- Suppose that two sides of the quadrilateral have been chosen, but Queen Dido may choose the angle between them and the other two sides. (Queen Dido still has a fixed loop of string.)
- Suppose that one edge of the quadrilateral has been chosen, but Queen Dido may decide the other three edges and the angles.
- What kind of quadrilateral maximizes the area given a fixed perimeter?

Return to Queen Dido's Original Problem

- Consider a hypothetical shape S and suppose that it has maximal area given a fixed perimeter L .
 - Can S be concave?
 - Can S have holes?

- Draw any line that divides L into two equal parts.
 - Can one “half” of S have greater area than the other?

Return to Queen Dido's Original Problem

- Draw a line dividing L into two equal parts, and call the points of intersection A and B .
- Choose another arbitrary point on the boundary of S and call it C .
- What must the value of $\angle ACB$ be if S has maximal area?

