#1 IRRATIONAL PROOF

A student gives you the following proof that $\sqrt{5}$ is irrational. Give a careful, full evaluation of this argument:
We use proof by contradiction. Assume that $\sqrt{5} = \frac{n}{k}$ *where* $\frac{n}{k}$ *is a fraction in lowest terms:*

$$\sqrt{5} = \frac{n}{k} \overset{()^2=()^2}{\Longrightarrow} 5 = \frac{n^2}{k^2} \Longrightarrow 5k^2 = n^2$$

which implies that k^2 *divides* n^2*. Thus* $n \cdot n = n^2$ *is divisible by* k *since the quotient is the integer* $5k$*. However,* k *and* n *have no factors in common so* k *and* n^2 *cannot have any factors in common either.*

#2 THE TAX MAN

The tax man is a game played solo against the fictitious tax man. You start with the numbers $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. You take turns picking valid numbers from this list and removing them. On each turn the tax man gets all divisors of the number you chose that still remain. You cannot choose any number that doesn't have at least one divisor still remaining. When no moves are possible, all remaining numbers go to the tax man. You win provided the sum of the numbers you have exceeds the sum of those held by the tax man. Here is a sample game where the tax man wins by a score of 56 to 22:

REMAINING	**YOURS**	**TAX MAN's**	**CHOICE**
$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$	$\{\}$	$\{\}$	12
$\{5, 7, 8, 9, 10, 11\}$	$\{12\}$	$\{1,2,3,4,6\}$	10
$\{7, 8, 9, 11\}$	$\{10,12\}$	$\{1,2,3,4,5,6\}$	**NO MOVES**
$\{\}$	$\{10, 12\} \Longrightarrow 22$	$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 11\} \Longrightarrow 56$	

Either find a way to beat the tax man or explain why this is impossible.

#3 SHUT THE BOX or SHUT UP

The game shut up is played solo with a pair of ordinary dice and the numbers $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. On each roll of the dice you may eliminate any set of numbers whose sum equals the number rolled. You continue rolling until you find a roll for which this cannot be done. Your score is then the sum of the numbers still remaining. **You want the lowest score possible - shutting the box is a score of zero!**. Here is a sample game:

ROLL	CHOOSE	**REMAINING**
6 & 4	2,3,5	$\{1,4,6,7,8,9,10,11,12\}$
3 & 1	4	$\{1,6,7,8,9,10,11,12\}$
3 & 6	9	$\{1,6,7,8,10,11,12\}$
5 & 6	1,10	$\{6,7,8,11,12\}$
4 & 5	**NO MOVES**	$\{6, 7, 8, 11, 12\} \Longrightarrow 44$

Suppose you are playing shut the box and the numbers remaining are just $1, 2, 6,$ *and* 7 *and you roll an* 5 *&* 3*. What should you do and why?*

#4 ARRANGING DIGITS

Use each digit $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ exactly once for the variables $a, b, c, d, e, f, g, h, i$ so that the formula below has the smallest possible absolute value. How do you know your answer is the smallest and what is this smallest value?

$$\frac{a}{b} + \frac{c}{d} + \frac{e}{f} + \frac{g}{h} - i$$

#5 ALPHABETIZING

Consider all the integers from 1 to 10^{10} and suppose we write each of these in English. For example, Three Billion, Four Million, Fifty Eight Thousand and Eighteen (i.e. 3,004,058,018). Ignoring all spaces, commas, hyphens (if included), and conjunctions, alphabetize this list. Thus 3,004,058,018 would be simply FourMillionFiftyEightThousandEighteen where the capitals are given for emphasis only. What is the first odd number in the list and where does this occur?

#6 CARD TRICK

You are placed in a completely dark room with no flashlight. In other words, you cannot see anything! On a table spread out in front of you are 52 playing cards. 21 of these are face up and 31 are face down. Your job is to create two piles of cards which both have the same number of face up cards. How can you do it?