Math Rumble Problems

American Mathematics Competitions

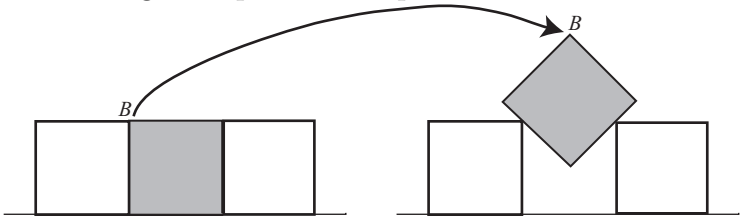
February 11, 2011

1. A wooden cube n units on a side is painted red on all six faces and then cut into n^3 unit cubes. Exactly one-fourth of the total number of faces of the unit cubes are red. What is n?

2. How many positive integers n satisfy the following condition:

$$(130n)^{50} > n^{100} > 2^{200} \ ?$$

3. Three one-inch squares are placed with their bases on a line. The center square is lifted out and rotated 45°, as shown. Then it is centered and lowered into its original location until it touches both of the adjoining squares. How many inches is the point B from the line on which the bases of the original squares were placed?

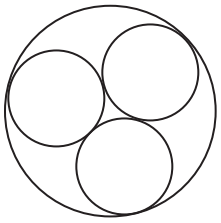


4. One fair die has faces 1, 1, 2, 2, 3, 3 and another has faces 4, 4, 5, 5, 6, 6. The dice are rolled and the numbers on the top faces are added. What is the probability that the sum will be odd?

5. In $\triangle ABC$, we have $AC = BC = 7$ and $AB = 2$. Suppose that D is a point on line AB such that B lies between A and D and $CD = 8$. What is BD?

6. Patty has 20 coins consisting of nickels and dimes. If her nickels were dimes and her dimes were nickels, she would have 70 cents more. How much are her coins worth?

7. Three circles of radius 1 are externally tangent to each other and internally tangent to a larger circle. What is the radius of the large circle?



8. Henry's Hamburger Heaven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties, and any collection of condiments. How many different kinds of hamburgers can be ordered?

9. Given that $-4 \leq x \leq -2$ and $2 \leq y \leq 4$, what is the largest possible value of $(x + y)/x$?

10. Coin A is flipped three times and coin B is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?