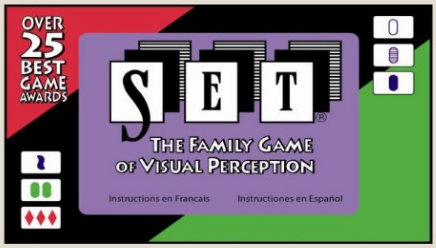
More Adventures in the Game of SET ® -- Transformations and Simulations

Mathfest 2021



Details available at:

- Developing Mathematical Reasoning Using Attribute Games: *Mathematics Teacher,* Vol 92 (9) December 1999, 768-775
- Paper available on the SET homepage:
  - https://www.playmonster.com/wp-content/uploads/2019/10/DEVELOPING-MATHEMATICAL-REASONING.pdf

Dr. Anne Quinn quinna@edinboro.edu

EDINBORO UNIVERSITY

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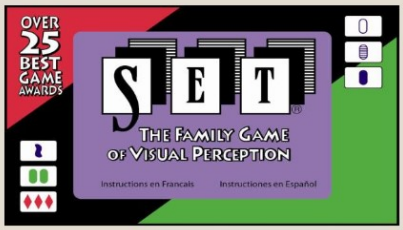
Transformations and Simulations on the Game of SET®

1. Introduction to the game – see setgame.com – QUICK REVIEW
2. Types of sets -- Theorem
3. Minimum, maximum, and average number of sets
4. Suggestions for playing with novices -- NEW
5. Matrix transformations and modular arithmetic
   - Ways to make new groups of cards
6. Using Excel to error check
7. Simulations on new dice versions of the game -- NEW
   - Differences between the two types of Set games will be highlighted.

Some ways to play SET

- Original Game
  - https://www.playmonster.com/product/set/

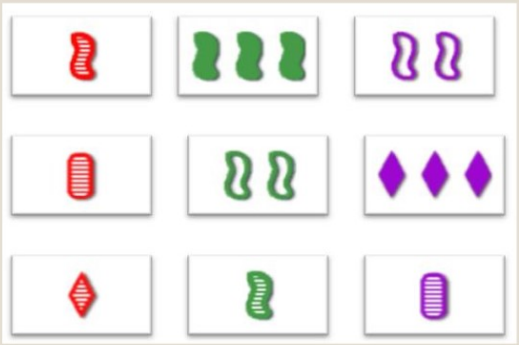


- Puzzle of the Day,
  - on www.setgame.com/welcome
  - or an app on the iPad

- Set Dice Game
  - https://www.youtube.com/watch?v=XLUvXIwrwpA
  - https://www.youtube.com/watch?v=bwcopSXqEKY
  - Game does not appear to be currently available

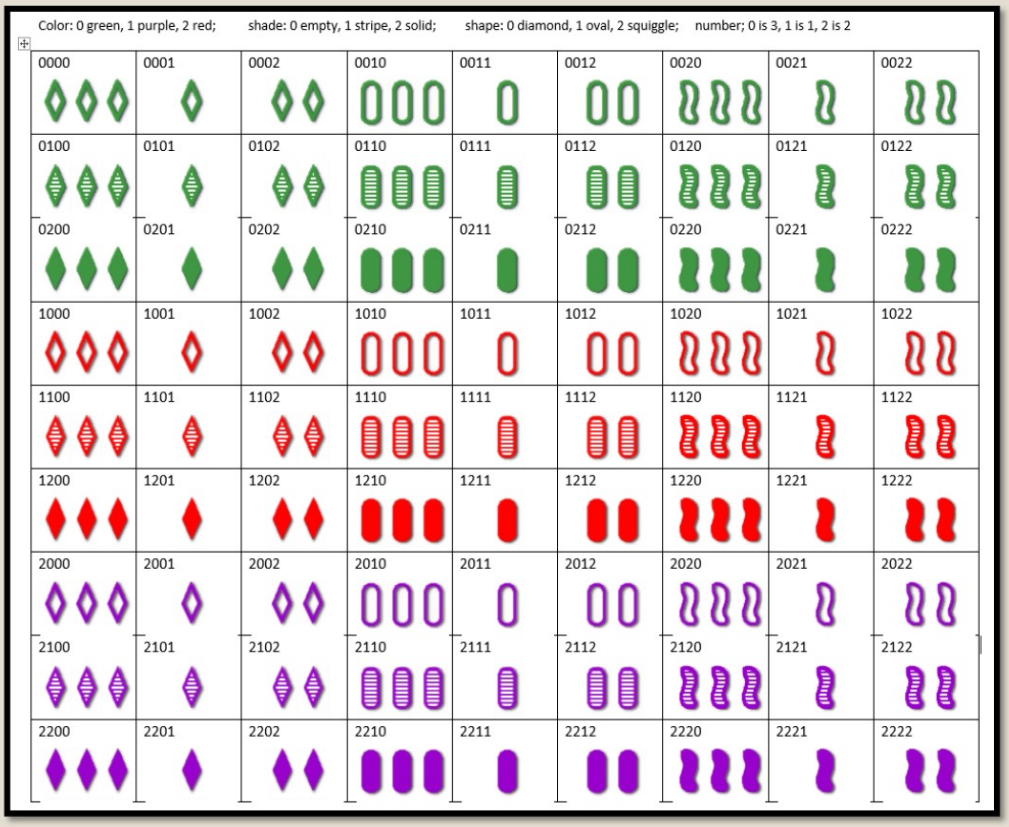
- Online: setwithfriends.com

Original Game- Summary Attributes



Color:	Red	Green	Purple
Number:	1	2	3
Shape:	Oval	Diamond	Squiggle
Shading:	Solid	Stripe	open

3*3*3*3=81 cards



Part 1: Introduction to SET

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Definition and Example

Definition of SET: A SET is a group of 3 cards where each attribute is ALL SAME or ALL DIFFERENT

Example of a SET:

color- DIFFERENT- 3 colors

number- SAME-all "1"s

shape- DIFFERENT- 3 shapes

shading- SAME- all stripes

When is it NOT a SET?

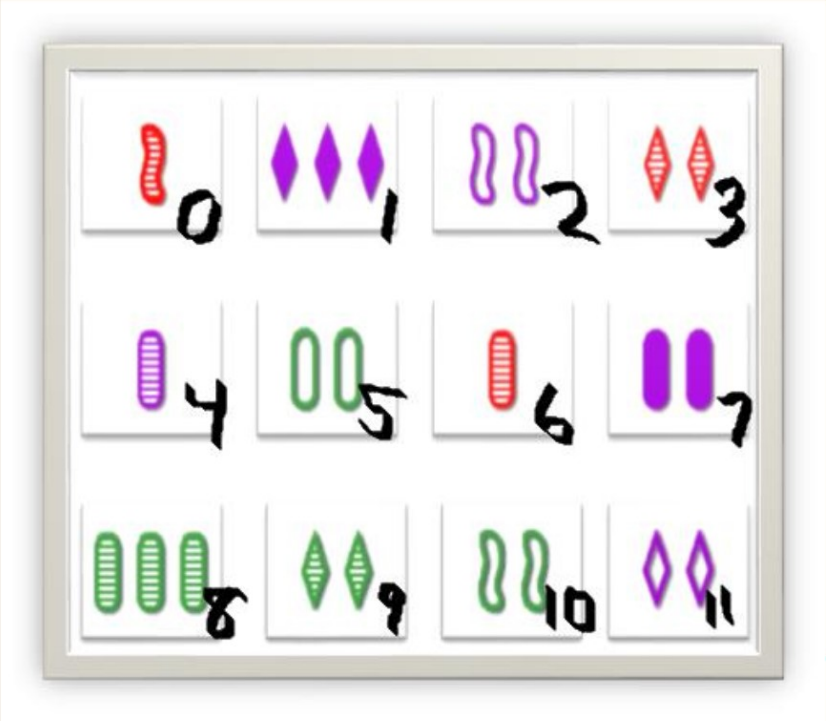
When there is TWO of a kind.

For example:

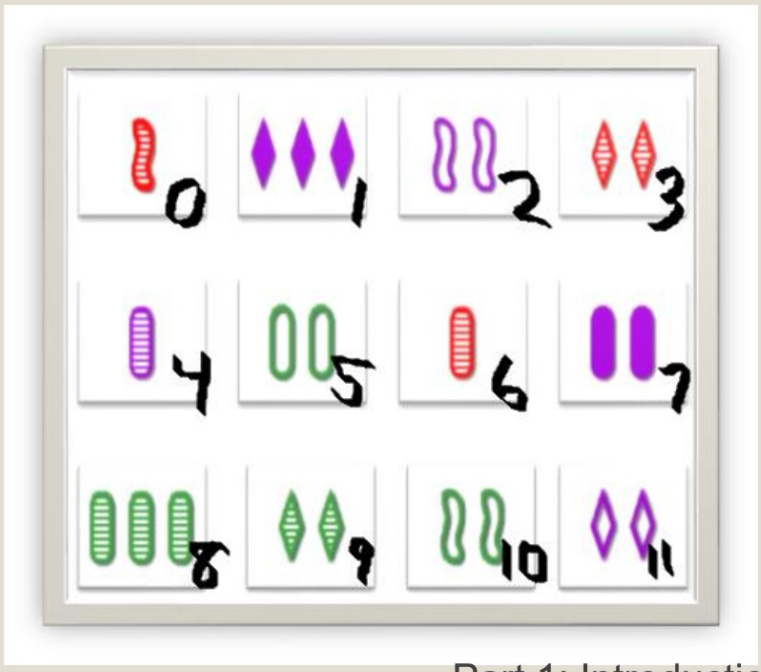
SET Questions-- Mathematics Teacher article (1999)

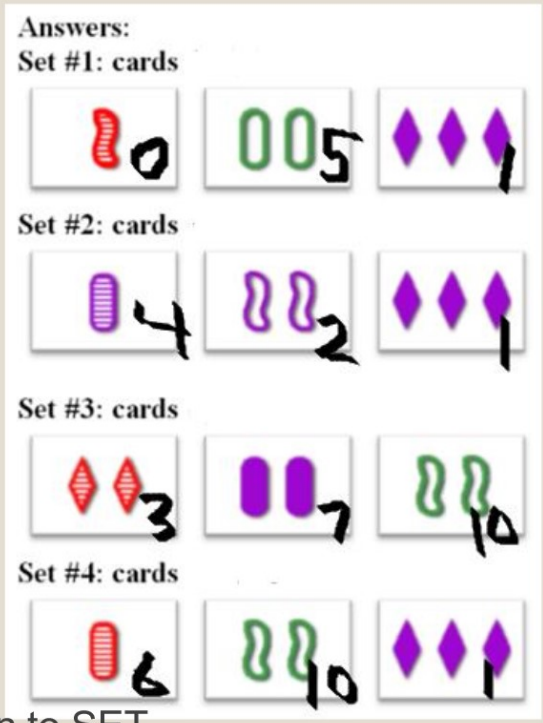
How many cards must be in a deck?
How many SETs (including overlapping ones) are possible in the deck?
What is the best strategy for searching for SETs? Which "type" are you most likely to find?
What is the AVERAGE number of set among 12 cards?
What is the MAXIMUM number of red cards that contain 0 sets?
Find the MAXIMUM number of cards that contain 0 sets.
Can only 3 cards be left at the end of the game?

Ex. #1: Find all SETs



Ex #1- Answers…
Question: Are there different "types"?





Ex #2:
"Daily Puzzle" always has 6 SETS

- Can find this on Set Mania app or www.setgame.com

0 Same

1 Same (#)

3 Same (#, color, shade)

1 Same (shade)

3 Same (color, shape, #)

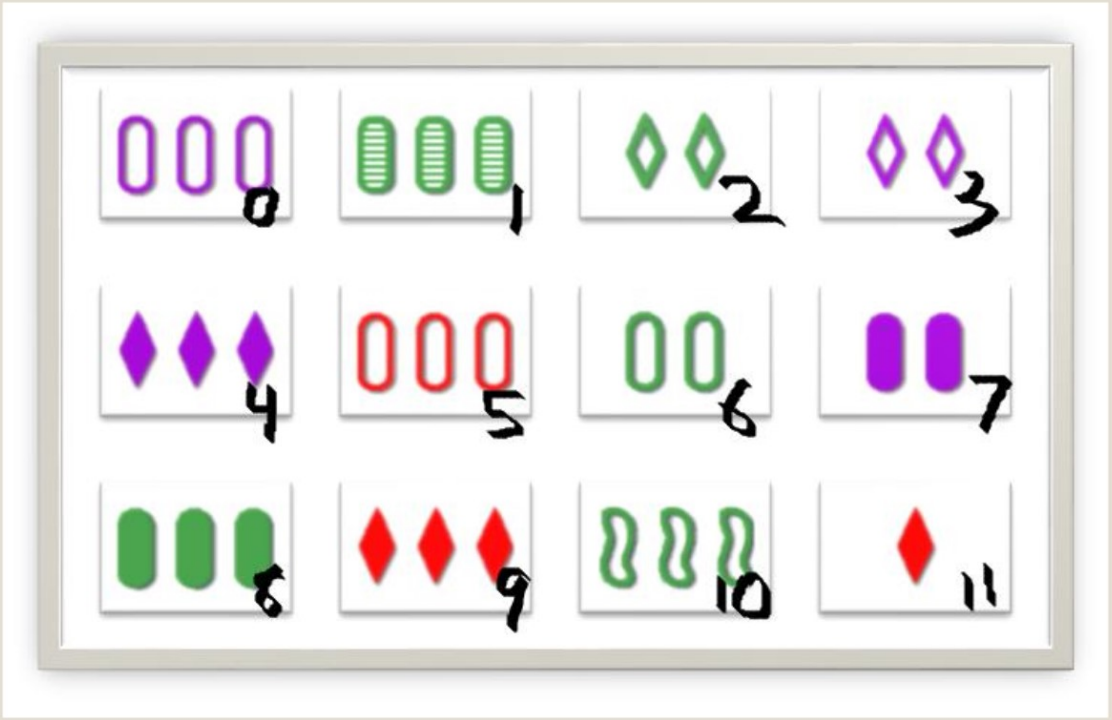
2 Same (shade, #)

Theorem: Types of SETs (NCTM, 1999)

TYPE OF SET	WAYS TO PICK 1st CARD	WAYS TO PICK THE ATTRIBUTE THAT IS SAME ON THE 2nd CARD	WAYS TO PICK THE 2nd CARD	WAYS TO PICK THE 3rd CARD	NUMBER OF SETS OF THIS TYPE	LIKELIHOOD OF THIS TYPE OF SET
4 different/ 0 same	81	${}_4C_0 = 1$	$2^4 = 16$	1	(81 * 16)/3! = 216	$=[81 * {}_4C_0 * 2^4 * 1]/3!$ =216/1080 =20%
3 different/ 1 same	81	${}_4C_1 = 4$	$2^3 = 8$	1	(81 * 4 * 8) /3! = 432	$=[81 * {}_4C_1 * 2^3 * 1]/3!$ =432/1080 =40%
2 different/ 2 same	81	${}_4C_2 = 6$	$2^2 = 4$	1	(81 * 6 * 4)/3! = 324	$=[81 * {}_4C_2 * 2^2 * 1]/3!$ =324/1080 =30%
1 different/ 3 same	81	${}_4C_3 = 4$	$2^1 = 2$	1	(81 * 4 * 2)/3! = 108	$=[81 * {}_4C_3 * 2^1 * 1]/3!$ =108/1080 =10%
TOTAL					1080	

- Hint: When playing, look for something in common among the cards.
- For details of proof, see www.setgame.com
  - https://www.playmonster.com/wp-content/uploads/2019/10/DEVELOPING-MATHEMATICAL-REASONING.pdf

Ex. #3: Find all SETs

- 

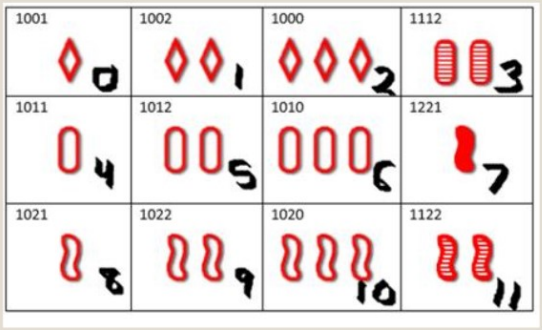
Part 3: Max, Min, Average

12

How many SETs in 12 cards:

- Minimum is 0 SETs
  - (see last example)

- Average is (1/79) * $_{12}C_3 \cong 2.79$ SETs
- "Puzzle of the day" always has 6 SETS
- Maximum is 14 SETs



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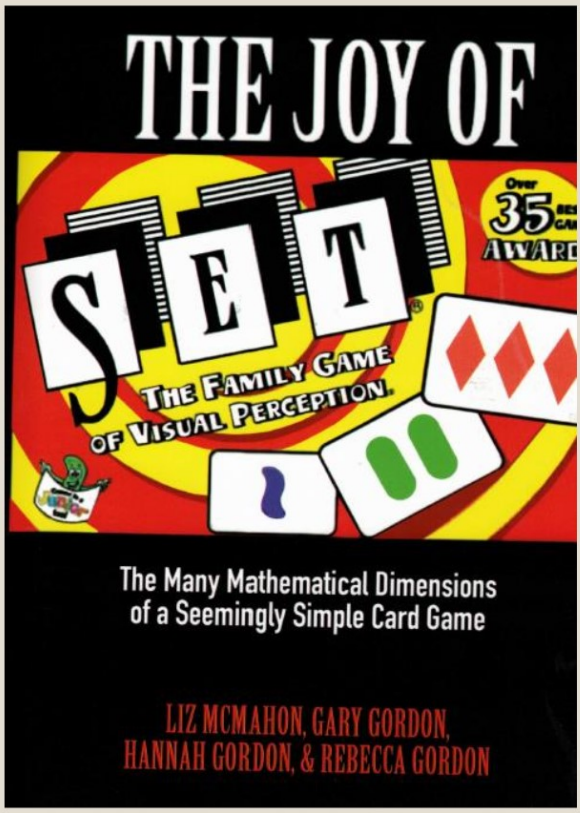

The Joy of SET (L. McMahon, G. Gordon, H. Gordon, R. Gordon, Princeton, 2017)

Part 3: Max, Min, Average

14

"Joy of SET" summary of 12 cards

- In a game with 12 cards:
  - Min number of SETs is 0
  - Average number of SETs is 2.79
  - Max number SETs is 14 (see later ex)
  - Probability of getting 6 SETs from 12 cards (like on the Puzzle of the Day): 2.3% of time
  - 81C12≅ $7.07x10^{13}$ layouts of 12 cards

# SETs in 12 cards	% in simulation	Layouts out of 81C12
0	3.2%	$2.26*10^{12}$
6	2.3%	$1.6*10^{12}$
14	0.000004%	2,828,973

Part 3: Max, Min, Average

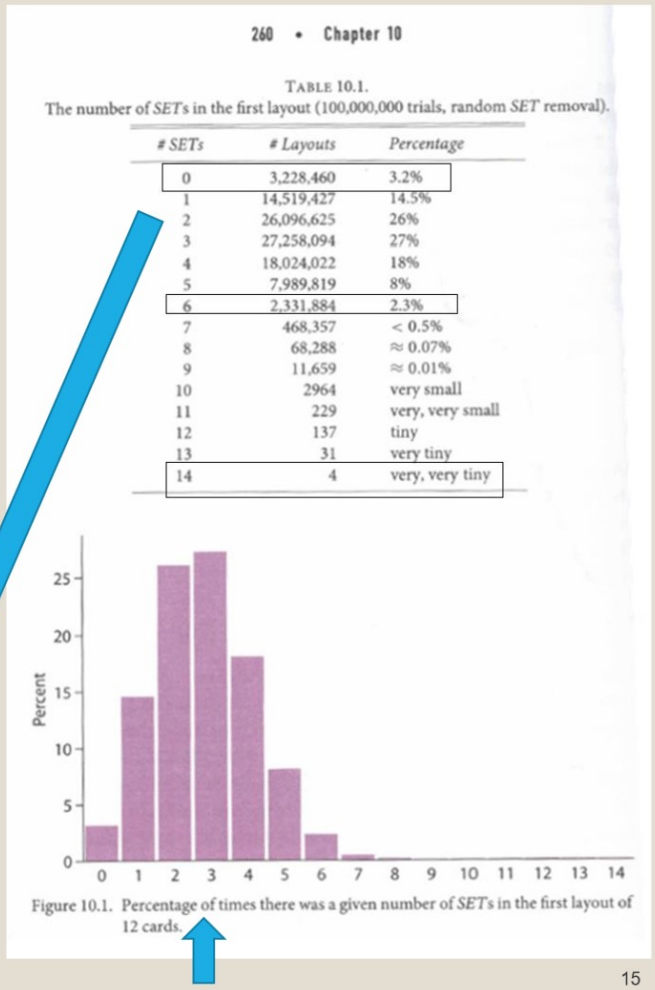

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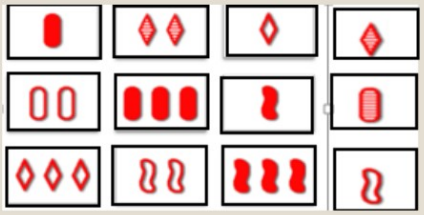
TABLE 10.1.
The number of *SET*s in the first layout (100,000,000 trials, random *SET* removal).

# SETs	# Layouts	Percentage
0	3,228,460	3.2%
1	14,519,427	14.5%
2	26,096,625	26%
3	27,258,094	27%
4	18,024,022	18%
5	7,989,819	8%
6	2,331,884	2.3%
7	468,357	< 0.5%
8	68,288	≈ 0.07%
9	11,659	≈ 0.01%
10	2964	very small
11	229	very, very small
12	137	tiny
13	31	very tiny
14	4	very, very tiny

Figure 10.1. Percentage of times there was a given number of *SET*s in the first layout of 12 cards.

Tips for playing with novices

- Start with just 3 attributes. For example, just use the Red cards



- Have the experienced players play "SET SET", or even SET^3.
  - Example on next slide

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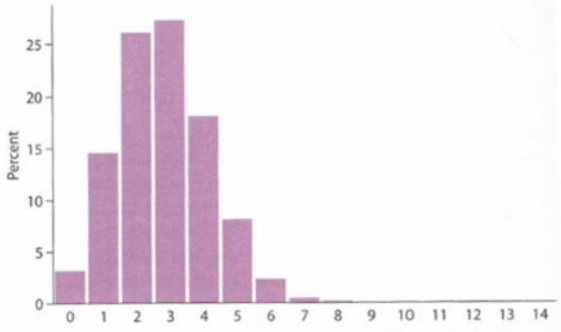
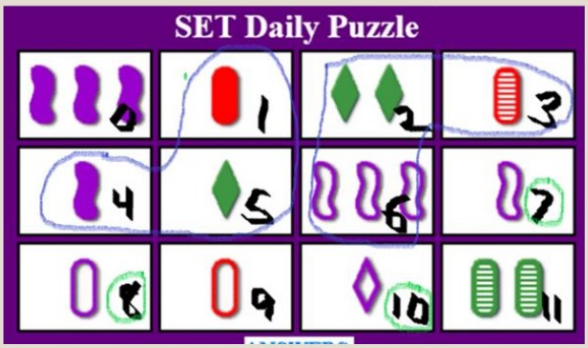


Figure 10.1. Percentage of times there was a given number of *SET*s in the first layout of 12 cards.

Part 4: Playing with Novices

Ex of "SET SET"–
from setgame.com

Non-overlapping



Overlapping

…Tips for playing with novices

- Have the experienced players announce a SET and allowing the novice 2 seconds to find it before claiming it.
- Let the experienced players only claim certain types (ex: all different).
  - See theorem
  - See example on next slide

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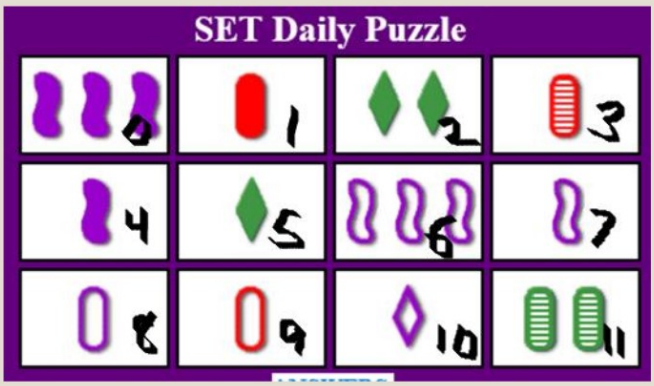
6	2,331,884	2.3%

TYPE OF SET	LIKELIHOOD OF THIS TYPE OF SET
4 different/ 0 same	$=[81 * {}_4C_0 * 2^4 * 1]/3!$ =216/1080 =20%
3 different/ 1 same	$=[81 * {}_4C_1 * 2^3 * 1]/3!$ =432/1080 =40%
2 different/ 2 same	$=[81 * {}_4C_2 * 2^2 * 1]/3!$ =324/1080 =30%
1 different/ 3 same	$=[81 * {}_4C_3 * 2^1 * 1]/3!$ =108/1080 =10%

Part 4: Playing with Novices

Ex:
-Expert can only claim from 2 sets
-Novice can claim all 6 sets



0 Same

1 Same (#)

3 Same (#, color, shade)

1 Same (shade)

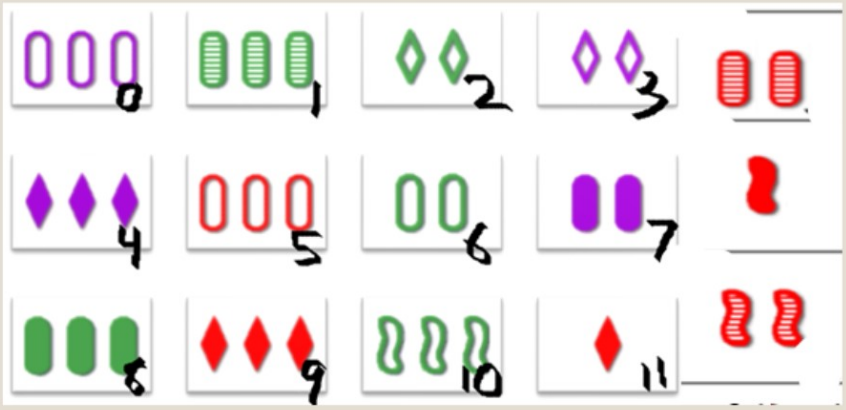
3 Same (color, shape, #)

2 Same (shade, #)

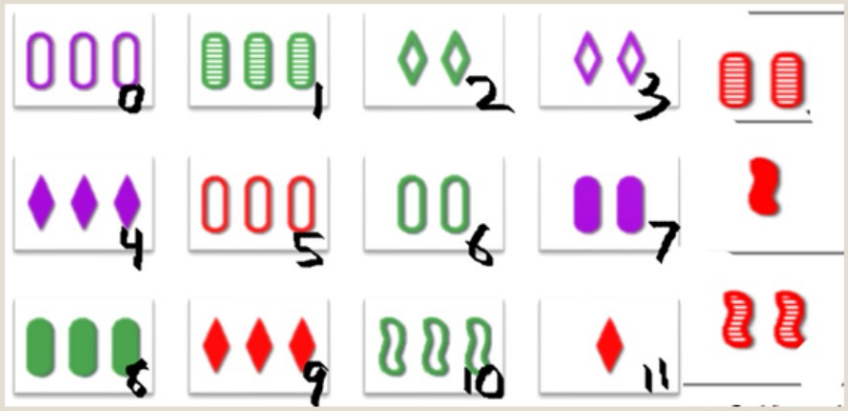
Part 4: Playing with Novices

…Tips for playing with novices

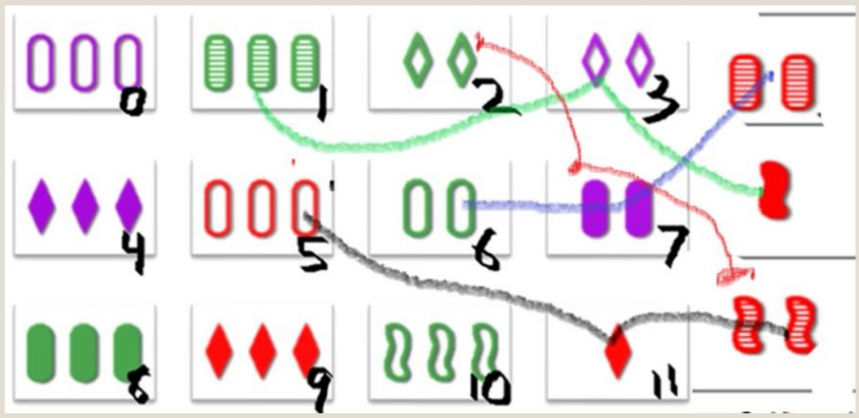
- If two novices don't see a SET (although an expert might), have them add 3 more cards. This will increase the numbers of sets.
  - Expected number of sets in first 12 cards is (1/79) * $_{12}C_3$ =(1/79)*220≅ 2.78
  - Expected number of sets in first 15 cards is (1/79) * $_{15}C_3$ =(1/79)*455≅ 5.76
  - Tell them to focus on making SETs with the 3 new ones



Part 4: Playing with Novices



The original 12 cards have 0 SETs…
Add 3 cards on the right.



At least 4 new SETs appear

Part 4: Playing with Novices

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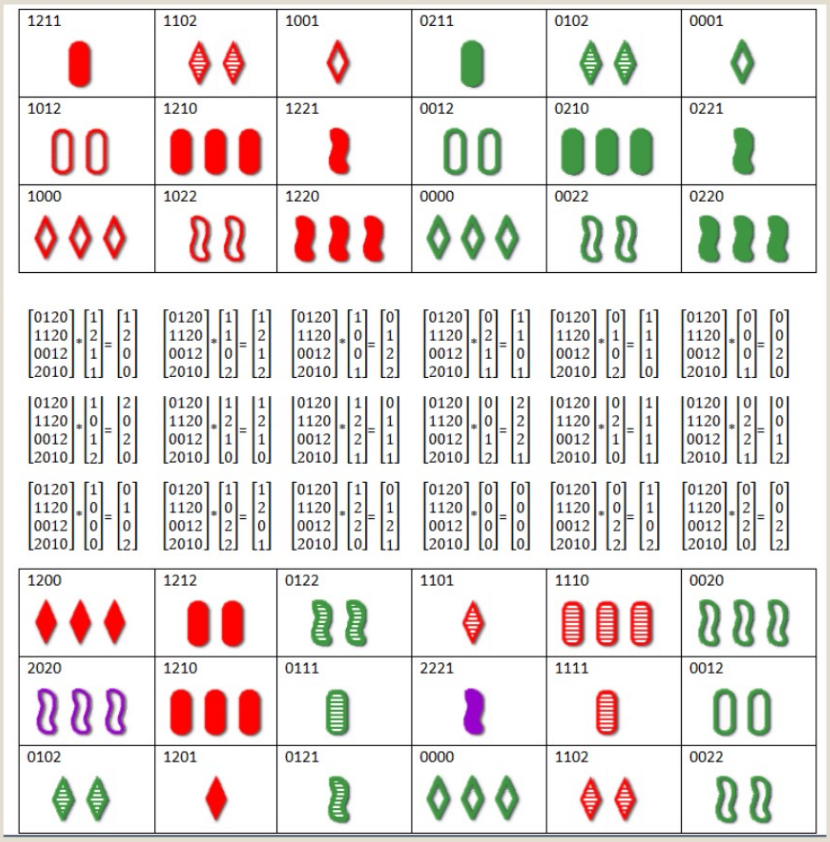
Transform to make a new group of 18 cards without a SET



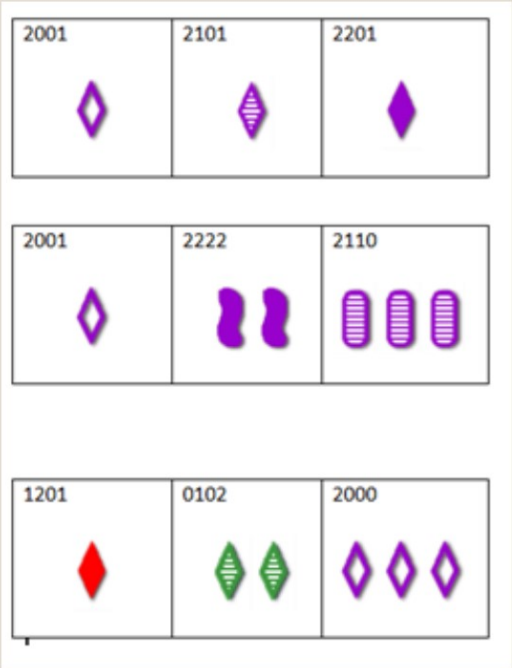
Part 5: Matrix transformations

Creating and checking on Excel

- We will use Excel on some of the MAX and MIN examples:
  - Check for SETs using Excel
    - For 9 cards, there are 9C3=84 possibilities
    - For 12 cards, there are 12C3=220 possibilities

Checking for SETs with Mods

Color: 0 green, 1 purple, 2 red;
shade: 0 empty, 1 stripe, 2 solid;
shape: 0 diamond, 1 oval, 2 squiggle;
number: 0 is 3, 1 is 1, 2 is 2



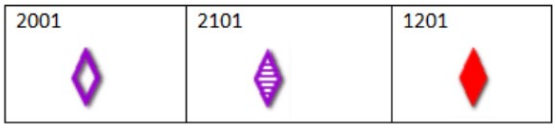
	Each term MOD 3	0 SUM is a SET
2001		
2101		
2201		
6303 is sum,	0000	0+0+0+0=0 SET
2001		
2222		
2110		
6333 is sum,	0000	0+0+0+0=0 SET
1201		
0102		
2000		
3303 is sum,	0000	0+0+0+0=0 SET

…checking for SETs with Mods

Color: 0 green, 1 purple, 2 red;
shade: 0 empty, 1 stripe, 2 solid;
shape: 0 diamond, 1 oval, 2 squiggle;
number: 0 is 3, 1 is 1, 2 is 2

Non-SETs will be non-zero,

And they will have 2 of same



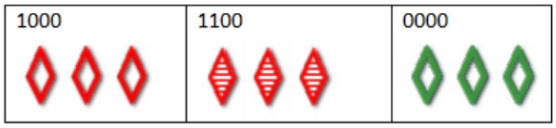
2001 (color has 2 same)

2101

1201

5303 is sum, 2000 2+0+0+0=2 non-SET

I



1000 (color has 2 same, as does shade)

1100

0000

2100 is sum, 2100 2+1+0+0=3 non-SET

(note: important to take each digit mod3 instead of summing digits)

New: 1. Puzzle SET Mania on iPad

2. Scramble SET Dice

Part 7: Simulations on new dice versions

New SET Dice Games

Box has 42 dice

- 28 choices:
  - 3 color* 3 shape* 3 number* 1 shade (solid)= 27
  - Plus 1 wild
- 28 choices * 9 occurrences = 252
- 252 / 6 sides of dice = 42 dice
  - So 9 of these 42 dice have a WILD side
- Questions
  - Average number of sets from 42
  - Probability of 0 sets from 12 dice…

Part 7: Simulations on new dice versions

Possible Coding: 000-222
number (0,1,2), Color (G, P, R), shape (D, O, S)

Sample toss of the 42 dice
Frequently there were 0 or 3 left at end.
Is it possible to make 0 sets?

Part 7: Simulations on new dice versions- 42 dice

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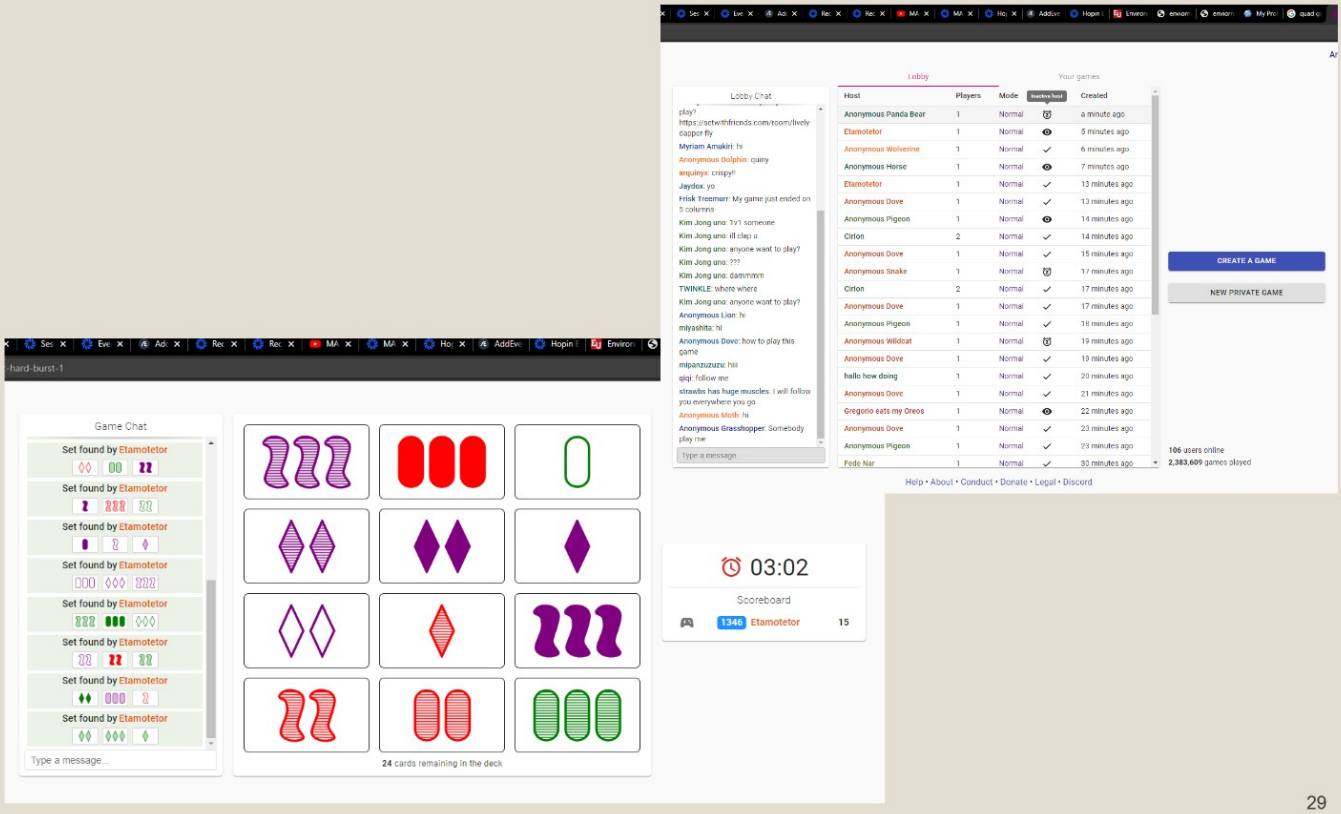
Setwithfriends.com



Contact info

- Dr. Anne Quinn, Chair, Professor,
  - Math/CS, Edinboro University of PA
- Email: quinna@edinboro.edu
- LinkedIn: linkedin.com/in/drannequinn/

- Article on Google Scholar: "Anne Quinn Edinboro"
- Find 1999 Mathematics Teacher article in the Skill Connections/ Teacher's Corner of: www.setgame.com

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