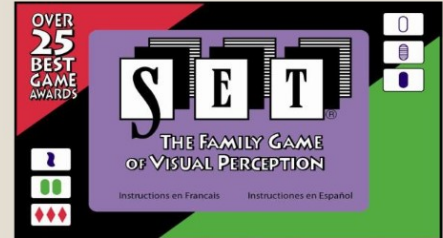


More Adventures in the Game of SET[®] -- Transformations and Simulations

Mathfest 2021



Details available at:

- Developing Mathematical Reasoning Using Attribute Games:
Mathematics Teacher, Vol 92 (9) December 1999, 768-775
- Paper available on the SET homepage:
 - <https://www.playmonster.com/wp-content/uploads/2019/10/DEVELOPING-MATHEMATICAL-REASONING.pdf>

Dr. Anne Quinn quinna@edinboro.edu



Transformations and Simulations on the Game of SET®

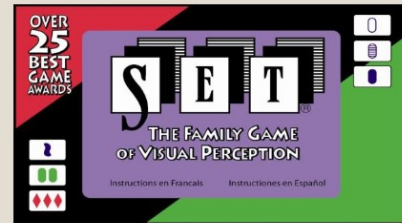
1. Introduction to the game – see setgame.com – QUICK REVIEW
2. Types of sets -- Theorem
3. Minimum, maximum, and average number of sets
4. Suggestions for playing with novices -- NEW
5. Matrix transformations and modular arithmetic
 - Ways to make new groups of cards
6. Using Excel to error check
7. Simulations on new dice versions of the game -- NEW
 - Differences between the two types of Set games will be highlighted.

Some ways to play SET

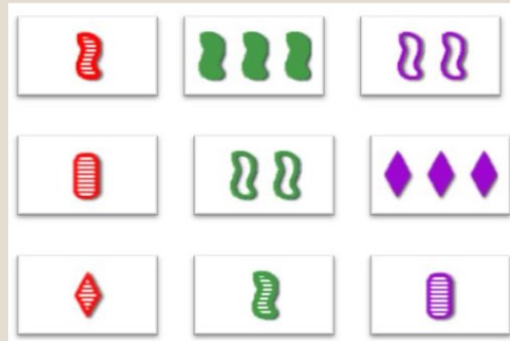
- Original Game
 - <https://www.playmonster.com/product/set/>
- Puzzle of the Day,
 - on www.setgame.com/welcome
 - or an app on the iPad



- Set Dice Game
 - <https://www.youtube.com/watch?v=XLUVXlwrrpA>
 - <https://www.youtube.com/watch?v=bwcopSXqEKY>
 - Game does not appear to be currently available
- Online: setwithfriends.com



Original Game- Summary Attributes



Color:	Red	Green	Purple
Number:	1	2	3
Shape:	Oval	Diamond	Squiggle
Shading:	Solid	Stripe	open

$3 \times 3 \times 3 \times 3 = 81$ cards

Color: 0 green, 1 purple, 2 red; shade: 0 empty, 1 stripe, 2 solid; shape: 0 diamond, 1 oval, 2 squiggle; number: 0 is 3, 1 is 1, 2 is 2

0000 	0001 	0002 	0010 	0011 	0012 	0020 	0021 	0022
0100 	0101 	0102 	0110 	0111 	0112 	0120 	0121 	0122
0200 	0201 	0202 	0210 	0211 	0212 	0220 	0221 	0222
1000 	1001 	1002 	1010 	1011 	1012 	1020 	1021 	1022
1100 	1101 	1102 	1110 	1111 	1112 	1120 	1121 	1122
1200 	1201 	1202 	1210 	1211 	1212 	1220 	1221 	1222
2000 	2001 	2002 	2010 	2011 	2012 	2020 	2021 	2022
2100 	2101 	2102 	2110 	2111 	2112 	2120 	2121 	2122
2200 	2201 	2202 	2210 	2211 	2212 	2220 	2221 	2222

Part 1: Introduction to SET

Definition and Example

Definition of SET: A SET is a group of 3 cards where each attribute is ALL SAME or ALL DIFFERENT

Example of a SET:



color- DIFFERENT- 3 colors

number- SAME-all "1"s

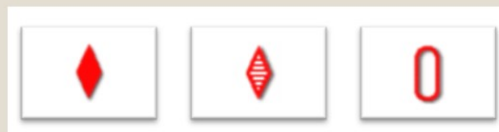
shape- DIFFERENT- 3 shapes

shading- SAME- all stripes

When is it NOT a SET?

When there is TWO of a kind.

For example:



Part 1: Introduction to SET

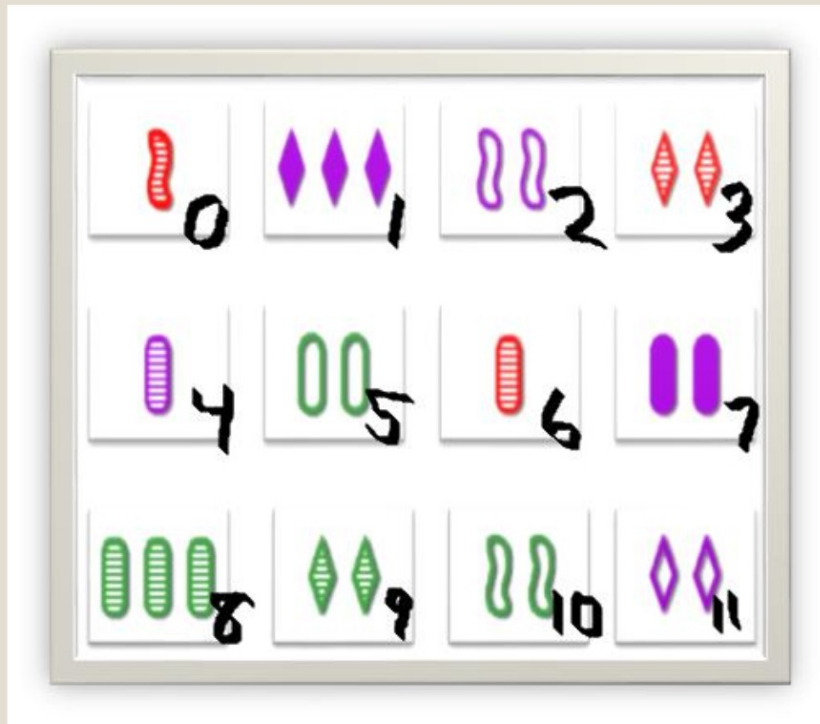
SET Questions-- Mathematics Teacher article (1999)

How many cards must be in a deck?
How many SETs (including overlapping ones) are possible in the deck?
What is the best strategy for searching for SETs? Which "type" are you most likely to find?
What is the AVERAGE number of set among 12 cards?
What is the MAXIMUM number of red cards that contain 0 sets?
Find the MAXIMUM number of cards that contain 0 sets.
Can only 3 cards be left at the end of the game?

Part 1: Introduction to SET

Ex. #1: Find all SETs

o

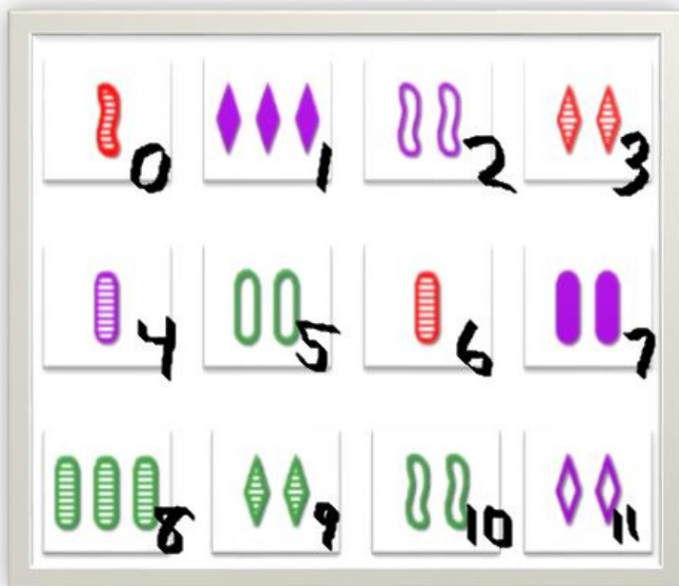


Part 1: Introduction to SET

8

Ex #1- Answers...

Question: Are there different “types”?



Answers:

Set #1: cards



Set #2: cards



Set #3: cards

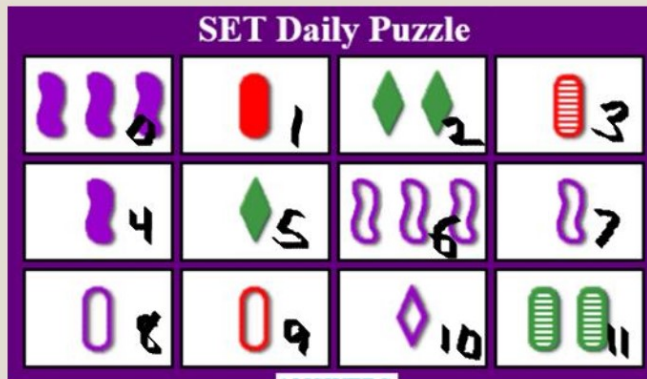


Set #4: cards

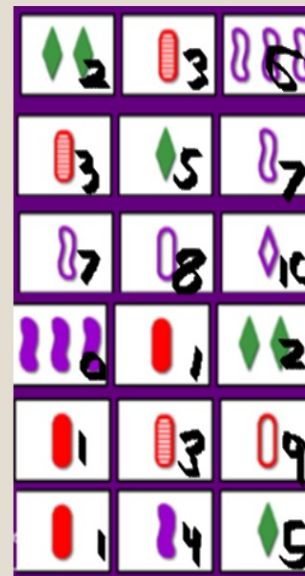


Part 1: Introduction to SET

Ex #2: “Daily Puzzle” always has 6 SETS



- Can find this on Set Mania app or www.setgame.com



0 Same

1 Same (#)

3 Same (#, color, shade)

1 Same (shade)

3 Same (color, shape, #)

2 Same (shade, #)

Part 1: Introduction to SET

10

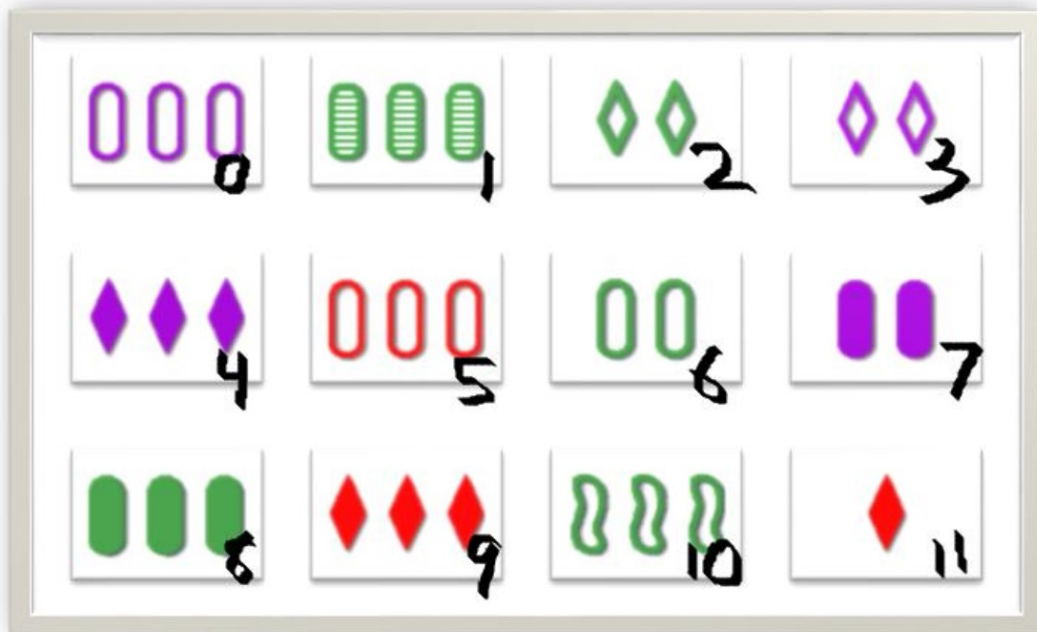
Theorem: Types of SETs (NCTM, 1999)

TYPE OF SET	WAYS TO PICK 1st CARD	WAYS TO PICK THE ATTRIBUTE THAT IS SAME ON THE 2nd CARD	WAYS TO PICK THE 2nd CARD	WAYS TO PICK THE 3rd CARD	NUMBER OF SETS OF THIS TYPE	LIKELIHOOD OF THIS TYPE OF SET
4 different/ 0 same	81	${}_4C_0 = 1$	$2^4 = 16$	1	$(81 \cdot 16)/3! = 216$	$= [81 \cdot {}_4C_0 \cdot 2^4 \cdot 1]/3! = 216/1080 = 20\%$
3 different/ 1 same	81	${}_4C_1 = 4$	$2^3 = 8$	1	$(81 \cdot 4 \cdot 8)/3! = 432$	$= [81 \cdot {}_4C_1 \cdot 2^3 \cdot 1]/3! = 432/1080 = 40\%$
2 different/ 2 same	81	${}_4C_2 = 6$	$2^2 = 4$	1	$(81 \cdot 6 \cdot 4)/3! = 324$	$= [81 \cdot {}_4C_2 \cdot 2^2 \cdot 1]/3! = 324/1080 = 30\%$
1 different/ 3 same	81	${}_4C_3 = 4$	$2^1 = 2$	1	$(81 \cdot 4 \cdot 2)/3! = 108$	$= [81 \cdot {}_4C_3 \cdot 2^1 \cdot 1]/3! = 108/1080 = 10\%$
TOTAL					1080	

- Hint: When playing, look for something in common among the cards.
- For details of proof, see www.setgame.com
 - <https://www.playmonster.com/wp-content/uploads/2019/10/DEVELOPING-MATHEMATICAL-REASONING.pdf>

Ex. #3: Find all SETs

o

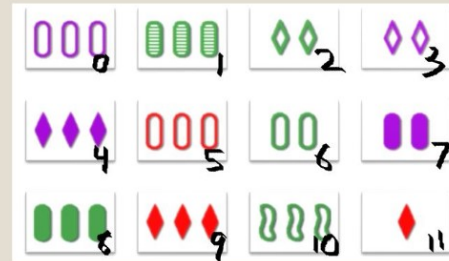


Part 3: Max, Min, Average

12

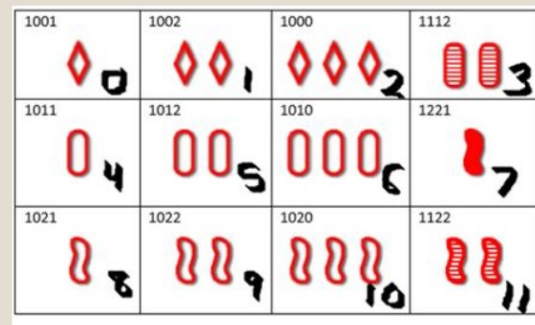
How many SETs in 12 cards:

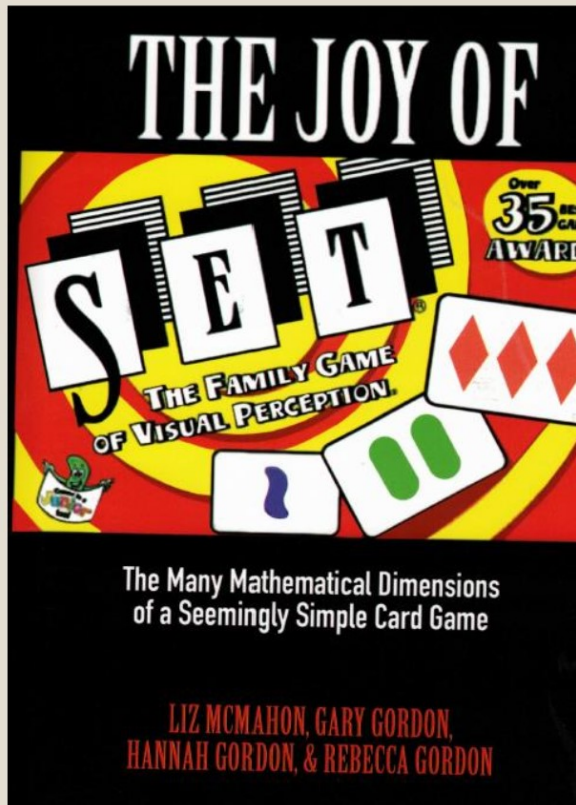
- Minimum is 0 SETs
 - (see last example)



- Average is $(1/79) * {}_{12}C_3 \cong 2.79$ SETs
- “Puzzle of the day” always has 6 SETs

- Maximum is 14 SETs





The Joy of SET (L. McMahon, G. Gordon, H. Gordon, R. Gordon, Princeton, 2017)

Part 3: Max, Min, Average

“Joy of SET” summary of 12 cards

- In a game with 12 cards:
 - Min number of SETs is 0
 - Average number of SETs is 2.79
 - Max number SETs is 14 (see later ex)
 - Probability of getting 6 SETs from 12 cards (like on the Puzzle of the Day): 2.3% of time
 - $81C12 \cong 7.07 \times 10^{13}$ layouts of 12 cards

# SETs in 12 cards	% in simulation	Layouts out of 81C12
0	3.2%	2.26×10^{12}
6	2.3%	1.6×10^{12}
14	0.000004%	2,828,973

Part 3: Max, Min, Average

260 • Chapter 10
TABLE 10.1.
The number of SETs in the first layout (100,000,000 trials, random SET removal).

# SETs	# Layouts	Percentage
0	3,228,460	3.2%
1	14,519,427	14.5%
2	26,096,625	26%
3	27,258,094	27%
4	18,024,022	18%
5	7,989,819	8%
6	2,331,884	2.3%
7	468,357	< 0.5%
8	68,288	$\approx 0.07\%$
9	11,659	$\approx 0.01\%$
10	2964	very small
11	229	very, very small
12	137	tiny
13	31	very tiny
14	4	very, very tiny

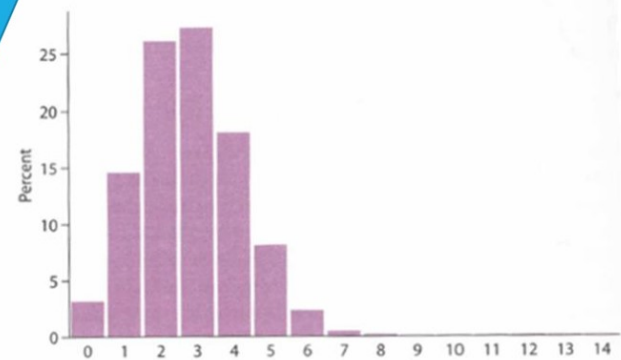
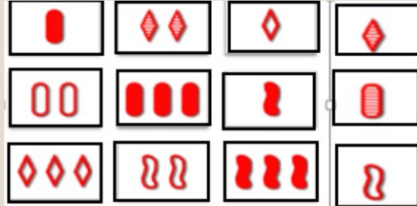


Figure 10.1. Percentage of times there was a given number of SETs in the first layout of 12 cards.

Tips for playing with novices

- Start with just 3 attributes. For example, just use the Red cards



- Have the experienced players play "SET SET", or even SET³.
 - Example on next slide

TABLE 10.1.
The number of SET's in the first layout (100,000,000 trials, random SET removal).

# SETs	# Layouts	Percentage
0	3,228,460	3.2%
1	14,519,427	14.5%
2	26,096,625	26%
3	27,258,094	27%
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5	7,989,819	8%
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14	4	very, very tiny

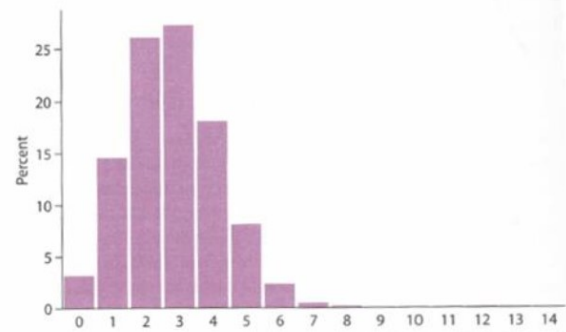
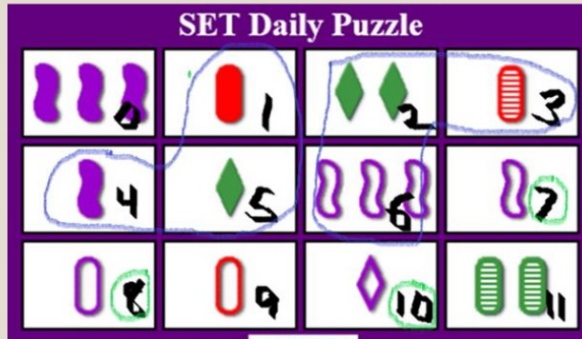


Figure 10.1. Percentage of times there was a given number of SET's in the first layout of 12 cards.

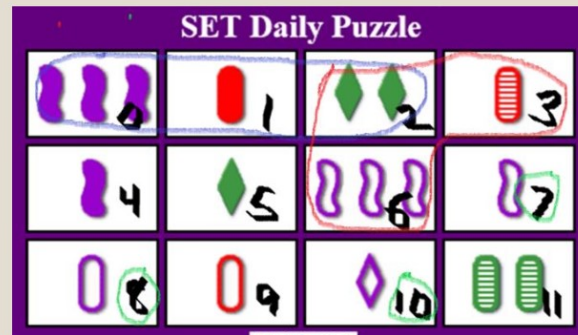
Part 4: Playing with Novices

Ex of “SET SET”– from setgame.com

Non-overlapping



Overlapping



Part 4: Playing with Novices

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...Tips for playing with novices

- Have the experienced players announce a SET and allowing the novice 2 seconds to find it before claiming it.
- Let the experienced players only claim certain types (ex: all different).
 - See theorem
 - See example on next slide

TABLE 10.1.
The number of SETs in the first layout (100,000,000 trials, random SET removal).

# SETs	# Layouts	Percentage
0	3,228,460	3.2%
1	14,519,427	14.5%
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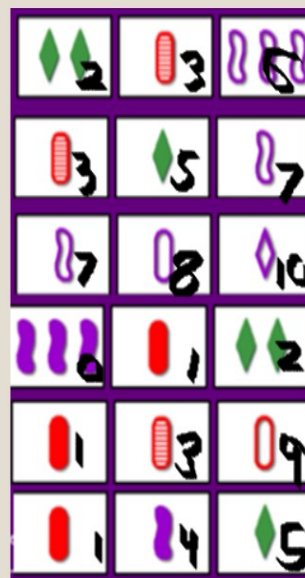
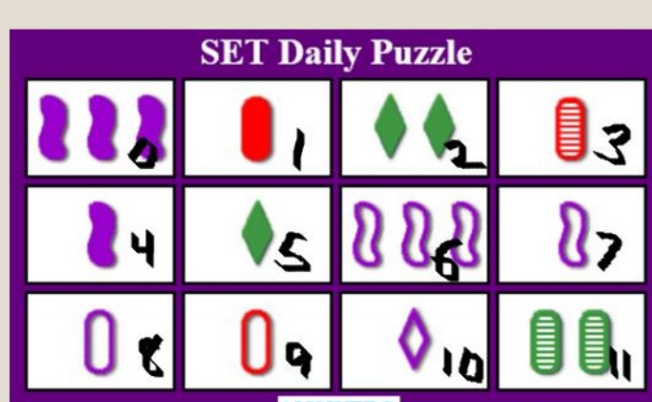
TYPE OF SET	LIKELIHOOD OF THIS TYPE OF SET
4 different/ 0 same	$= [81 \cdot {}_4C_0 \cdot 2^4 \cdot 1]/3!$ $= 216/1080$ =20%
3 different/ 1 same	$= [81 \cdot {}_4C_1 \cdot 2^3 \cdot 1]/3!$ $= 432/1080$ =40%
2 different/ 2 same	$= [81 \cdot {}_4C_2 \cdot 2^2 \cdot 1]/3!$ $= 324/1080$ =30%
1 different/ 3 same	$= [81 \cdot {}_4C_3 \cdot 2^1 \cdot 1]/3!$ $= 108/1080$ =10%

Part 4: Playing with Novices

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Ex:

- Expert can only claim from 2 sets
- Novice can claim all 6 sets



0 Same

1 Same (#)

3 Same (#, color, shade)

1 Same (shade)

3 Same (color, shape, #)

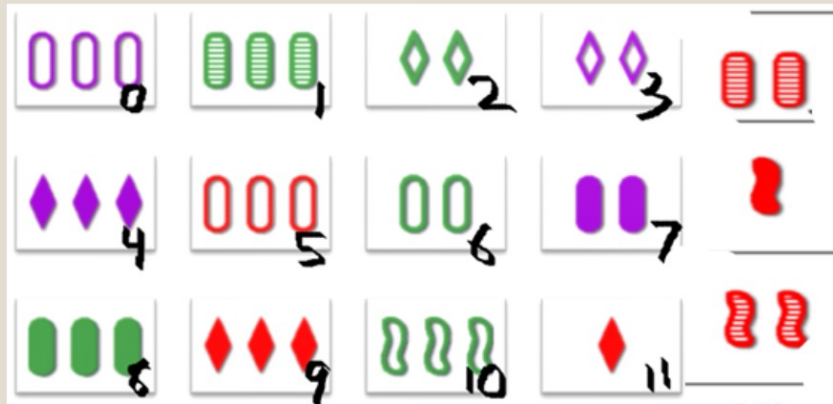
2 Same (shade, #)

Part 4: Playing with Novices

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...Tips for playing with novices

- If two novices don't see a SET (although an expert might), have them add 3 more cards. This will increase the numbers of sets.
 - Expected number of sets in first 12 cards is $(1/79) * {}_{12}C_3$
 $= (1/79) * 220 \approx \underline{2.78}$
 - Expected number of sets in first 15 cards is $(1/79) * {}_{15}C_3$
 $= (1/79) * 455 \approx \underline{5.76}$
 - Tell them to focus on making SETs with the 3 new ones



Part 4: Playing with Novices



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Transform to
make a new
group of 18
cards without
a SET

1211 	1102 	1001 	0211 	0102 	0001
1012 	1210 	1221 	0012 	0210 	0221
1000 	1022 	1220 	0000 	0022 	0220

$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}$
$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}$
$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 0120 \\ 1120 \\ 0012 \\ 2010 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix}$

1200 	1212 	0122 	1101 	1110 	0020
2020 	1210 	0111 	2221 	1111 	0012
0102 	1201 	0121 	0000 	1102 	0022

Part 5: Matrix transformations

Creating and checking on Excel

- We will use Excel on some of the MAX and MIN examples:
 - Check for SETs using Excel
 - For 9 cards, there are $9C3=84$ possibilities
 - For 12 cards, there are $12C3=220$ possibilities

Checking for SETs with Mods

Color: 0 green, 1 purple, 2 red;
 shade: 0 empty, 1 stripe, 2 solid;
 shape: 0 diamond, 1 oval, 2 squiggle;
 number: 0 is 3, 1 is 1, 2 is 2

2001 	2101 	2201
2001 	2222 	2110
1201 	0102 	2000

Each term MOD 3 0 SUM is a SET

2001

2101

2201

6303 is sum, 0000 0+0+0+0=0 SET

2001

2222

2110

6333 is sum, 0000 0+0+0+0=0 SET

1201

0102

2000

3303 is sum, 0000 0+0+0+0=0 SET

Part 6: Error check on Excel

24

...checking for SETs with Mods

Color: 0 green, 1 purple, 2 red;
 shade: 0 empty, 1 stripe, 2 solid;
 shape: 0 diamond, 1 oval, 2 squiggle;
 number: 0 is 3, 1 is 1, 2 is 2

Non-SETs will be non-zero,

And they will have 2 of same

2001	2101	1201

1000	1100	0000

2001 (color has 2 same)

2101

1201

5303 is sum, 2000 $2+0+0+0=2$ non-SET

1000 (color has 2 same, as does shade)

1100

0000

2100 is sum, 2100 $2+1+0+0=3$ non-SET

(note: important to take each digit mod3 instead of summing digits)

New: 1. Puzzle SET Mania on iPad



Overall SET Achievement		
SET Master		
Classic	Seconds per SET	Advanced Deck
Master	6.00	★ ★ ★
		Basic Deck
Master	1.22	★ ★ ★
Puzzle	Seconds per SET	Advanced Deck
Master	4.00	★ ★ ★
		Basic Deck
Master	1.75	★ ★ ★
Sprint	Seconds per SET	Advanced Deck
Master	5.62	★ ★ ★
		Basic Deck
Master	1.88	★ ★ ★
Marathon	Seconds per SET	Advanced Deck

2. Scramble SET Dice



Part 7: Simulations on new dice versions

New SET Dice Games

Box has 42 dice

- 28 choices:
 - $3 \text{ color} * 3 \text{ shape} * 3 \text{ number} * 1 \text{ shade (solid)} = 27$
 - Plus 1 wild
- $28 \text{ choices} * 9 \text{ occurrences} = 252$
- $252 / 6 \text{ sides of dice} = 42 \text{ dice}$
 - So 9 of these 42 dice have a WILD side
- Questions
 - Average number of sets from 42
 - Probability of 0 sets from 12 dice...



Part 7: Simulations on new dice versions

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Possible Coding: 000-222
number (0,1,2), Color (G, P, R), shape (D, O, S)

Sample toss of the 42 dice
Frequently there were 0 or 3 left at end.
Is it possible to make 0 sets?



Part 7: Simulations on new dice versions- 42 dice

Setwithfriends.com

The screenshot displays the Setwithfriends.com interface, which is divided into two main sections: a game lobby and a game in progress.

Game Lobby:

- Lobby Chat:** A chat window on the left showing messages from various users, including "play?", "https://setwithfriends.com/room/levely-clapper-fly", "Myram Amakiri: hi", "Anonymous Dolphin: qamq", "enigma: cteqy!", "Jaydex: yo", "Frik Treemarr: My game just ended on 5 columns", "Kim Jong uno: Tu't someone", "Kim Jong uno: (I clap u", "Kim Jong uno: anyone want to play?", "Kim Jong uno: ???", "Kim Jong uno: dammm", "TWINKLE: where where", "Kim Jong uno: anyone want to play?", "Anonymous Lion: hi", "Anonymous Dove: how to play this game", "mipanzuzuzuz: hi!", "qiqi: follow me", "strawbs has huge muscles: i will follow you everywhere you go", "Anonymous Maki: hi", and "Anonymous Grasshopper: Somebody play me".
- Lobby Table:** A table listing games in the lobby. The columns are Host, Players, Mode, and Created. The games listed are: Anonymous Panda Bear (1 player, Normal, 1 minute ago), Etamototor (1 player, Normal, 5 minutes ago), Anonymous Wolverine (1 player, Normal, 6 minutes ago), Anonymous Horse (1 player, Normal, 7 minutes ago), Etamototor (1 player, Normal, 13 minutes ago), Anonymous Dove (1 player, Normal, 13 minutes ago), Anonymous Pigeon (1 player, Normal, 14 minutes ago), Citron (2 players, Normal, 14 minutes ago), Anonymous Dove (1 player, Normal, 15 minutes ago), Anonymous Snake (1 player, Normal, 17 minutes ago), Citron (2 players, Normal, 17 minutes ago), Anonymous Dove (1 player, Normal, 17 minutes ago), Anonymous Pigeon (1 player, Normal, 18 minutes ago), Anonymous Wildcat (1 player, Normal, 18 minutes ago), Anonymous Dove (1 player, Normal, 19 minutes ago), halo how doing (1 player, Normal, 20 minutes ago), Anonymous Dove (1 player, Normal, 21 minutes ago), Gregorio eats my Oreos (1 player, Normal, 22 minutes ago), Anonymous Dove (1 player, Normal, 23 minutes ago), Anonymous Pigeon (1 player, Normal, 23 minutes ago), and Fede Nar (1 player, Normal, 33 minutes ago).
- Buttons:** "CREATE A GAME" and "NEW PRIVATE GAME" buttons are visible on the right.

Game in Progress:

- Game Chat:** A chat window on the left showing messages from various users, including "Set found by Etamototor", "Set found by Etamototor", "Set found by Etamototor", "Set found by Etamototor", "Set found by Etamototor", "Set found by Etamototor", "Set found by Etamototor", and "Set found by Etamototor".
- Game Board:** A 3x3 grid of cards. The cards are: Row 1: Purple 2, Red 3, Green 1; Row 2: Purple 4, Purple 5, Purple 6; Row 3: Purple 7, Red 8, Purple 9.
- Scoreboard:** A scoreboard showing a timer of 03:02, a score of 1346, and a player named Etamototor with a score of 15.
- Buttons:** "Type a message..." and "24 cards remaining in the deck" are visible at the bottom.

Contact info



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- LinkedIn: [linkedin.com/in/drannequinn/](https://www.linkedin.com/in/drannequinn/)
- Article on Google Scholar: "Anne Quinn Edinboro"
- Find 1999 Mathematics Teacher article in the Skill Connections/ Teacher's Corner of: www.setgame.com