Superfactorials and Perfect Squares

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University of Colorado **Denver**

Implementation Settings

- At least 7 times at various Math Teachers' Circles

  » Rocky Mountain MTC

  » Southwest Colorado MTC

  » Emory MTC

  » Heartland MTC (twice)

  » East Texas MTC

- Participants

  » Elementary Teachers through High School Teachers

  » Higher Education Faculty

- Sessions of 1.5 to 5 hours (latter over multiple days)

Problem

- Can you remove one item from the list 1!, 2!, 3!, … , 98!, 99!, 100! so that the product of the remaining terms is a perfect square?

- Source: Jeremy Kun (Cal Poly Puzzles, 2011)

- See: mathteacherscircle.org – write-up by Estelle Basor

Spoiler Alert/Trigger Warning

The next slide violates the Math Circle pledge and will violate your Math Circle rights not to have the answer be told to you!

Out of sensitivity for those who do not want to be permanently damaged mathematically by having their own sense of inquiry violated, I will not read or say the next slide aloud. Feel free to take photos or enjoy 1 minute of silence!

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Slick Solution

sf(100) = 1!·2!·3!·…·100!

= (1!)^2 ·2·(3!)^2·4·(5!)^26·…·(99!)^2·100 = (2·4·6·…·100) ·(1!)^2 ·(3!)^2 ·(5!)^2 ·…·(99!)^2 = 50!·2^50 ·(1!)^2 ·(3!)^2 ·(5!)^2 ·…·(99!)^2 = 50! · (perfect square)

Generalization: Works for any multiple of 4 – remove half the largest number…

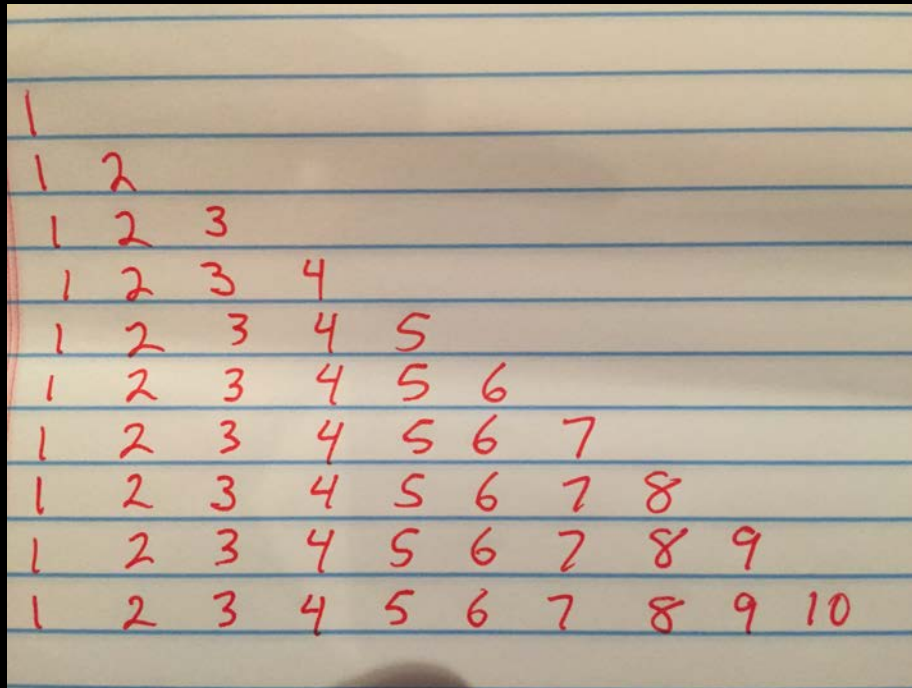University of Colorado **Denver**

What do Teachers Do?
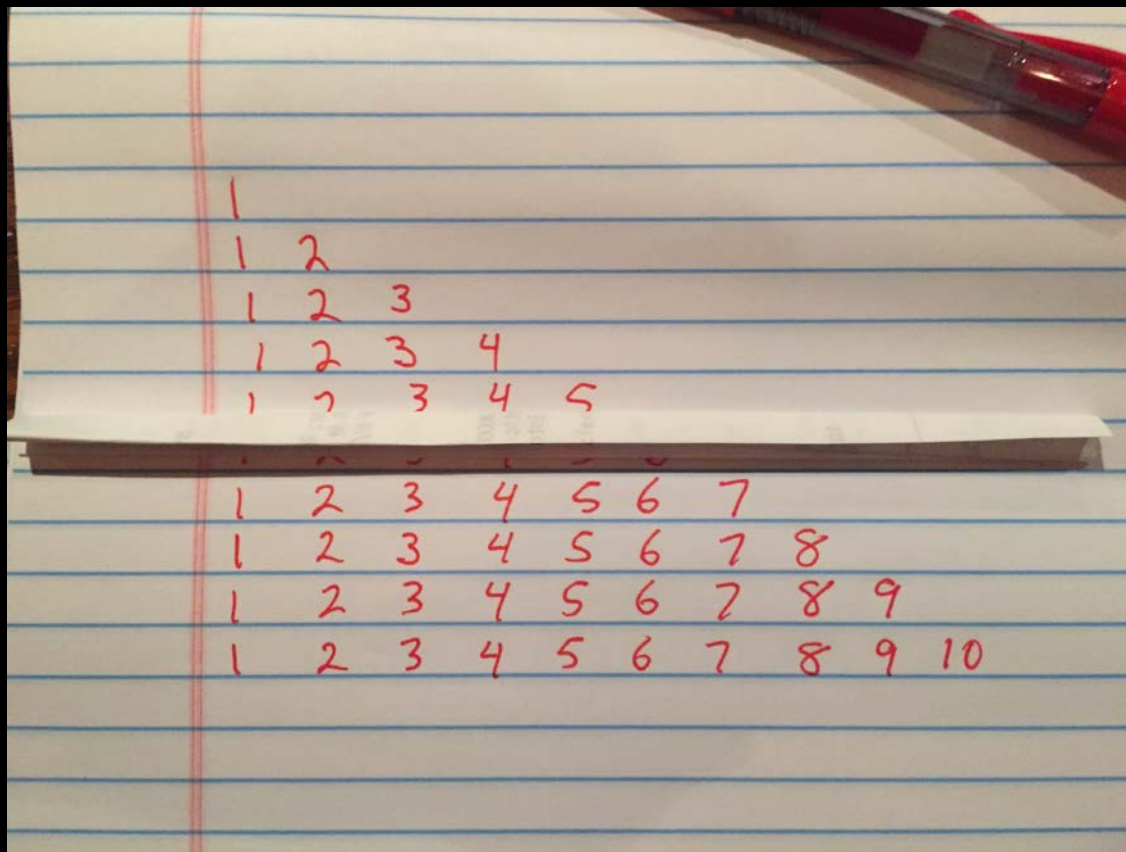
- Calculator – ACK!

  » Brings up eventual discussion of structure

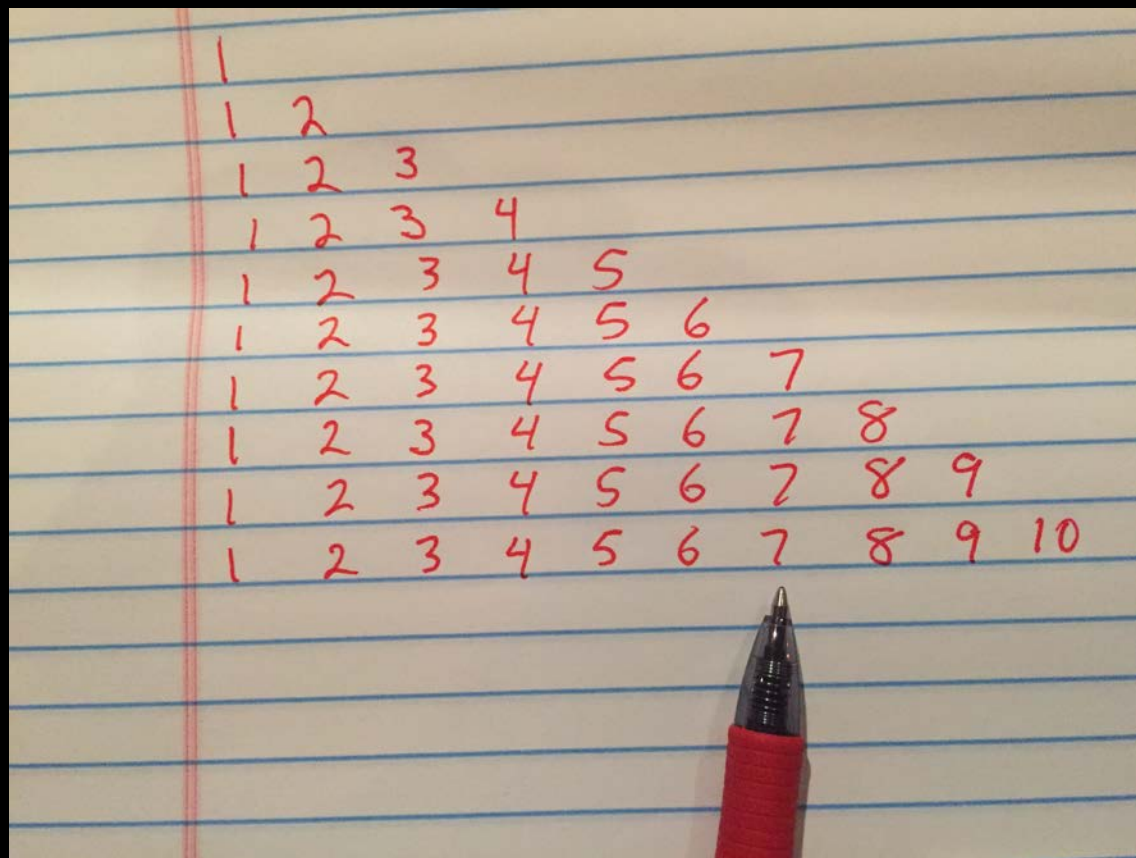- Ask a simpler problem – often up to 10!
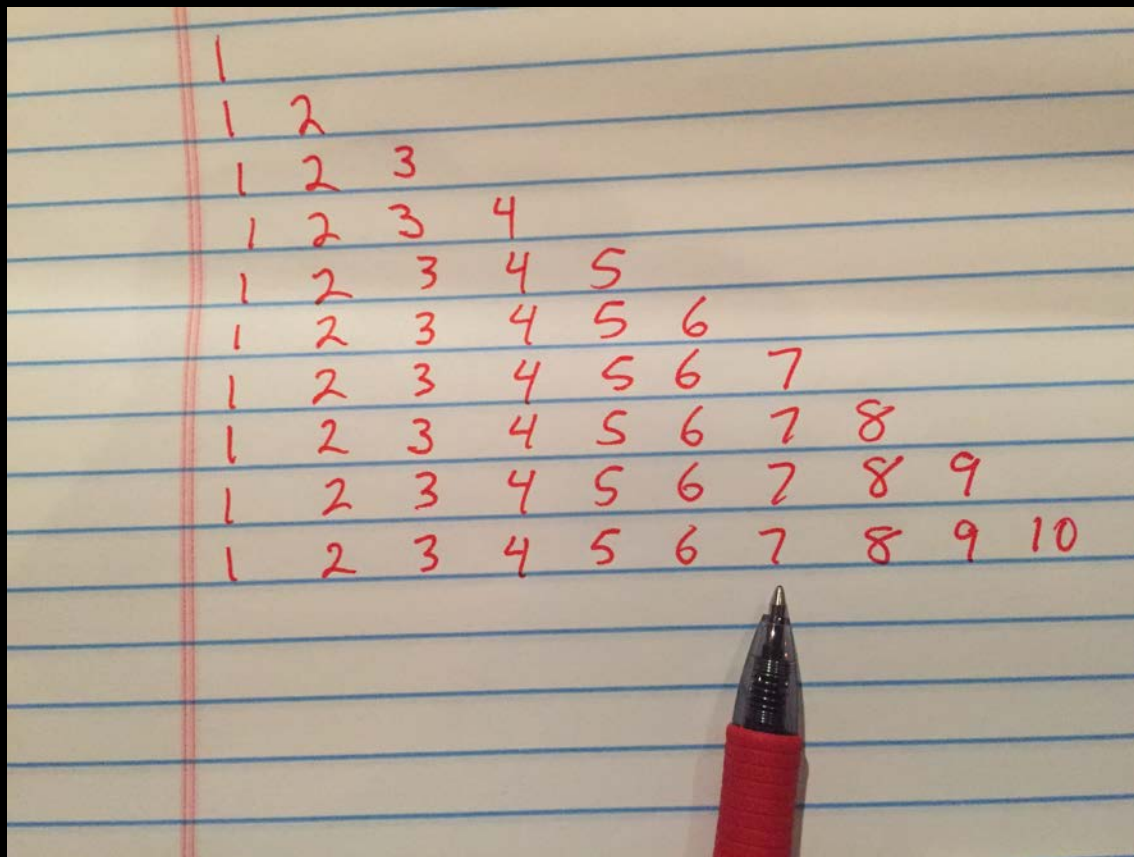
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Participant Approaches

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Emergent Themes

- Structure of numbers, prime factorization, primes

- Product of perfect squares is a perfect square

- Gather data and try to generalize

- Problem posing

- It's ok to leave things open ended and not come to a conclusion – partial results are common in mathematics!

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Questions?

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