

3. Let's increase to two nails! Assume two nails are nailed into a wall and lined up horizontally. Draw two (creative) examples of wrapping the string around two nails where the picture hangs when you let go.

4. From your two examples in problem 3., what happens if you remove the left nail? Does the picture still hang?

5. From your two examples in problem 3., what happens if you remove the right nail? Does the picture still hang?

Picture Hanging.... With Yarn!

Get in groups of 3 – 4. You need two people to be nails and at least one picture hanger. This role can be rotating.

1. Find a picture hang (the picture initially hangs) on two nails that satisfies: (each is a separate picture hang)
 - a. If the left nail is removed first, then the picture falls
 - b. If the left nail is removed first, then the picture falls BUT the picture still hangs if the right nail is removed first
 - c. If the right nail is removed first, then the picture falls
 - d. If the right nail is removed first, then the picture falls BUT the picture still hangs if the left nail is removed first.
 - e. The picture falls if either the left or right nail is removed first.

2. Find a picture hang on three nails on three nails that satisfies: (each is a separate picture hang)
 - a. Removing the left and middle nail makes the picture fall, but removing any one nail leaves the picture hanging
 - b. Removing the left and right nail makes the picture fall, but removing any one nail leaves the picture hanging
 - c. Removing the middle and right nail makes the picture fall, but removing any one nail leaves the picture hanging
 - d. (Challenge) Removing every combination of two nails makes the picture fall, but removing any one nail leaves the picture hanging

3. Find a picture hang on three nails that satisfies ALL of the following at once
 - a. Removing the left nail makes the picture fall
 - b. Removing the middle and right nail (in any order) makes the picture fall
 - c. The picture still hangs after removing the middle nail first
 - d. The picture still hangs after removing the right nail first

4. (Challenge) Find a picture hang on three nails that satisfies ALL of the following at once
 - a. Removing the left nail makes the picture fall
 - b. Removing the middle nail makes the picture fall
 - c. Removing the right nail makes the picture fall

Free Group Notation Worksheet

These following symbols will be our elements: 1,a,b,c

We will multiply these elements together to make a word. Some examples:

- $abbca1bbacba$
- $aaaaaa$
- bc
- 1

Each element will have a multiplicative inverse denoted by a “hat”: $\hat{a}, \hat{b}, \hat{c}$
(The multiplicative inverse of 1 will be 1)

Example: $1bb\hat{a}c\hat{b}ba1$

What do we mean by “multiplicative inverse”

$$a\hat{a} = \hat{a}a = 1$$

$$b\hat{b} = \hat{b}b = 1$$

$$c\hat{c} = \hat{c}c = 1$$

Other rules:

- Any element multiplied by 1 is equal to itself (including 1 times 1 is 1)
- Multiplication is associative (For example $(ab)c = a(bc)$)
- 1 commutes with other symbols and an element commutes with its inverse, but the other symbols do not commute! (For example $1a = a1$, but $ab \neq ba$)

Simplify the following words as much as possible:

- $ab\hat{b}ca\hat{c}1cbaa1b1$

- $a\hat{a}ab\hat{b}bc\hat{c}c$

- $a\hat{b}c\hat{c}b\hat{a}$

Free Groups and Picture Hanging

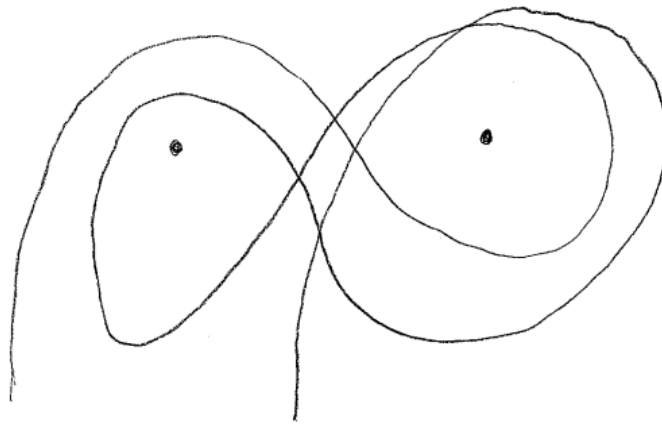
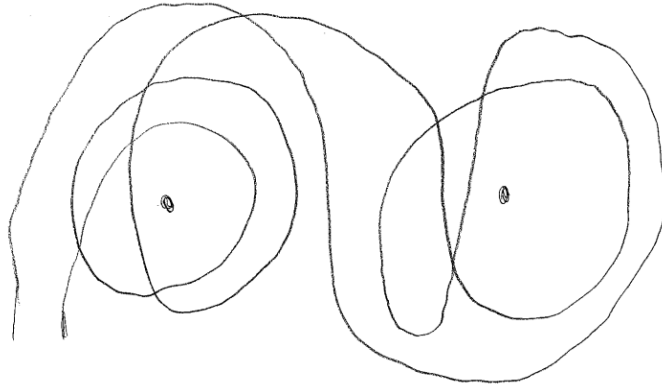
1. For the following words draw the associated picture hang.

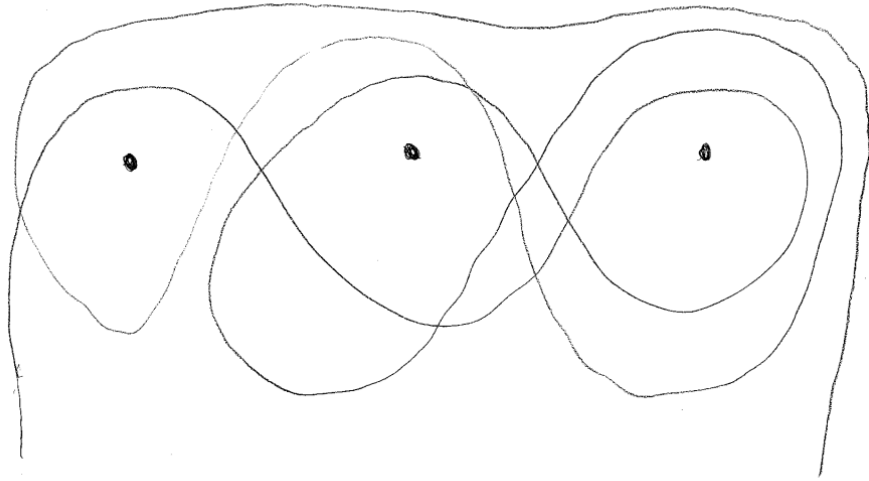
- $aabb\hat{a}b\hat{a}b$

- $\hat{a}b\hat{a}\hat{b}a\hat{a}b\hat{a}\hat{b}a\hat{a}$

- $abc\hat{a}\hat{b}\hat{c}bca$

2. For the following picture hang diagrams, find the associated word





3. If we remove a nail, all instances of the associated symbol and its inverse are removed from the word. For the word

$aab\hat{a}bb\hat{a}\hat{a}bba$

a) Remove the “a” nail and simplify as much as possible.

b) Remove the “b” nail and simplify as much as possible.

4. For the word

âbcâcabâcâabâcba

- a) Remove the “a” nail and simplify as much as possible
- b) Remove the “b” nail and simplify as much as possible.
- c) Remove the “c” nail and simplify as much as possible.
- d) Remove the “a” nail first, then the “c” nail. Simplify as much as possible.

5. If the word simplifies to 1, the picture falls. Otherwise, the picture hangs.
Does removing the “a” nail leave the picture hanging or does it make the picture fall?

a) $a\hat{b}\hat{a}baabb\hat{a}$

b) $a\hat{b}\hat{a}b\hat{a}b\hat{a}b\hat{a}$

c) $aba\hat{c}aca\hat{b}a$

6. From the **Picture Hanging.... With Yarn** worksheet, do the following:

Find a word that solves each problem, verify that your word solves the problem, and check with your group that your word works physically by performing that picture hang/nail removal with the yarn.