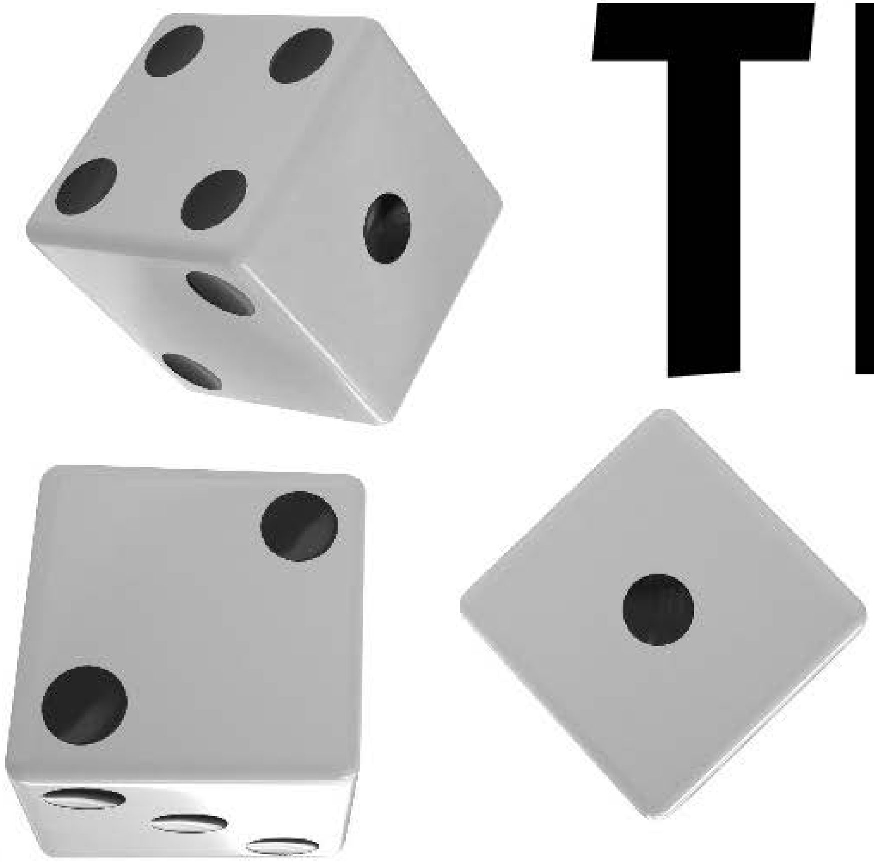
How to win at

TENZI



A (not so) simple game of dice

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How to Play Tenzi

Choose a **target number** before you begin

Roll all 10 dice

Set aside your matches

Roll the rest of your dice

Repeat until all 10 dice match

Yell out “TENZI”!

Questions to Consider

How much time is spent at each stage of the game?

How many rolls does it take (on average) for a player to get Tenzi?

If there are n players, how many rolls would it take (on average) for someone to win?

When one player wins, how many of the remaining players will have just one die left?

If you were to play a relative speed x faster than your opponent, how likely are you to win the game?

Markov Chain

States: Number of dice that match the target number

$$T_{0,1} = B(10,1) = \binom{10}{1}\left(\frac{1}{6}\right)^{1}\left(\frac{5}{6}\right)^{9} \approx 0.3230$$

$$T_{6,10} = B(4,4) = \binom{4}{4}\left(\frac{1}{6}\right)^{4}\left(\frac{5}{6}\right)^{0} \approx 0.0008$$

$$T_{ij} = B(10-i, j-i) = \binom{10-i}{j-i}\left(\frac{1}{6}\right)^{j-i}\left(\frac{5}{6}\right)^{10-j}$$

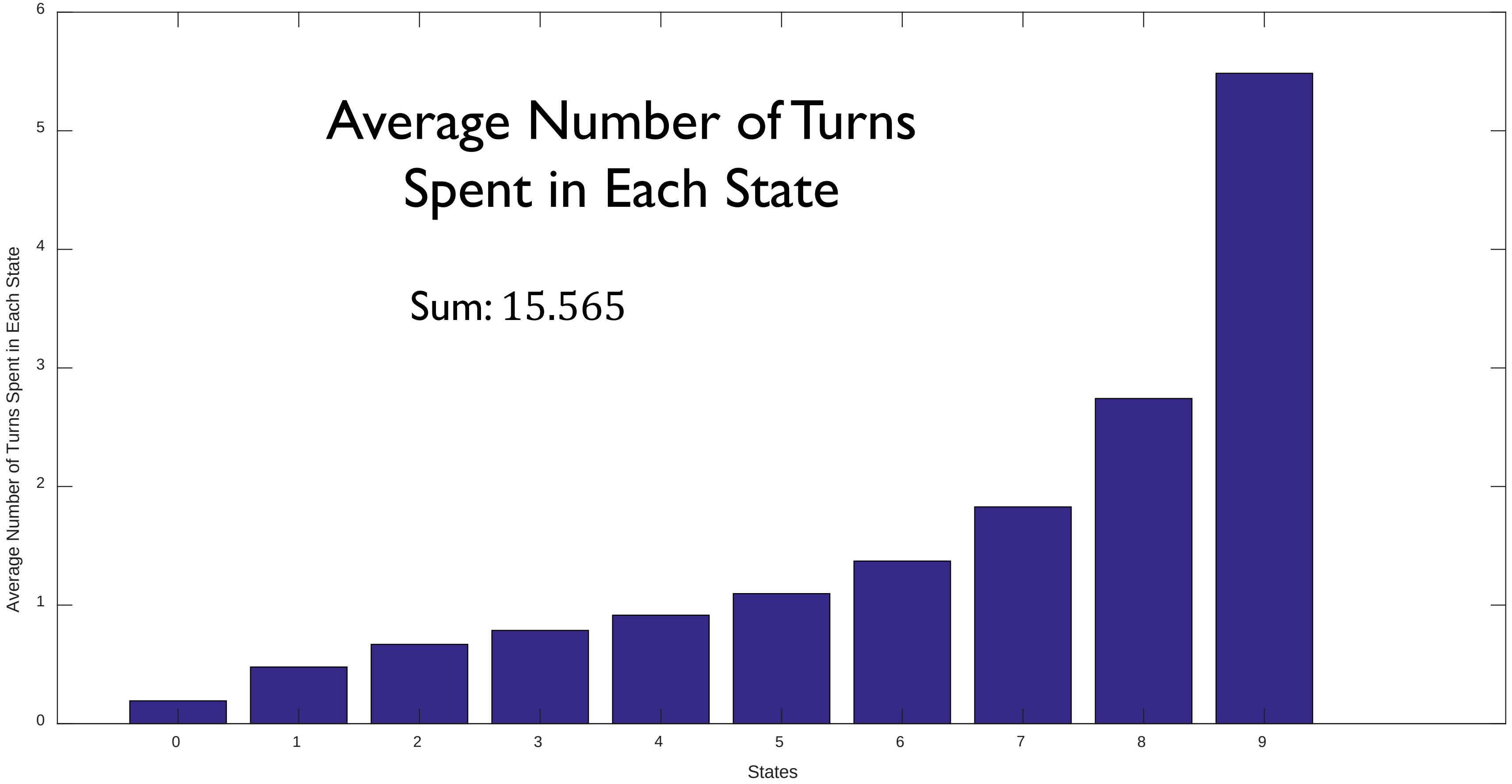
for $0 \leq i, j \leq 10$ and $i \leq j$

T	0	1	2	3	4	5	6	7	8	9	10
0	0.1615	0.3230	0.2907	0.1550	0.0543	0.0130	0.0022	0.0002	0.0000	0.0000	0.0000
1	0.0000	0.1938	0.3489	0.2791	0.1302	0.0391	0.0078	0.0010	0.0001	0.0000	0.0000
2	0.0000	0.0000	0.2326	0.3721	0.2605	0.1042	0.0260	0.0042	0.0004	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.2791	0.3907	0.2344	0.0781	0.0156	0.0019	0.0001	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.3349	0.4019	0.2009	0.0536	0.0080	0.0006	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.4019	0.4019	0.1608	0.0322	0.0032	0.0001
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4823	0.3858	0.1157	0.0154	0.0008
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5787	0.3472	0.0694	0.0046
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6944	0.2778	0.0278
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8333	0.1667
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

Fundamental Matrix

$$T = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}$$

$$F = \sum_{k=0}^{\infty} Q^k = (I - Q)^{-1}$$

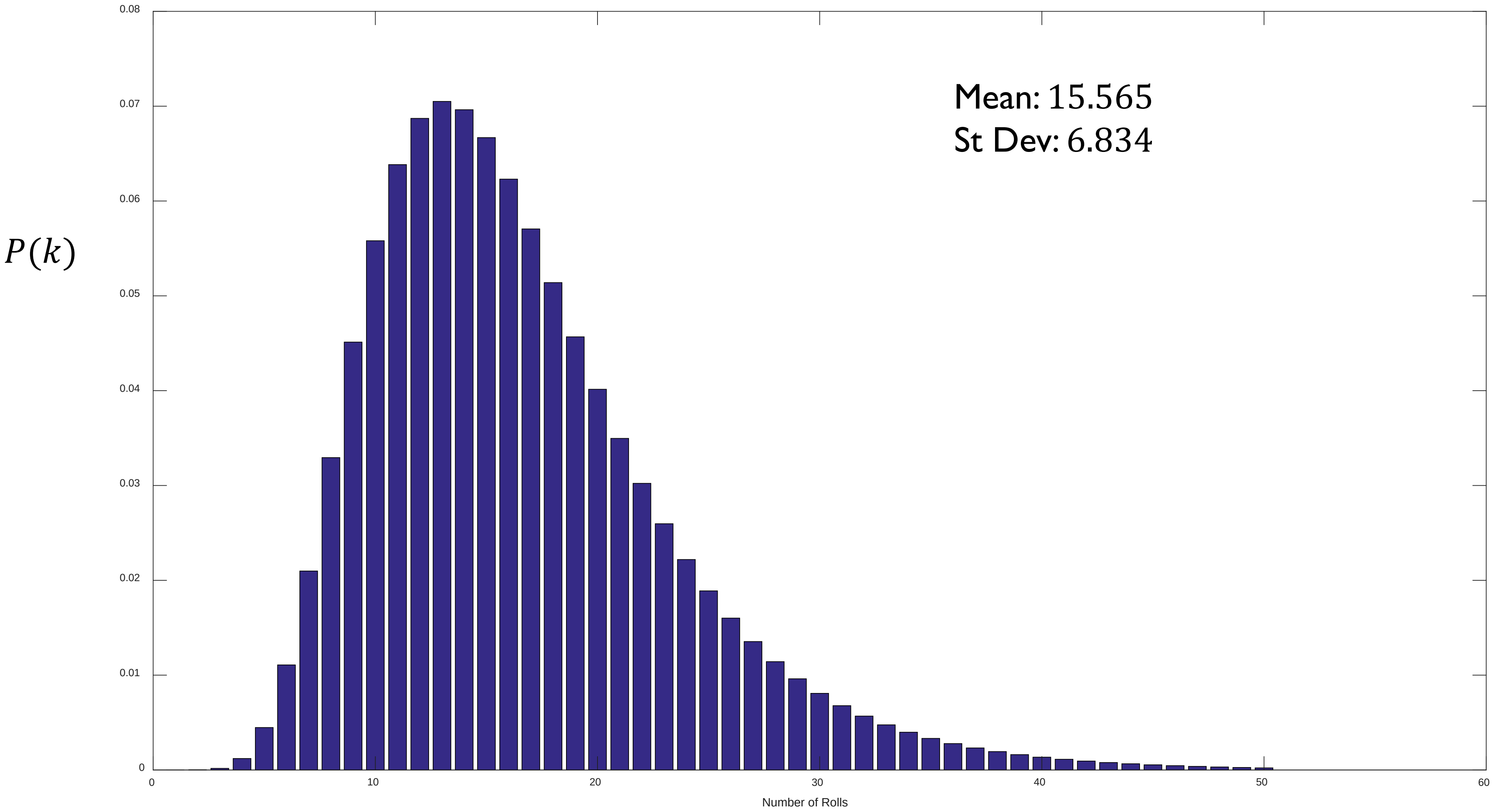


Probability of being in each state after *k* rolls

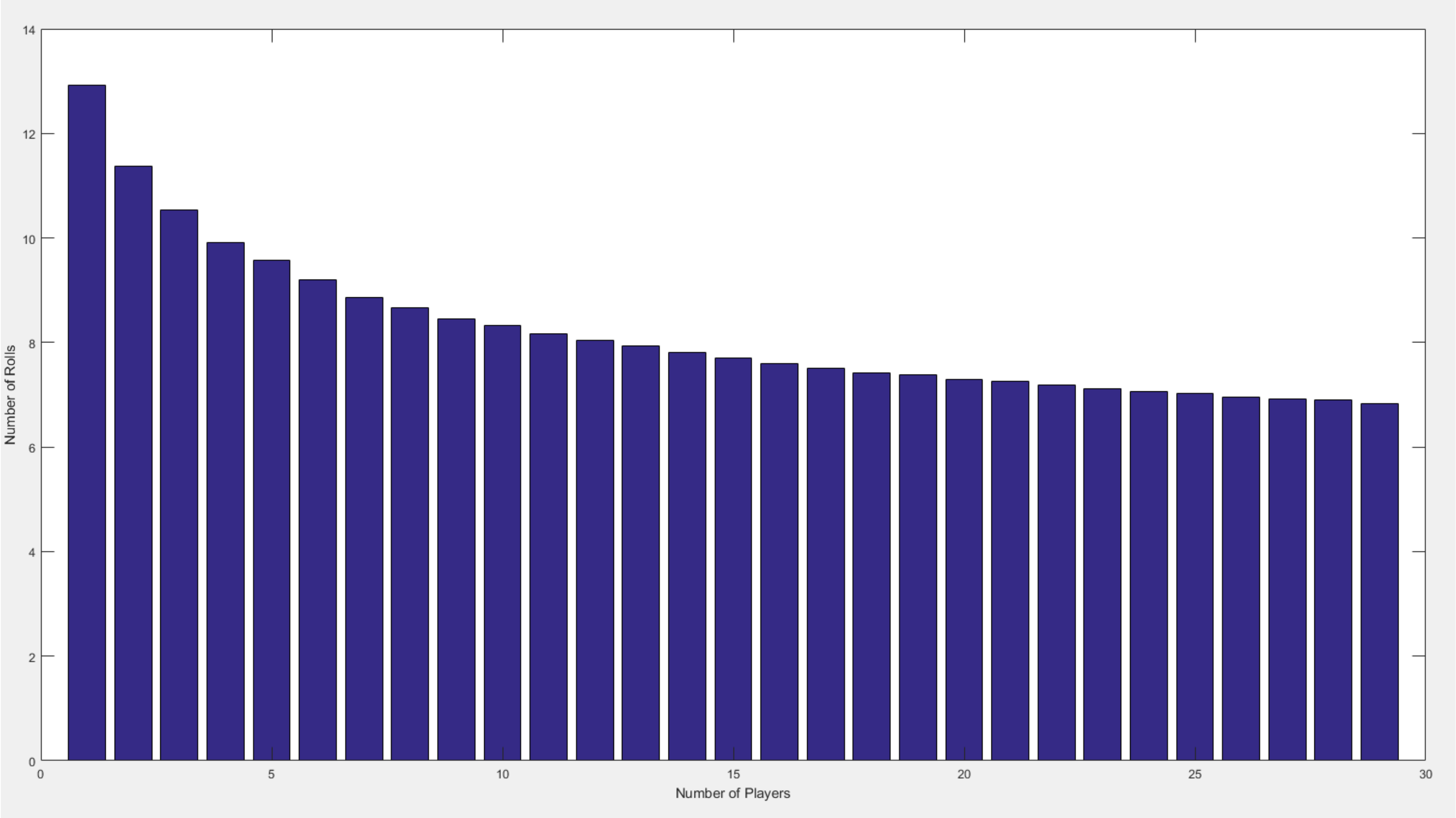
$$S_k = S_0 \times T^k$$

		0	1	2	3	4	5	6	7	8	9	10	Probability of winning in exactly *k* rolls
0 Rolls	$S_0 =$	1	0	0	0	0	0	0	0	0	0	0	
1 Roll	$S_1 =$	0.1615	0.3230	0.2907	0.1550	0.0543	0.0130	0.0022	0.0002	0.0000	0.0000	0.0000	
⋮													
4 Rolls	$S_4 =$	0.0007	0.0073	0.0353	0.1010	0.1898	0.2445	0.2188	0.1342	0.0540	0.0129	0.0014	
5 Rolls	$S_5 =$	0.0001	0.0016	0.0110	0.0435	0.1132	0.2022	0.2508	0.2133	0.1190	0.0394	0.0059	0.0045
⋮													
19 Rolls	$S_{19} =$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0029	0.0342	0.2351	0.7276	
20 Rolls	$S_{20} =$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0018	0.0248	0.2056	0.7677	0.0401

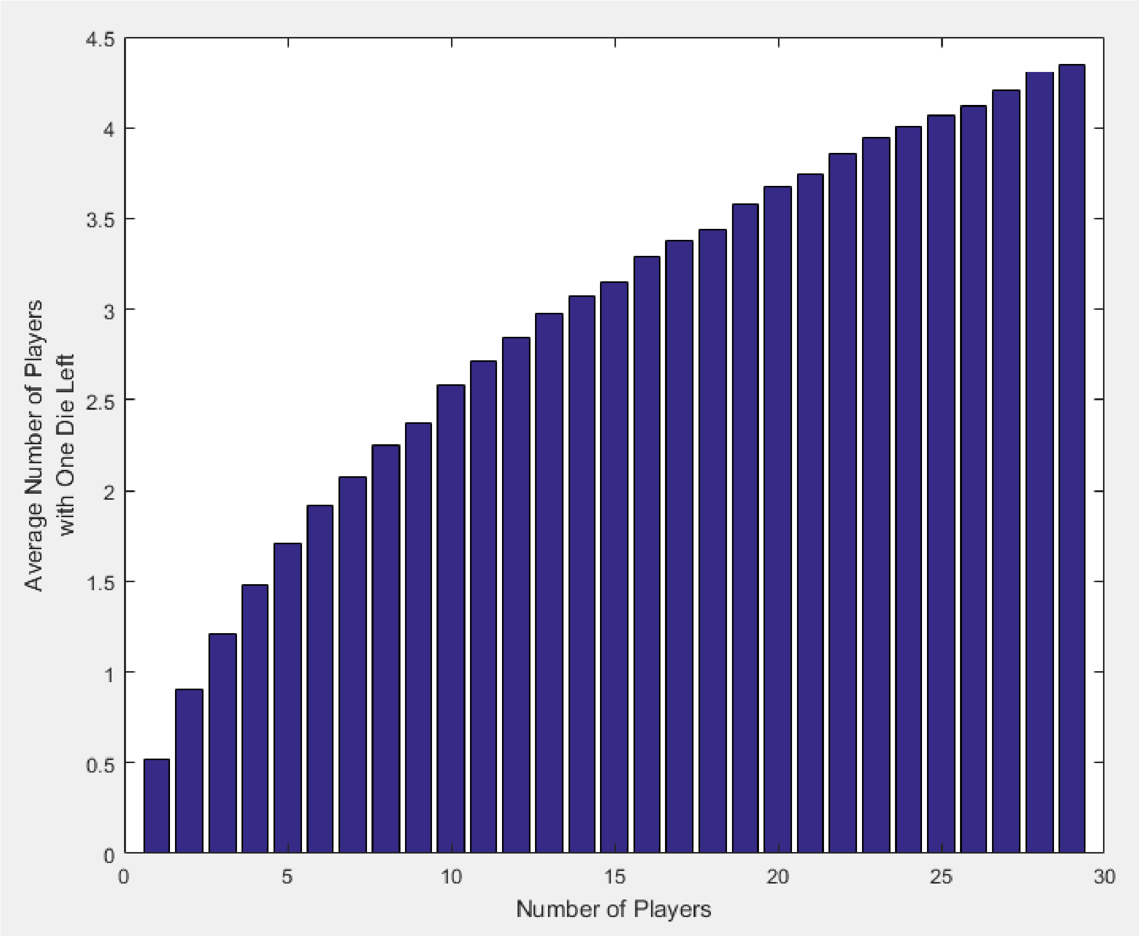
Probability of Winning in Exactly k Rolls

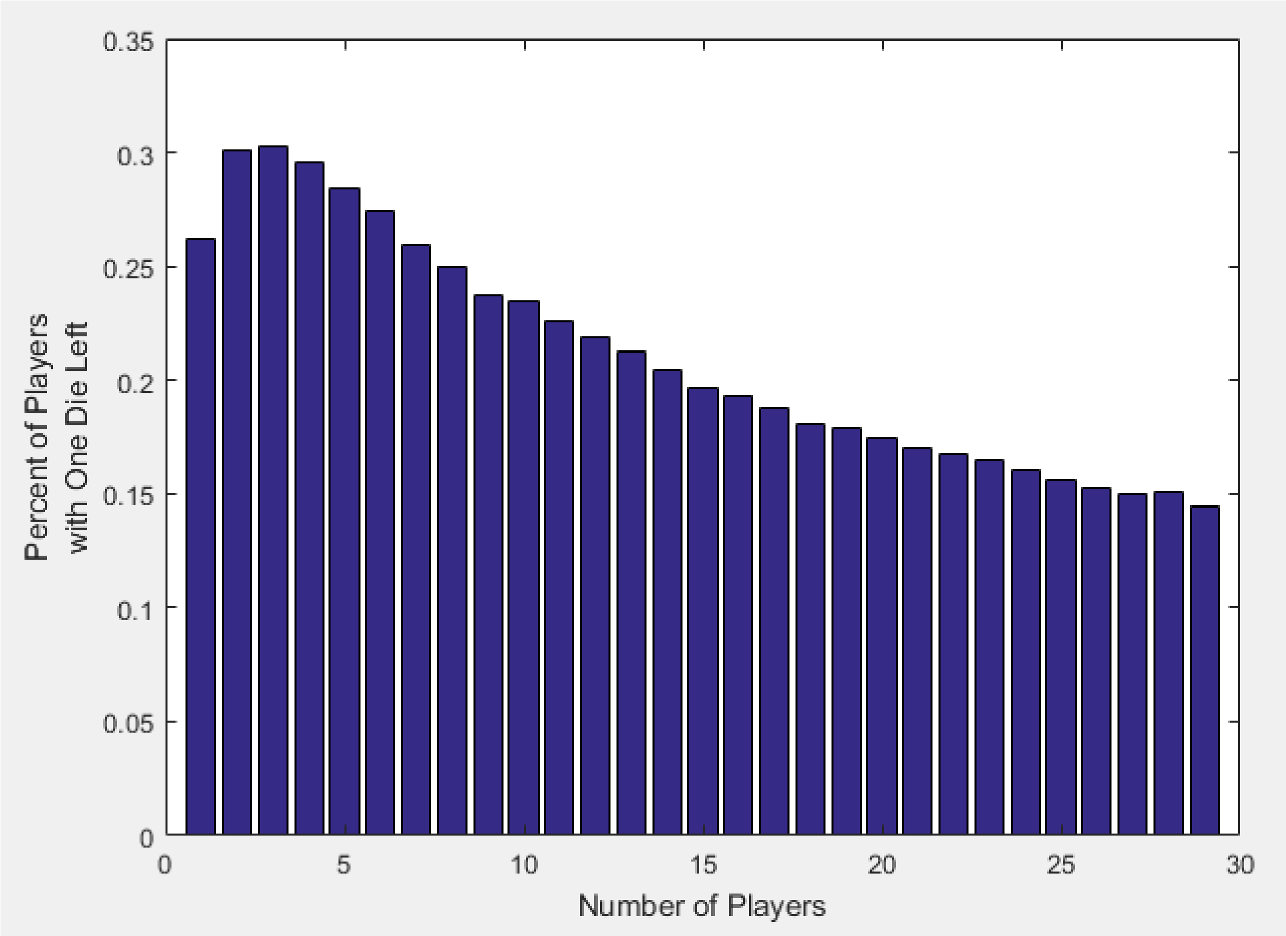


How many rolls would it take (on average) for someone to win in a game with n players?



When one player wins,
how many of the remaining players will have just one die left?



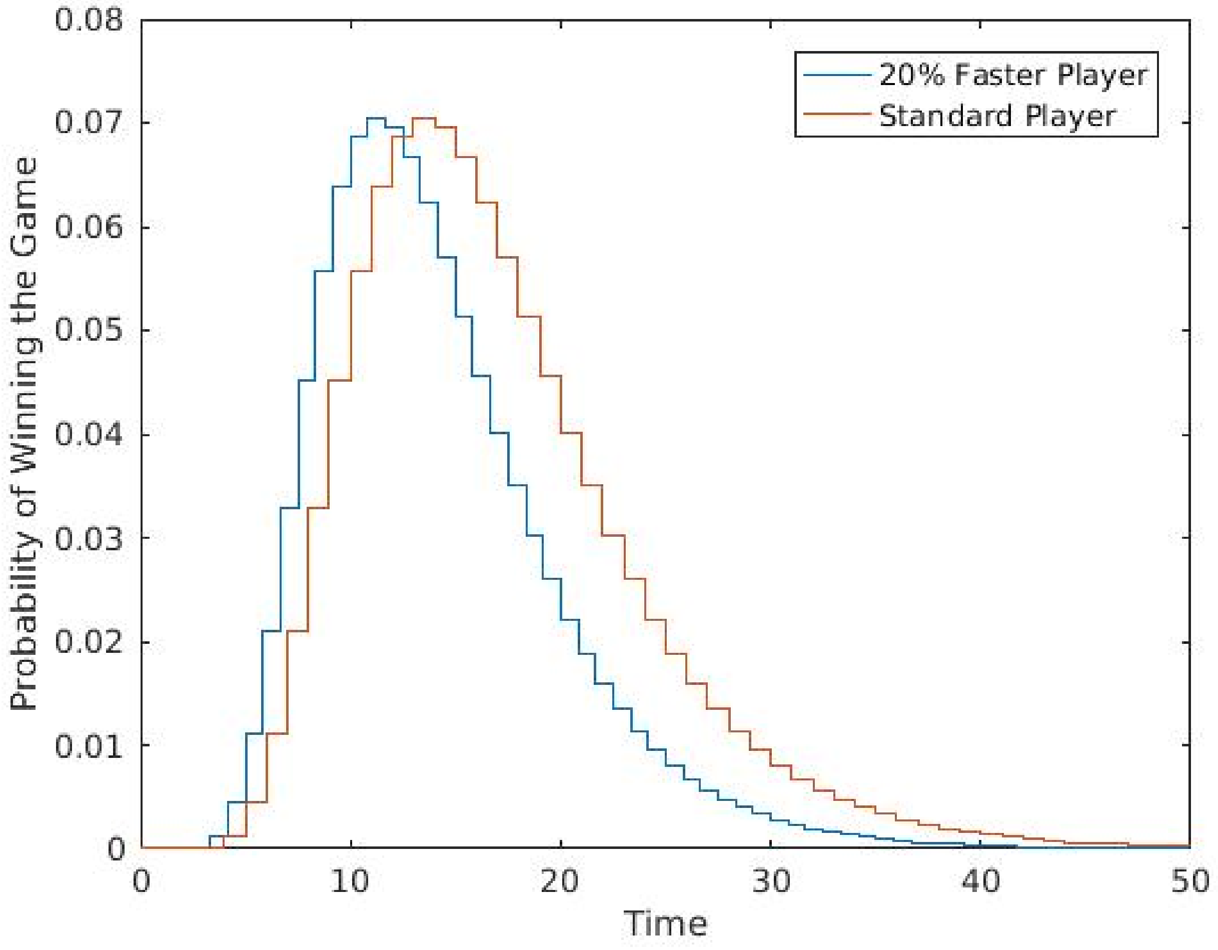


So… how do you win at Tenzi?

Be luckier than your opponents,

or

play *FASTER* than your opponents.



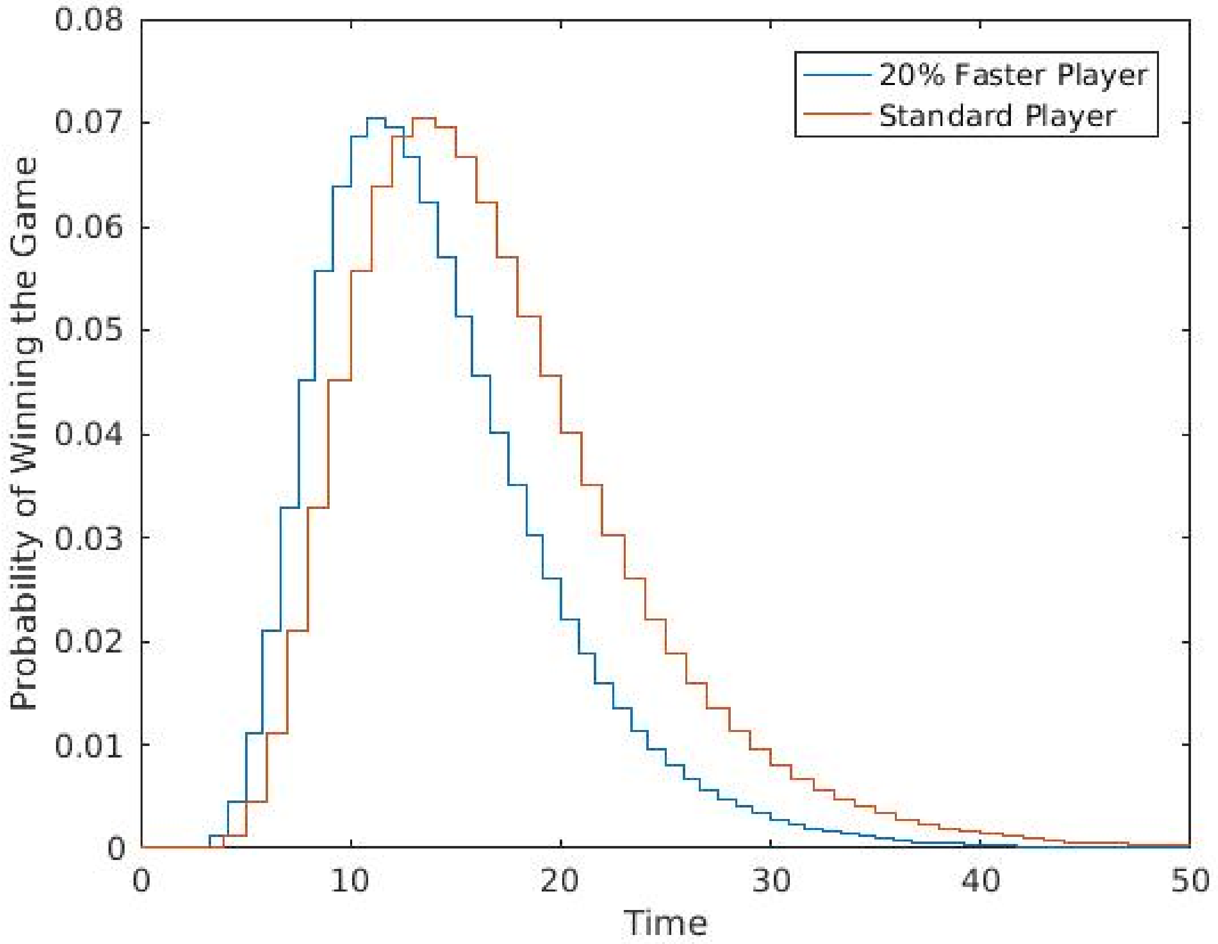
What is your advantage when playing speed x faster than your opponent?

The probability that you finish on roll k, and your opponent hasn't already won

$$P(x) = \sum_{k=1} \left(P(k) \cdot \left(1 - \sum_{h=1}^{\left\lfloor \frac{k}{x} \right\rfloor} P(h) \right) \right)$$

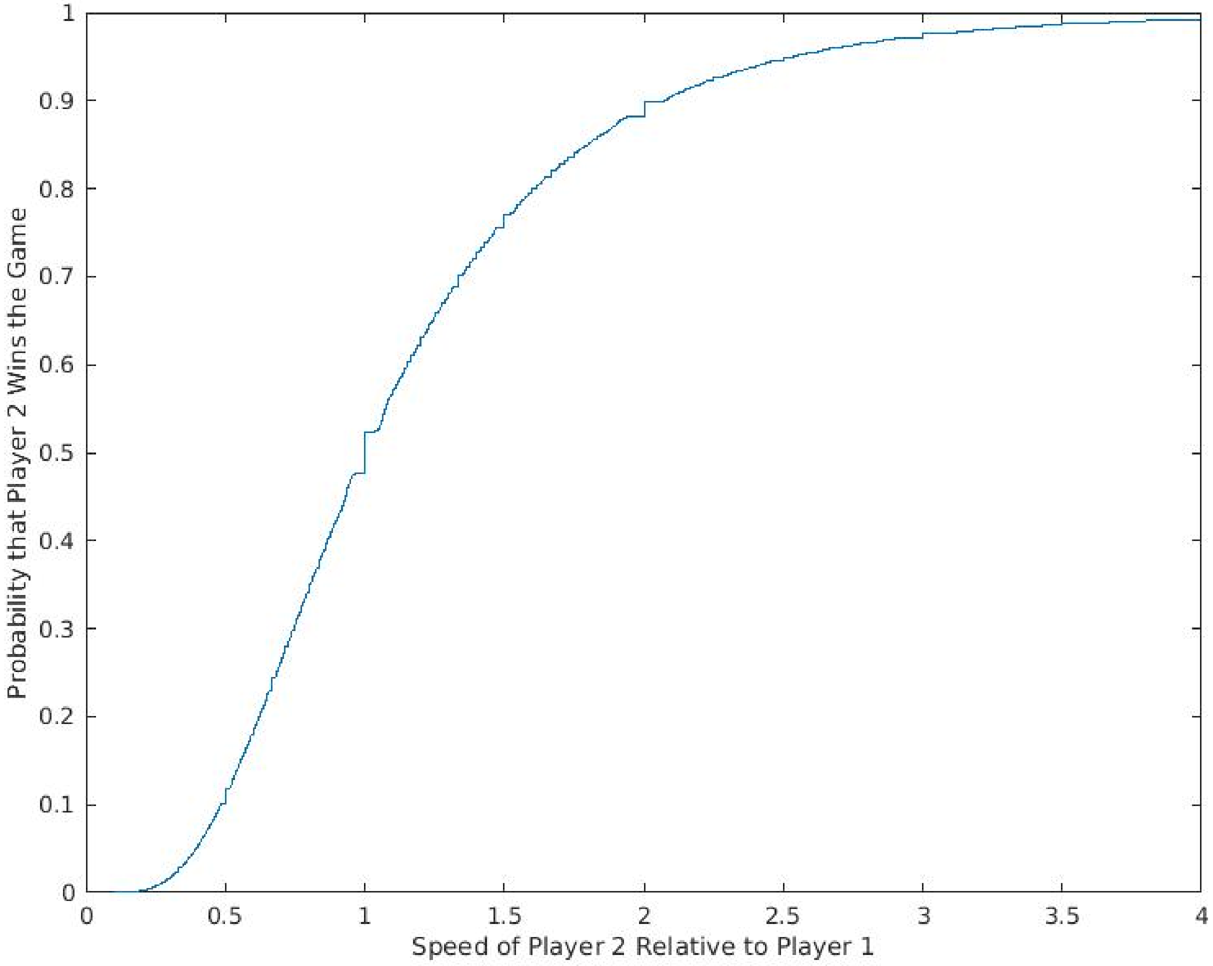
You finish on roll k

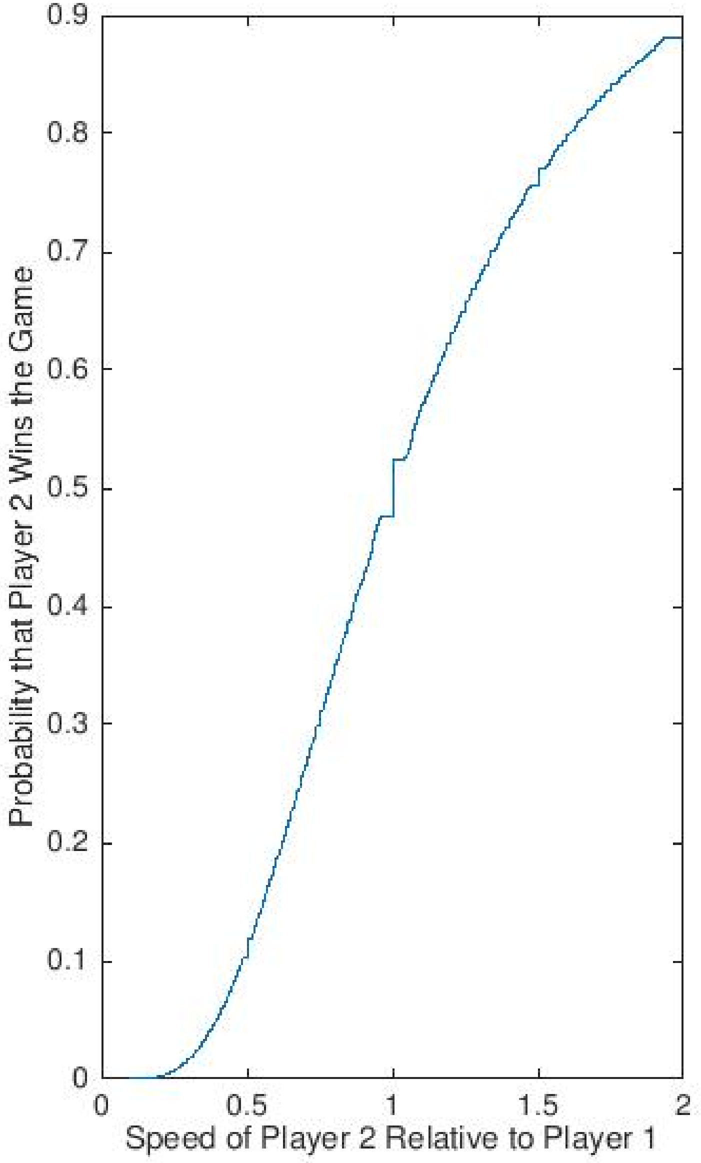
Your opponent hasn't already won

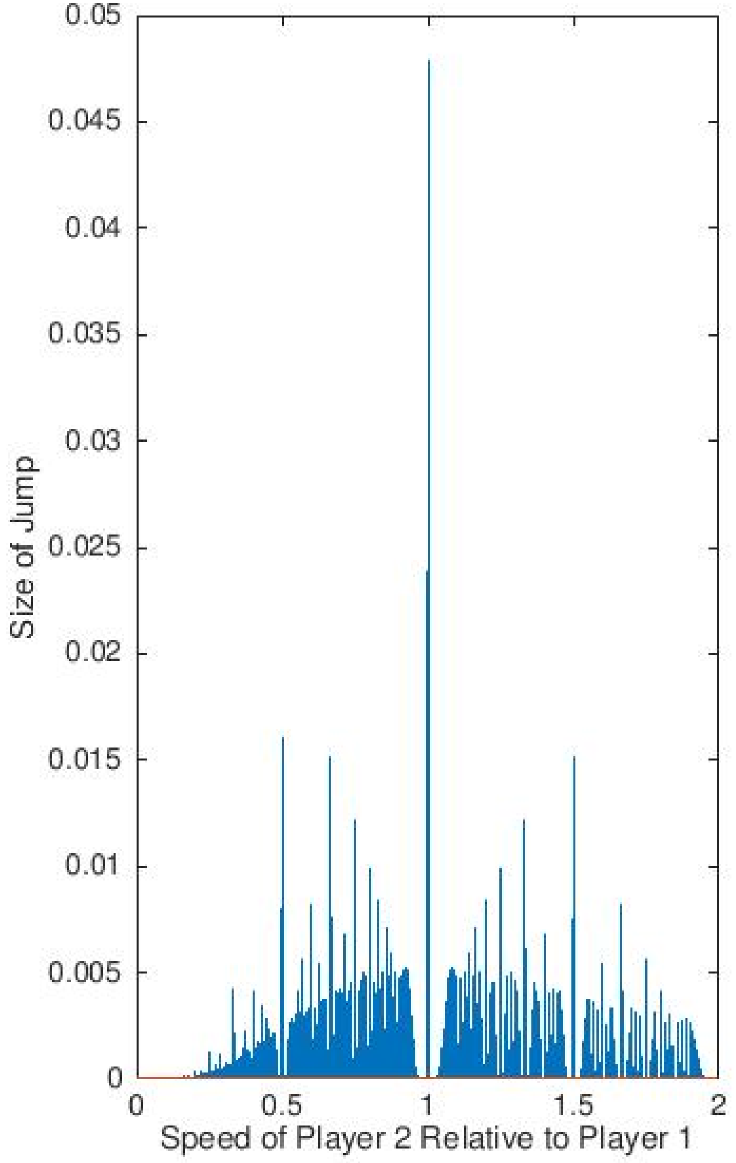


Advantage when playing speed x faster than your opponent

$$P(x) = \sum_{k=1}\left(P(k)\cdot\left(1-\sum_{h=1}^{\left\lfloor \frac{k}{x}\right\rfloor}P(h)\right)\right)$$







The size of the jump at x is equal to the probability of a **TIE** playing at speed x.

Jumps occur at every **rational** value of x.

The function $P(x)$ is **discontinuous** at the rationals and **continuous** at the irrationals.

Questions/Comments?

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