Projective Geometry Hidden Inside:

Can You Spot It?

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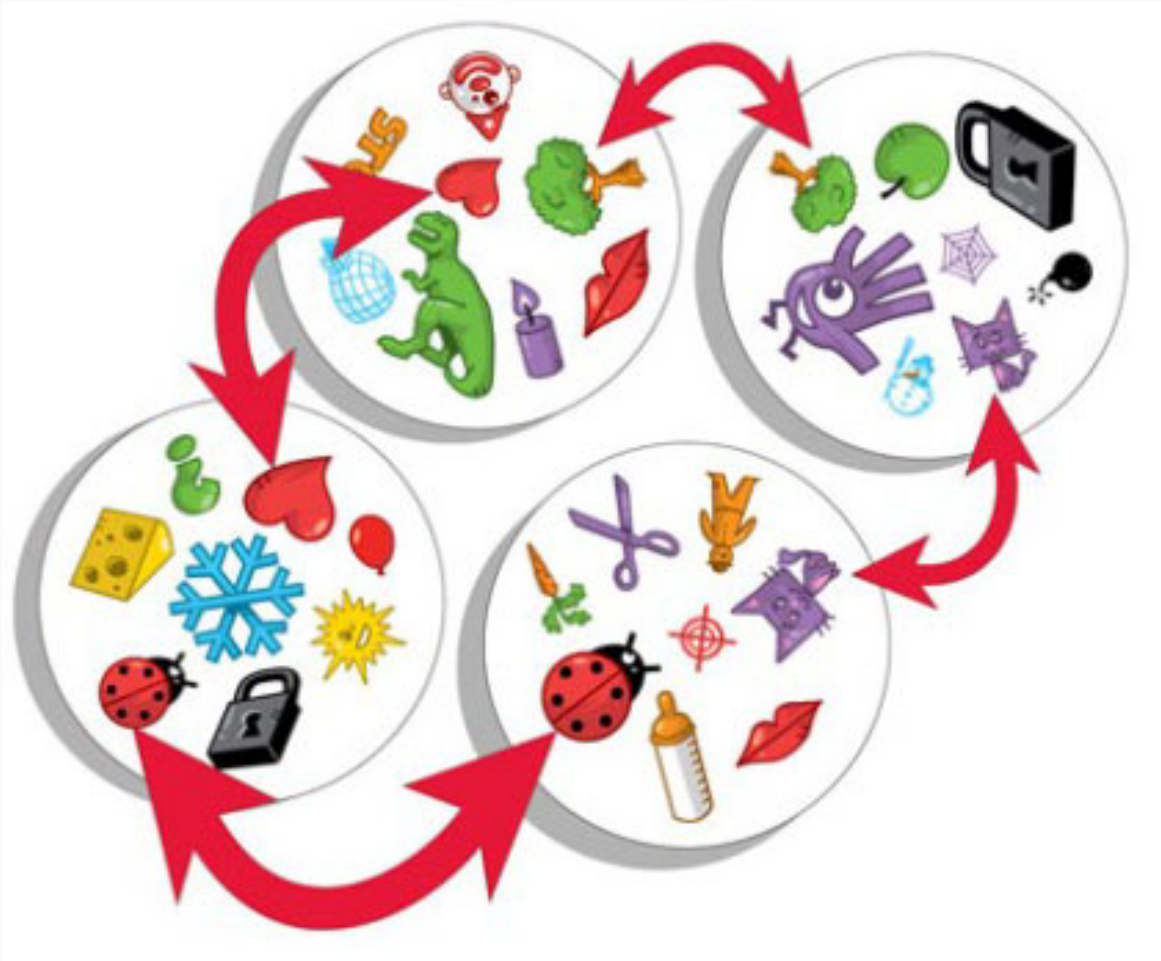
an introduction to spot it!

What is Spot It!^{TM}?

Basic Idea

- Every card has exactly one match with every other card.
- No “filler” symbols.



My Spot It!^{TM} Story

- Thanks to Cal Jongsma for bringing the problem to my attention.

Curiosity

Question

What mathematical questions come to mind that you might ask about this game?

- How many cards are in the deck? Could there be more/less?
- How many symbols are on each card? Is it the same for each card?
- How many total symbols are used? Could there be more/less?
- How many times does each symbol appear? Is it the same for each symbol?
- How many possible "decks" can one make?
- How hard is it to make a Spot It!^{TM} deck of a particular size/type?

possible mathematical directions

Build Your Own Game

Problem Solving Theme: Special Cases / Simplify Problem / Patterns

- Attempt to answer the questions by solving the case with 1, 2, 3, and 4 symbols per card.
- Look for numerical patterns in that case and attempt to generalize and/or look for explanations of those patterns e.g., $s + (s-1)^2 = s^2 - s + 1$ cards given s symbols per card.
- Analyze the Spot It!^{TM} and Spot It! Junior^{TM} games and look for patterns.

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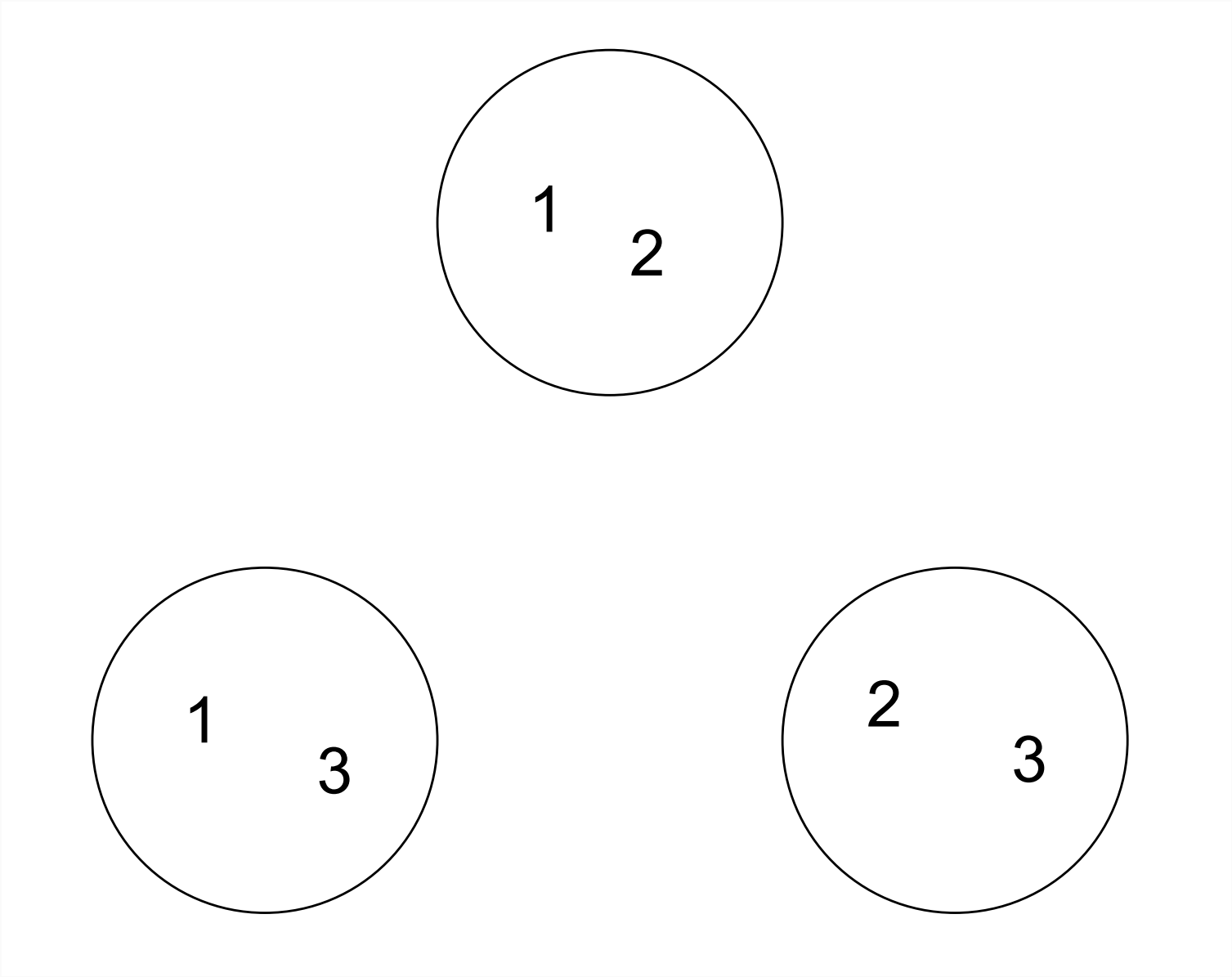
When I ran this session at my Math Teachers' Circle, this is the direction I started with. I pivoted to projective geometry when we hit questions we couldn't answer and answers we couldn't explain.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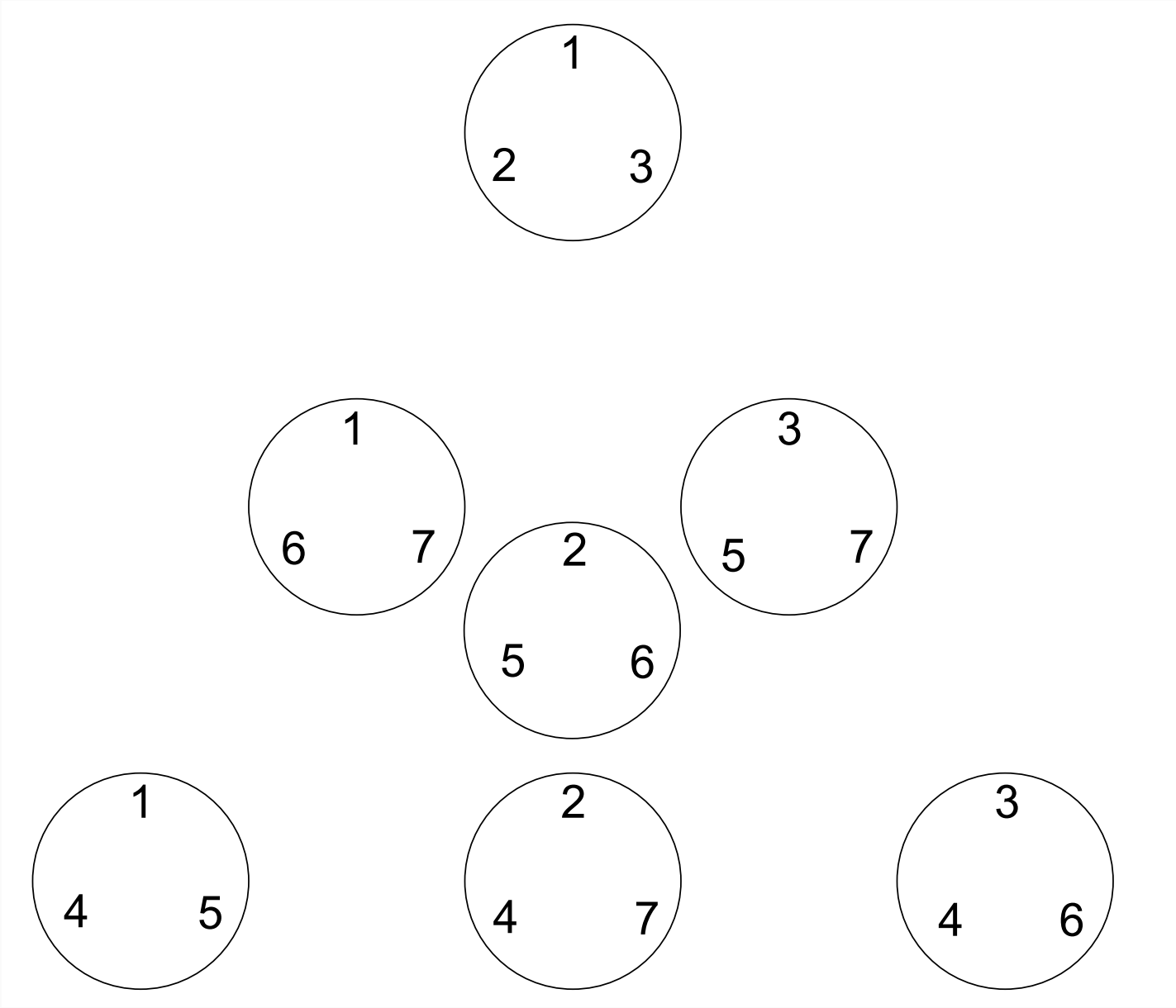
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Game Rules as Mathematical Axioms

Problem Solving Theme: Modeling / Using Precision

Question

What makes a good game? What makes Spot It! fun?

- Given any two cards, there is always a match between them. (Function)
- Each card has same number of symbols. (Fairness)
- Symbols are equally "popular". (Fairness)

Projective Geometry Axioms

- Any two points are on exactly one line
- Any two lines have exactly one point in common.
- Four points must exist such that no three of them are on the same line.

Question

How do these axioms relate to Spot It!^{TM}?

Spot It!^{TM} Axioms

Definition

We define a **point** to be a Spot It!^{TM} card and a **line** to be a set of cards that contain a common symbol.

- Any two cards must must have exactly one symbol in common (are on a "line").
- For any pair of symbols, there is a card which contains those two symbols.
- All cards contain the same number of symbols.
- There is at least one card.
- Every card contains at least three symbols.
- Every symbol is on at least three cards.

Deeper Questions

Question

Can you make a Spot It!^{TM} deck with 7 symbols on every card?

Question

What about 9, 10, 11, 12, 13, ... symbols per card?

Secret Code

For those interested/skilled with computer programming there are some interesting directions to investigate Spot It!^{TM}

- Write a program that generates a deck of Spot It!^{TM} cards with a given number of symbols per card.
- Count the number of possible Spot It!^{TM} "decks" that can be made with a particular bank of symbols.

wrapping up...

Thanks For Your Attention!

To see any of my materials contact me at Tom.Clark@dordt.edu.

References

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