Projective Geometry Hidden Inside:
Can You Spot It?

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an introduction to spot it!
What is Spot It!™?
Basic Idea

- Every card has exactly one match with every other card.
- No “filler” symbols.
Thanks to Cal Jongsma for bringing the problem to my attention.
What mathematical questions come to mind that you might ask about this game?

- How many cards are in the deck? Could there be more/less?
- How many symbols are on each card? Is it the same for each card?
- How many total symbols are used? Could there be more/less?
- How many times does each symbol appear? Is it the same for each symbol?
- How many possible “decks” can one make?
- How hard is it to make a Spot It!™ deck of a particular size/type?
possible mathematical directions
Problem Solving Theme: Special Cases / Simplify Problem / Patterns

- Attempt to answer the questions by solving the case with 1, 2, 3, and 4 symbols per card.
- Look for numerical patterns in that case and attempt to generalize and/or look for explanations of those patterns e.g., \( s + (s - 1)^2 = s^2 - s + 1 \) cards given \( s \) symbols per card.
- Analyze the Spot It!™ and Spot It! Junior™ games and look for patterns.
Build Your Own Game

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When I ran this session at my Math Teachers’ Circle, this is the direction I started with. I pivoted to projective geometry when we hit questions we couldn’t answer and answers we couldn’t explain.
Problem Solving Theme: Modeling / Using Precision

Question

What makes a good game? What makes Spot It! fun?

- Given any two cards, there is always a match between them. (Function)
- Each card has same number of symbols. (Fairness)
- Symbols are equally “popular”. (Fairness)
Any two points are on exactly one line
Any two lines have exactly one point in common.
Four points must exist such that no three of them are on the same line.

Question

How do these axioms relate to Spot It!™?
Spot It!™ Axioms

Definition

We define a **point** to be a Spot It!™ card and a **line** to be a set of cards that contain a common symbol.

- Any two cards must have exactly one symbol in common (are on a “line”).
- For any pair of symbols, there is a card which contains those two symbols.
- All cards contain the same number of symbols.
- There is at least one card.
- Every card contains at least three symbols.
- Every symbol is on at least three cards.
Deeper Questions

Question

Can you make a Spot It!™ deck with 7 symbols on every card?

Question

What about 9, 10, 11, 12, 13, ... symbols per card?
For those interested/skilled with computer programming there are some interesting directions to investigate Spot It!™

- Write a program that generates a deck of Spot It!™ cards with a given number of symbols per card.
- Count the number of possible Spot It!™ “decks” that can be made with a particular bank of symbols.
wrapping up...
Thanks For Your Attention!

To see any of my materials contact me at Tom.Clark@dordt.edu.

References