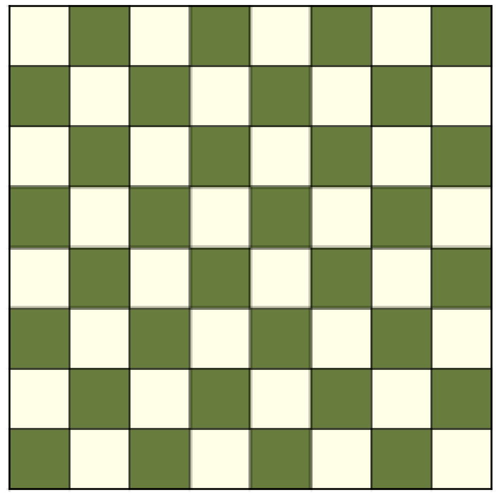
Julia Robinson Mathematics Festival

Directed By Joshua Zucker

Tiling Torment

The problem

There are many problems that involve tiling (covering) all the squares on a chessboard (or similar board) with tiles of various sizes. The chessboard may be 8x8, 7x7 or other sizes and may or may not have squares missing. The tiles can be dominoes (2x1) or tiles of other sizes.



Questions

The Basics

1. Is it possible to tile a 7x7 board with 2x1 tiles?
2. In general, is it possible to tile an nxn board with 2x1 tiles? If so, which boards can you tile and why?

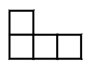
Taking it Further

3. Now consider the 7x7 board again. If you remove one square, is it possible to tile the board? If so, does it matter which square you remove? Describe completely.
4. In general, if n is odd, is it possible to tile an nxn board with 2x1 tiles if one square is covered with a 1x1 tile? Does it matter which square is covered?
5. Remove two diagonally opposite corners of a chessboard. Is it possible to tile this shape with 31 2x1 tiles?
6. In general, if n is even, is it possible to tile an nxn board with 2x1 tiles if two squares are removed? Does it matter which two squares are removed?
7. Is it possible to tile an 8x8 board with 21 “L-shaped” tiles of three squares and one 1x1 tile? If so, how? Describe all possible locations for the 1x1 tile. If not, why not?

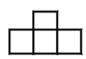
8. Is it possible to tile an 8x8 board with 21 3x1 tiles and one 1x1 tile? If so, how? Describe all possible arrangements. If not, why not?

Impossible Cases

9. Prove that an 8x8 chessboard cannot be covered without overlapping by fifteen 4x1 polyominoes and the single polyomino shown below:



10. Prove that a 10x10 board cannot be covered without overlapping by the polyominoes show below:



11. Prove that a 102 x 102 board cannot be covered without overlapping by 4x1 tiles.

12. Add your own problems, using different shaped tiles and different size boards.