Favorite problems from the UWM Math Circle

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UWM Math Circle

- Started in September 2011
- Three faculty members from UWM Math Department:

  Chris Hruska and Boris Okun

- Graduate and undergraduate student helpers
- Students in grades 5-12, two groups
- Small circle — open ended problem solving
- Weekly meetings

A few problems with a surprise

- Ping pong games

- More ping pong

- Athletics

- Chess

- Fishing

- Witch hunt

Ping-pong games

Three friends (A, B and C) are playing ping pong. They play the usual way: two play at a time, the winner stays on, and the loser waits his/her turn again. At the end of the day, they summarize the number of games that each of them played:

A played 10 games

B played 15 games

C played 17 games

Who lost the second game?

More ping-pong (A. Pickford, NYT Numberplay)

Dan and Avery love playing ping-pong. They love playing ping-pong so much that they devised a new rule to make games last longer. Scoring and play is normal, except that the score is "reduced" whenever possible. In other words, the scores are divided by the greatest common factor. So if Dan is ahead 7-4 and wins a point, instead of going to 8-4 the score becomes 2-1. Like in normal ping-pong, games go to 21. Note: If Avery is leading 20-7 and scores a point, he does not win. The score would go to 3-1.

What questions would you ask?

Athletics

Mr White, Mr Green and Mr Black took part in an athletic competition (they were the only participants). There were a few events during the competition and for every event the winner was awarded g points, the runner-up s points and the last competitor (i.e., a competitor who took the third place) b points. Of course, $g > s > b > 0$, and all these numbers were integers. There were no ties. Mr White's total for all the events was 22 points and the totals of Mr Green and Mr Black were the same at 9 points each. Mr Green won the long jump. Who was second in the 400 meter race?

Chess

Some of the students in grade 11 organized a chess tournament. Everyone played with everyone else exactly once. The results were interesting:

to any two participants there was at least one other participant who they both beat.

What is the smallest number of students that could have participated in the tournament?

Fish in buckets   (Joel David Hamkins)

This is a two-player game played with finitely many buckets in a line on the beach, each containing a finite number of fish. There is also a large supply of additional fish available nearby, fresh off the boats.

Taking turns, each player selects a bucket and removes exactly one fish from it and then, if desired, adds any finite number of fish from the nearby supply to the buckets to the left.

The winner is whoever takes the very last fish from the buckets, leaving them empty.

Fish in bases

Start with a natural number k. Write k in base 2. Interpret the digits as a base 3 number. Subtract one, and rewrite in base 3. Interpret the digits as a base 4 number. Subtract one, and rewrite as a base 4 number.

...

Continue, increasing the base by one each time.

The ever increasing bases make the numbers appear to grow at an alarming rate. What is the 'fate' of this sequence?

Witch hunt

Four women are tied to stakes before you. In front of them is a very puzzled inquisitor carrying an equally puzzled lizard. The inquisitor says:

"Our mayor, as you can plainly see, has been turned into a lizard by two women, one a terrible witch and the other her assistant. The mayor says only these two women were present in the town hall when he was transformed. For this evil act, the witch deserves the standard burning-pyre treatment, and her assistant, since she is not a witch, will be banished. The other two women are completely innocent of sorcery, but we have learned that they have knowledge of the powers of witches.

Each made statements earlier, and I know that evil witches are incapable of telling the truth. The witch's assistant may or may not be telling the truth; even if one of her statements is true, another may be false. The other two women would have no reason to lie, so I'm certain their statements are absolutely accurate. Since I'm hesitant to incinerate two innocents, please help me determine which must burn, and which go free.

These are their statements."

Heddy: "I am not a witch."
"Through a window, I saw Wilhelmina and Beatrix turn the mayor into a lizard."
"Witches cannot change to appear like other women."

Wilhelmina: "I am not a witch."
"Through a window, I saw Heddy and Beatrix turn the mayor into a lizard."
"Witches can change to appear like other women."

Rosemary: "I am not a witch."
"I was outside the town hall with Wilhelmina when the mayor was changed."
"Witches cannot change to appear like other women."

Beatrix: "I am not a witch."
"I was alone when the mayor was changed."
"Witches can change to appear like other women."

THANK YOU !

Handout with the list of problems is available

or

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