

DRAWING ON EFFECTIVE PROFESSIONAL DEVELOPMENT RESEARCH:

Using a Pedagogical-Content Model in
the Philadelphia Area Math Teachers Circle (PAMTC)

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Hi! My name is Josh Taton and I am a PhD candidate at the University of Pennsylvania Graduate School of Education, and I am also a founder of the Philadelphia Area Math Teachers' Circle. My research focuses mostly on teachers' knowledge and use of curriculum materials.



In the New York Times Magazine, last summer, Elizabeth Green wrote a piece with the somewhat-provocative title “Why Do Americans Stink at Math?” In the article, she summarized the last 20-30 years of research in math education for a wide audience.

In doing so, she tells a story from the 1980’s. Has anyone heard this story—do you know where I am going? So, in the 80’s the A&W restaurant chain introduced a new burger to compete with McDonald’s Quarter Pounder. The A&W burger had a third of a pound of beef and it was priced comparably to the Quarter Pounder. In taste tests, it also won, hands-down. BUT people weren’t buying it! Why not? After conducting research with focus groups, A&W learned that Americans generally thought that the Quarter Pounder was a better deal—because the “4” in Quarter Pounder was larger than the “3” in A&W third-pounder.

Of course, for a long time, we’ve had concerns about Americans’ abilities to do math (before and after the ‘80’s). In her article, Elizabeth Green makes the argument that one thing that has remained largely stagnant, over the course of many generations, is our teaching. (There are broader, cultural factors involved, too, but I am not going to go into these.)

[If anyone asks, cite Jo Boaler’s Atlantic Monthly article.]

And for just as long, a simple argument has been made: if we can improve teachers’ knowledge of math, then we can improve students’ achievement. My aim is to poke holes in that argument, just a bit.

$$\begin{array}{r} 35 \\ \times 25 \\ \hline \end{array}$$

Ball & Bass (2003); Ball, Hill, & Bass (2005)

In the article, Green also tells the story of Deborah Ball (who was a math teacher and is now a prominent math education researcher) and Hyman Bass (who is a mathematician). She describes their research in the 1990's and early 2000's, in which they presented mathematicians and math teachers with a few math problems. Like this one... 35×25 .

$$\begin{array}{r} 35 \\ \times 25 \\ \hline 875 \end{array}$$

$$\begin{array}{r} 35 \\ \times 25 \\ \hline 1055 \end{array}$$

$$\begin{array}{r} 35 \\ \times 25 \\ \hline 245 \end{array}$$

Ball & Bass (2003); Ball, Hill, & Bass (2005)

They also showed incorrect answers that elementary school students provided. [Ask if they know which is the right one and which is the wrong one?]

$$\begin{array}{r} 35 \\ \times 25 \\ \hline 875 \end{array}$$


$$\begin{array}{r} 35 \\ \times 25 \\ \hline 1055 \end{array}$$

$$\begin{array}{r} 35 \\ \times 25 \\ \hline 245 \end{array}$$

Ball & Bass (2003); Ball, Hill, & Bass (2005)

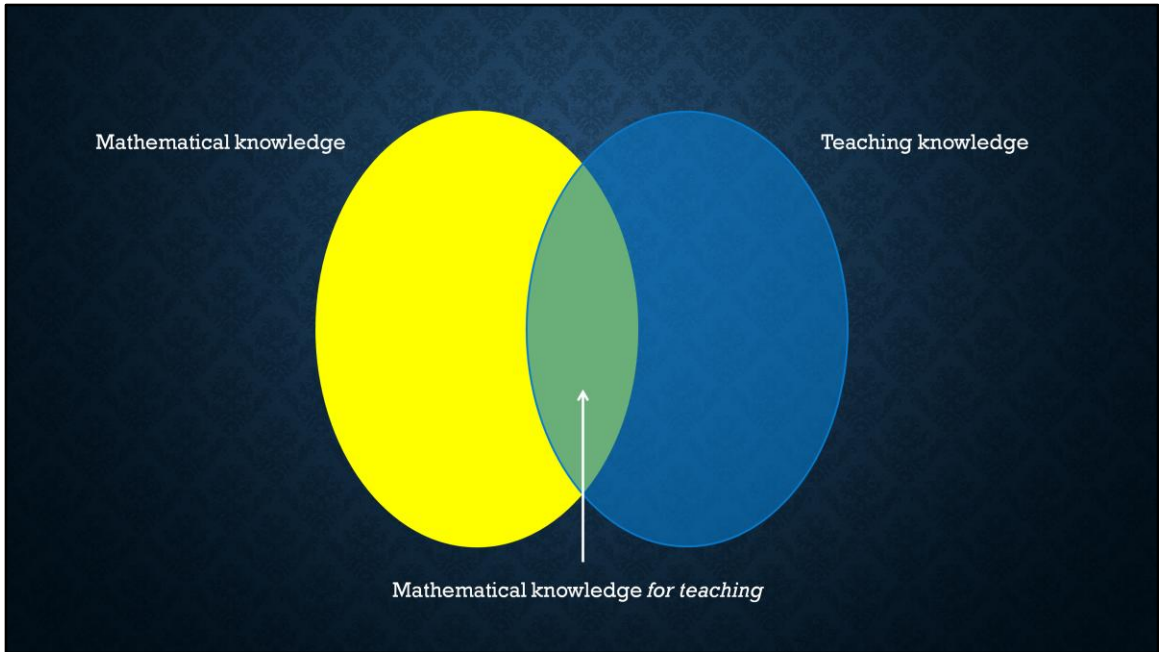
Yes, the one on the left is correct. But can you tell the mistake made in the others? In their research, Deborah Ball and Hyman Bass asked mathematicians and math teachers to explain the mistakes that students were making. I realize that I probably should not say this to this audience, but the mathematicians could NOT explain students' mistakes but expert math teachers COULD.

For example, can anyone see the mistake being made on the far right?

$$\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ + 70 \\ \hline 245 \end{array}$$


Ball & Bass (2003); Ball, Hill, & Bass (2005)

[I see people mouthing / motioning answers.] Yes, the student did not move the 70 over to the right. What this suggests, beyond a misapplication of the algorithm is a fundamental misunderstanding about the operation of multiplication and base-10 place value.



Where am I going with this? Well, their research demonstrates (broadly) the existence of three types of knowledge: mathematical knowledge, teaching knowledge, and a special kind of knowledge known as **mathematical knowledge for teaching.** Mathematical knowledge for teaching includes an understanding of helpful examples of problems that promote insight into larger ideas, useful representations of math, the ability to identify and troubleshoot errors, and so on.

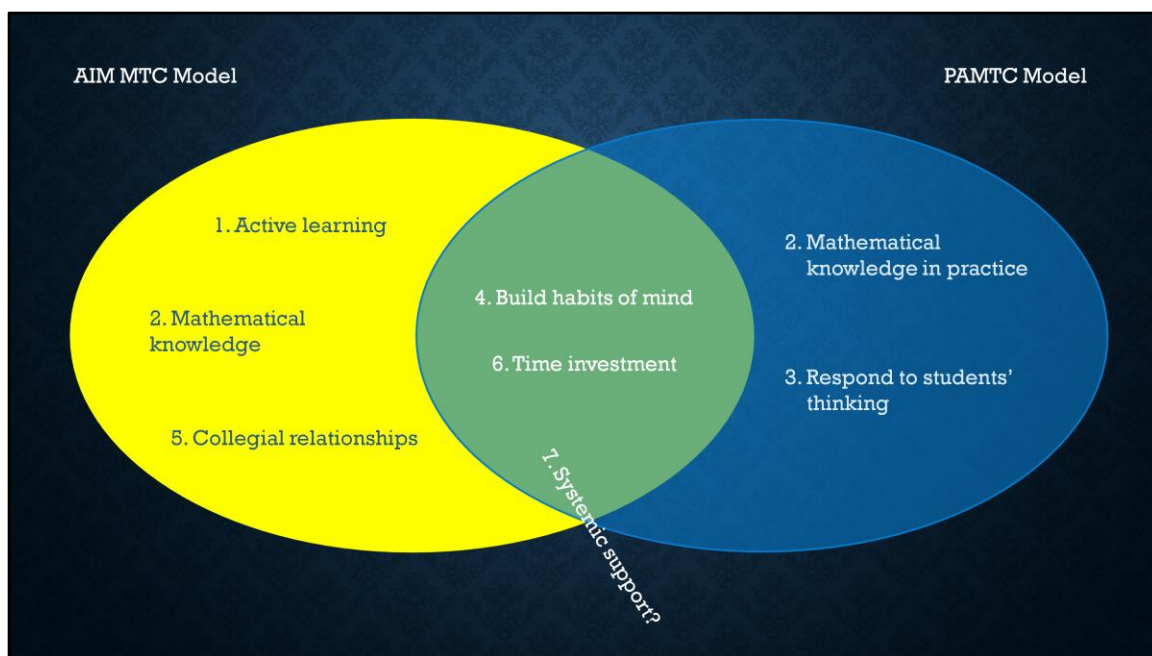
Building Mathematical Knowledge for Teaching

1. Active learning
2. Build mathematical knowledge & capacity to use it in practice
3. Build capacity to notice, analyze, & respond to students' thinking
4. Build habits of mind
5. Build collegial relationships
6. Continuous, ongoing learning (time investment)
7. Systemic support

Doerr, Goldsmith, & Lewis (2010)

The good news is that mathematical knowledge for teaching can be developed; it's not something you're born with. Research on teachers' learning is showing that these are the conditions under which it can be developed:

- 1) Provide opportunities for active learning
- 2) Build teachers' mathematical knowledge and their capacity to use it in practice
- 3) Build teachers' capacity to notice, analyze, and respond to students' thinking
- 4) Build teachers' productive habits of mind
- 5) Build collegial relationships
- 6) Provide opportunities and structures that support continued learning (i.e., a significant time investment is needed)
- 7) Provide systemic support (i.e., institutional and logistical factors that permit the other opportunities)



Before I talk about how we have taken up these research-based recommendations in the Philadelphia Area Math Teachers' Circle, I want to compare the hallmarks of the AIM Math Teachers' Circles as presented in the training sessions we received in Washington, DC and the areas that we've also decided to focus on.

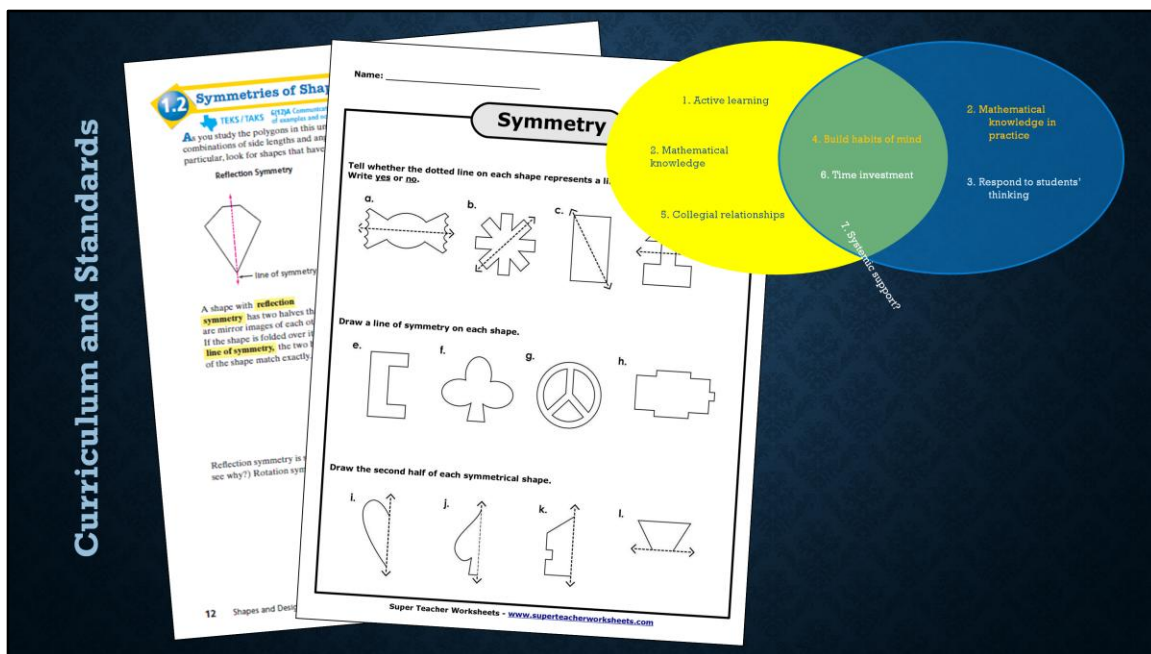
First, we think that by having teachers do mathematical problem-solving, that this is obviously a quintessential example of active learning. (I've located that in the AIM model.)

Second, since teachers are also encountering sophisticated mathematical ideas and making connections among mathematical topics, we believe there is evidence that teachers' are also building their mathematical knowledge (i.e., content knowledge).

The capacity to use mathematical knowledge in the practice of teaching is somewhat more complex. Research undertaken by Diana White suggests that teachers *are* building mathematical knowledge used in teaching by participating in MTCs. We think, though, that it is important to go beyond modeling high-quality teaching practices—and so we also try to make this knowledge for teaching transparent in our sessions. I'll talk more about this in just a minute.

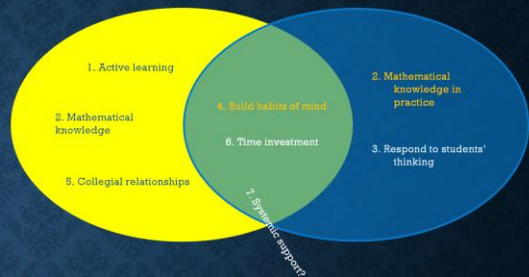
In the Philadelphia group, we also try to spend time talking about how to elicit and interpret students' thinking in the classroom. This is a teaching capacity, that is often overlooked; research has shown, though, that it is essential to help K-12 students develop deep understanding. I'll talk more about this in just a minute, too.

We think that both the AIM model and our model focus on supporting teachers in



First, with regard to building “pedagogical habits of mind” and “mathematical knowledge in practice,” we ask teachers to bring in examples of curriculum materials that they use and we take a look at them in the context of the problem that we solve together. This slide shows material for a lesson on symmetry, and we had just solved a problem on symmetry in our circle. The problems shown here are not very rigorous, as they simply require students to understand the meaning of line symmetry and to identify it in several figures.

So we talk about whether the problems are rigorous, how they are scaffolded, the language that is used and whether or not it would be confusing for students, and so forth. This, we feel, supports teachers in building the habit of analyzing curriculum materials and thinking about pedagogical representations of the math that we’ve studied.



“Coat of Many Bits” Problem

Students need to make costumes for the school play, and they need to know how much fabric to buy. In this activity, they measure the surface area of each other’s coats and use symmetry to simplify their work.

Math Trailblazers, Gr. 3

[http://www.teachertube.com/viewVideo.php?title=Coat of Many Bits 2&video_id=214796](http://www.teachertube.com/viewVideo.php?title=Coat%20of%20Many%20Bits%202&video_id=214796)

We don’t necessarily provide teachers with activities that they can take back to their classrooms, but when we can, we provide examples of curriculum materials that foster active inquiry and making connections. This is an example of a problem that makes use of symmetry in a real-world context, instead of the generic worksheet examples on the previous slide.

Grade 4

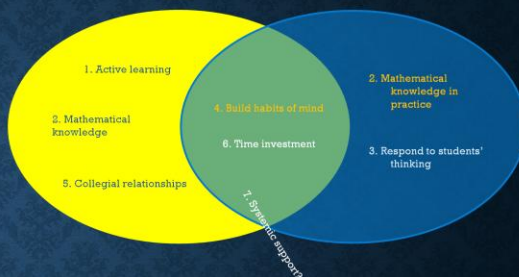
CCSS.Math.Content.4.G.A.3:

Recognize lines of symmetry, identify symmetric figures, and draw lines of symmetry.

Grade 8

CCSS.Math.Content.8.G.A.2: Understand congruency of two-dimensional figures via symmetry (e.g., via rotations, reflections, translations)

CCSS.Math.Content.8.G.A.4: Understand similarity via symmetry (e.g., rotations, rotations, translations, and dilations).



And, as I said, we also try to help teachers interpret the state content standards (or Common Core Standards) and how the problems that we discuss can help students make progress towards meeting them. (We also talk about the Practice Standards.)

Linking Ideas

Q. Why is it important to link ideas? How do you?

Three types of linking:

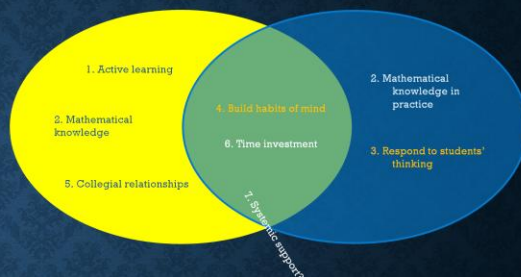
1. *Concepts before & after*
2. *Mathematical goals*
3. *Students' ideas*

Examples:

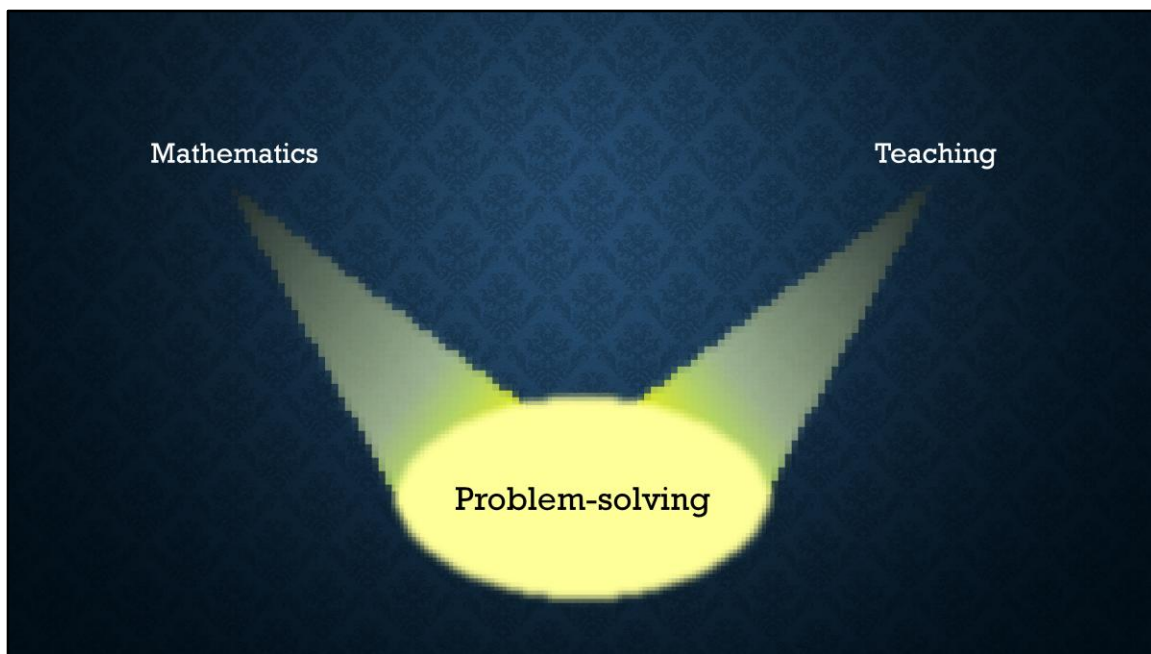
“Ana, can you repeat what Jay said in your own words?”

“Fulano, I heard you say ... but how is what Cameron said different?”

“Who agrees and who disagrees? Why?”



With regard to building “pedagogical habits of mind” and “capacities to interpret and respond to students’ thinking,” we talk about the importance of planning and analyzing classroom discourse, or speech moves. This is a slide from a workshop in which we talked about the importance of linking ideas, both mathematical ideas and encouraging students’ to make connections between their peers’ contributions.



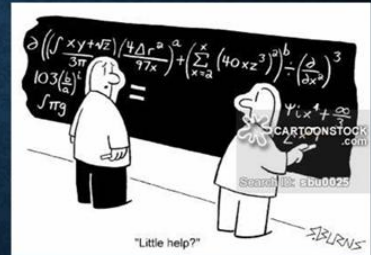
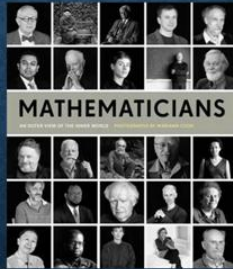
Here's another way to think about our model.

In our teacher ed program at Penn, we asked students at the beginning of the semester to say what mathematics is. Many of them said math is problem-solving. At the end of the semester we asked them to reflect on what they had learned. One student said, with many others nodding, "I've learned that not only is math problem-solving, but teaching is problem-solving, too."

I found this very instructive, because it means that we need to throw light on both mathematics *and* on *teaching* mathematics. Teachers need to know mathematics, for sure, but they also need to know specific strategies and capacities for teaching mathematics. It's not just about knowing more math. That's why we try to make *transparent* to our participants how we in the Philadelphia Area Math Teachers' Circle *enact* mathematical learning experiences...and why we highlight important aspects of pedagogy, as well.

I'm now going to turn this over to Cathryn Anderson, who is going to talk about this approach from our teachers' and mathematicians' perspectives.

JOINING AND CREATING PAMTC

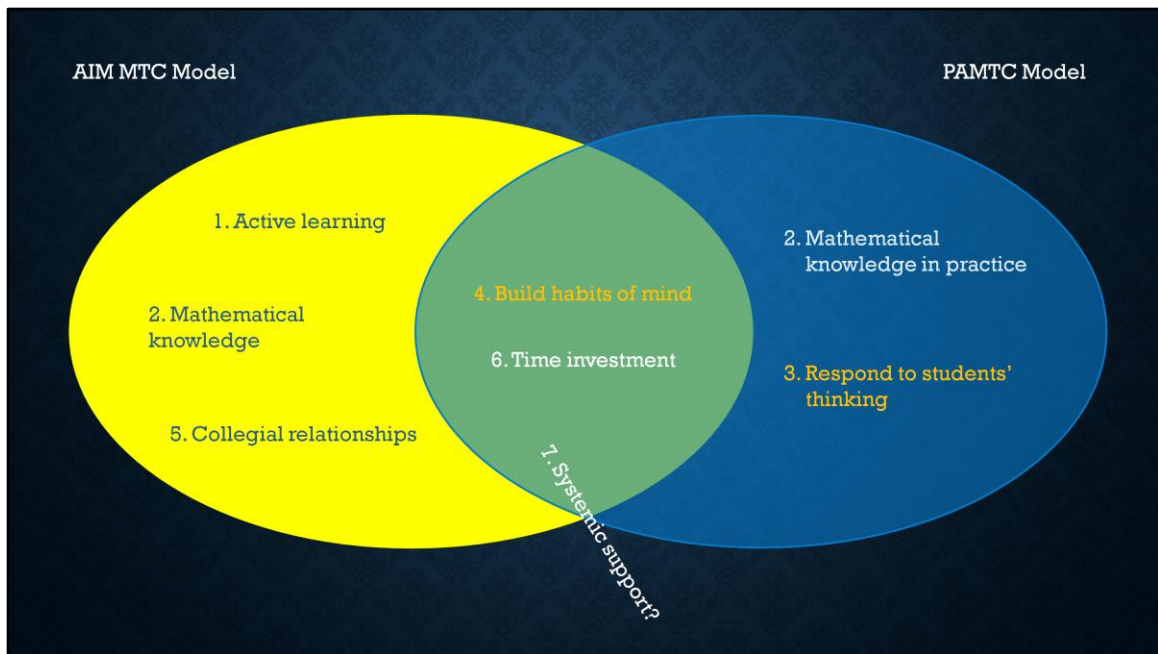


Share background

Not a math person

Knew I wanted to be a teacher but lacked of confidence in my skills

Not a mathematician - I did not see myself in these pictures



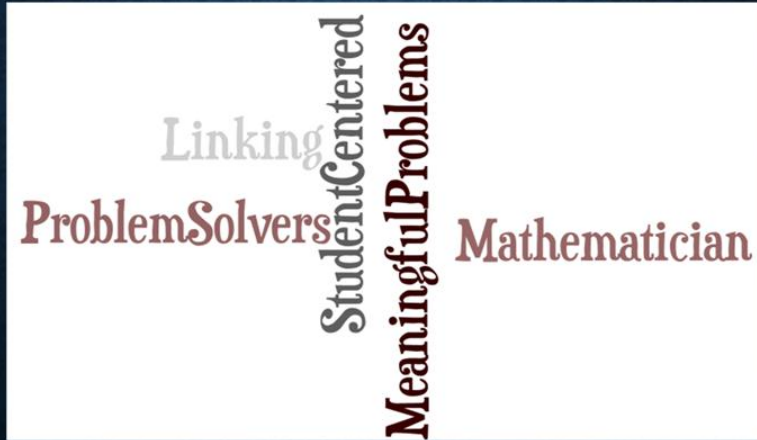
Relate back to Josh

The two effective strategies that I believe our Math teachers circle has helped me with is 3 and 4.

We take time each session to explicitly address student thinking and create positive habits of mind.

[JAT Note: You also had math knowledge in practice (2) and collegial relationships (5) highlighted in yellow.]

BUILDS TEACHERS' PRODUCTIVE HABITS OF MIND



Change the meaning of the word mathematician in the classroom

Preservers

Able to describe thinking in words or with a model

Tries various angles to begin a problem.

Listens to others

Reflection on strategies that work or don't work in the classroom

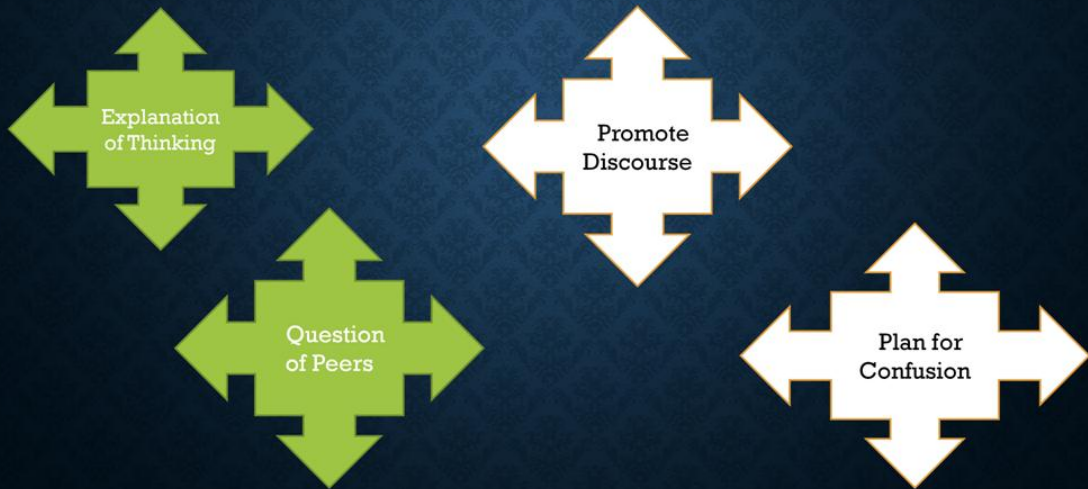
Letting the room go – create a student centered classroom

Taking ideas from the other teachers.

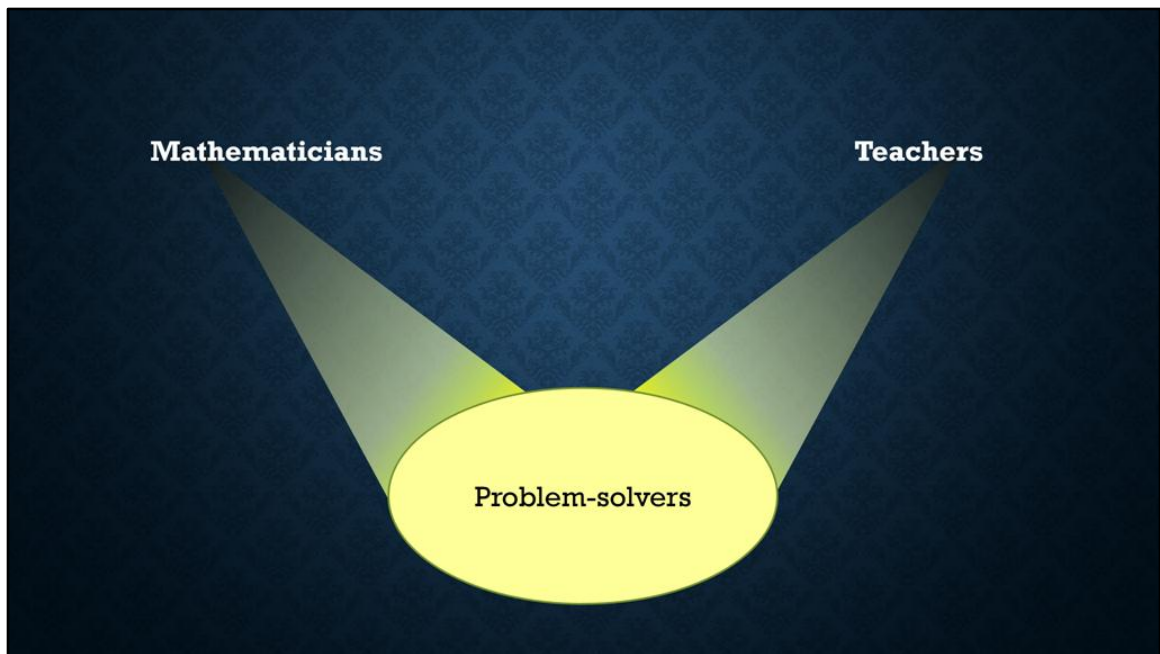
Creating meaningful problems for students

Discussing and implementing Mathematical Practices in the classroom

BUILDS TEACHERS' CAPACITY TO NOTICE, ANALYZE, AND RESPOND TO STUDENTS' THINKING

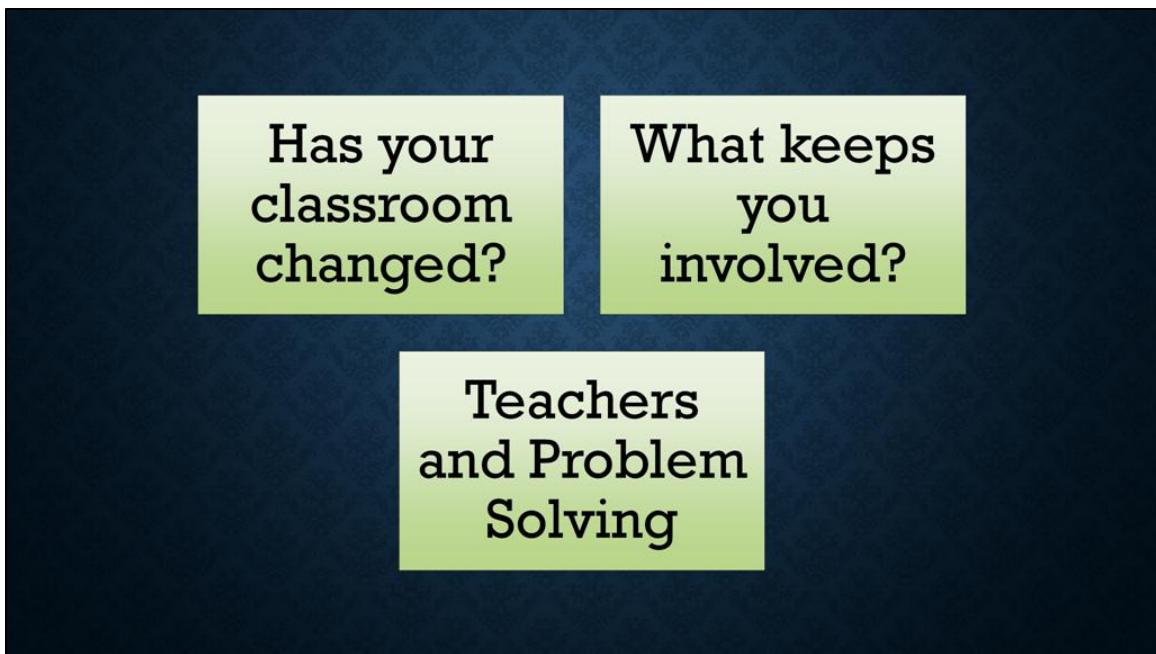


- Notes from meeting with Josh -
 - speak on the math and then pedagogy
- Math -
 - As we work through problems in our circle both during the planning and then monthly sessions I get an experience of how to notice , analyze and respond to the thinking of my peers
 - Math
 - explaining my thinking
 - asking follow up questions to understand their view of a problem
 - knowledge of common errors with certain math problems
- Pedagogy
 - the goal to have a student centered classroom.
 - explicitly show how to develop questions to promote discourse
 - we encourage our participants to ask questions of each other. One person shares and the other members of the group.
 - As we discuss what strategies the planning team uses to create discussion teachers are sharing what strategies have worked in their classroom. Or what has not worked.
- Immediate understanding of how some students feel in Math Classroom



Teachers have a special knowledge unique to them. Openly discussing as do provides an opportunity for teachers to share teaching knowledge with mathematicians. The mathematicians involved with our circle have shared some of the ways this has impacted their collegiate classrooms or knowledge of teachers.

Ball calls this mathematical knowledge for teaching—a special type of math knowledge used in teaching.



Classroom Change: We are more conscious of the language and process of problem-solving, and try to explicitly reference it when working with students in and outside of the classroom. The “magic” of scaffolding. Allowing to struggle.

Involvement: This is a meaningful professional development opportunity for teachers. We also enjoy the sense of community and the fun atmosphere.

Teachers: The wrap up reminds me that open-ended problems have many different solutions, and can lead in many different directions. I can anticipate a number of them, but teachers always find new ones. We already knew how hard teachers work, but this experience has brought our understanding their commitment to new heights

There is a strong benefit for teacher participation in the circle along with mathematician participation.

References

- Ball, D. L. & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis and E. Simmt (Eds.), *Proc Ann Meeting of Canadian Math Ed St Group* (2002) (pp. 3-14). Edmonton, AB: CMESG/GDEDM
- Ball, D.L., Hill, H., & Bass, H. (2005). *Knowing Mathematics for Teaching. American Educator*, Fall 2005, 14-22, 43-46.
- Doerr, H.M., Goldsmith, L.T., Lewis, C.C. (2010). Mathematics professional development (Research brief). Reston, VA: NCTM. Retrieved on January 7, 2015 from www.nctm.org/uploadedFiles/Research_News_and_Advocacy/
- Green, E. (2014 Jul). Why do Americans stink at math? *New York Times Magazine*. Retrieved on January 7, 2015 from nytimes.org/2014/07/27/magazine/why-do-americans-stink-at-math.html

Thank you!

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