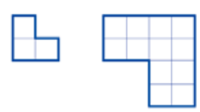
The wrangle at JMM 2017 was snowed out!
So these problems were saved for Math Fest

JMM 2017 MATH WRANGLE QUESTIONS

1 Let $g(x) = e^x + e^{-x}$. Define real numbers a and k so that $g(a) = k$. Determine k so that for each natural number n, $g(na)$ is constant.

2 A group of airplanes is based on a small island. The tank of each plane holds just enough fuel to take it halfway around the world. Any desired amount of fuel can be transferred from the tank of one plane to the tank of another while the planes are in flight. The only source of fuel is on the island, and we assume that there is no time lost in refueling either in the air or on the ground. What is the smallest number of planes that will ensure the flight of one plane around the world on a great circle, assuming that the planes have the same constant ground speed and rate of fuel consumption and that all planes return safely to the island base?

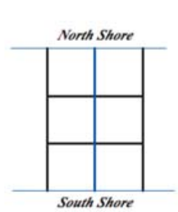
3 Define a *size-n tromino* to be the shape you get when you remove one quadrant from a $2n \times 2n$ square. In the figure below, a size-1 tromino is on the left and a size-2 tromino is on the right.



We say that a shape can be *tiled with size-1 trominos* if we can cover the entire area of the shape—*and no excess area*—with *non-overlapping* size-1 trominos. For example, it is easy to see that a 2×3 rectangle can be tiled with size-1 trominos, but a 3×3 square cannot be tiled with size-1 trominos.

Can a size-2017 tromino be tiled by size-1 trominos?

4 *Bridges*. A system of 13 bridges, shown below, connects the north shore of a river to the south shore. For each bridge, there is a 50% probability that a protest march will block traffic across that bridge, and these probabilities are independent (imagine that each bridge flips a coin). What is the probability that it is possible to cross from one shore to the other?



5 Consider a deck of cards with the numbers #1 through #20 on them. Three cards are picked at random. Find the probability that at least one of the following is true of the hand chosen:

(A) All three cards are odd.

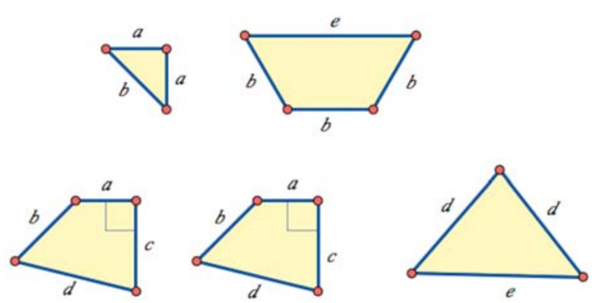
(B) All three cards are prime numbers.

(C) None of the cards are divisible by 3.

6 Colorings of the edges of two congruent regular tetrahedra are said to be *equivalent* if it is possible to perform a rotation that turns one coloring into the other. For example, if we color edges with two colors (depicted by thick or thin lines below), the two leftmost tetrahedra have equivalent colorings, but the third tetrahedron's coloring is not equivalent to the first two.

In how many different (non-equivalent) ways can one color the edges of a regular tetrahedron so that two edges are red, two are black, and two are green?

7 Consider the five shapes below, consisting of an isosceles right triangle, an isosceles trapezoid, an isosceles triangle, and two congruent quadrilaterals that contain one right angle (marked). The lengths of these shapes are $a = 1, b = \sqrt{2}, c = 2, d = \sqrt{5}, e = \sqrt{8}$.



Find the volume of the polyhedron formed when these five shapes are fitted together.

8 Let $F_1, F_2, F_3, \ldots$ be the *Fibonacci sequence*, the sequence of positive integers satisfying

$$F_1 = F_2 = 1 \quad \text{and} \quad F_{n+2} = F_{n+1} + F_n \text{ for all } n \geq 1.$$

Does there exist an $n \geq 1$ for which F_n is divisible by 2017?