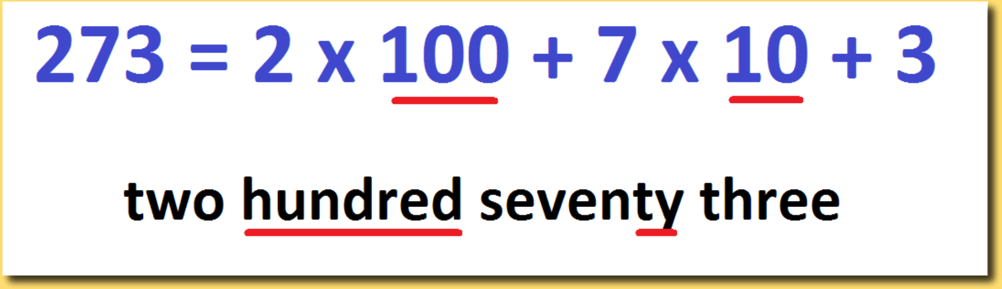
WEIRD WAYS
TO MULTIPLY

(Have you ever noticed that the spelling of “weird” is weird?)

James Tanton
Mathematical Association of America

Before we multiply numbers, are we clear on how we write them?



$$273 = 2 \times \underline{100} + 7 \times \underline{10} + 3$$

two hundred seventy three

We use base ten. Why ten?

Base 20?

Base 12?

Base 60?

Why are the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called *digits* ?

Our first weird multiplication:

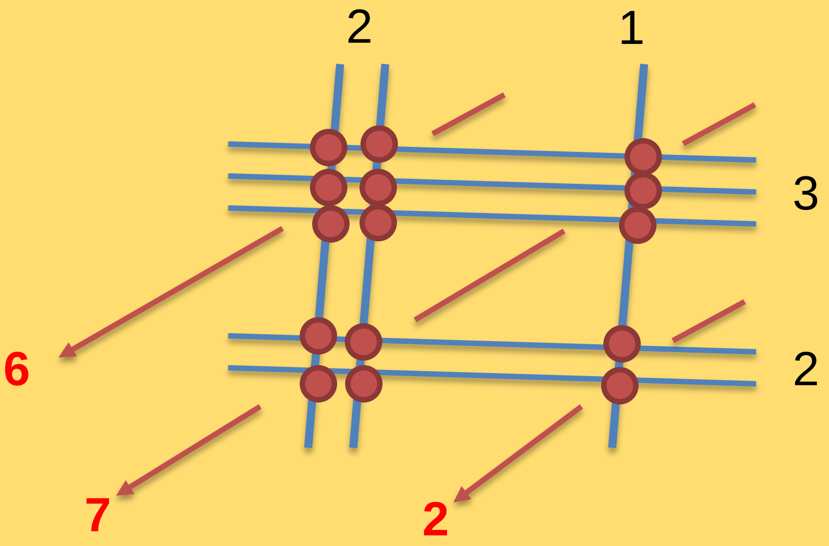
FINGER MULTIPLICATION!

Fingers and toes!

Our second weird multiplication:

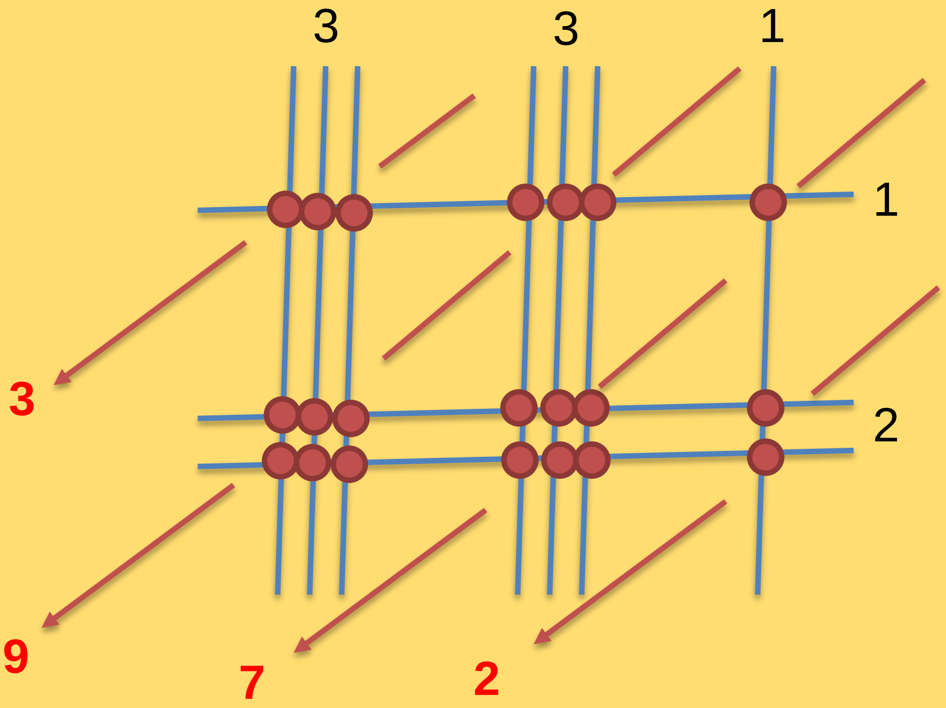
Line Multiplication!

e.g. 21 x 32



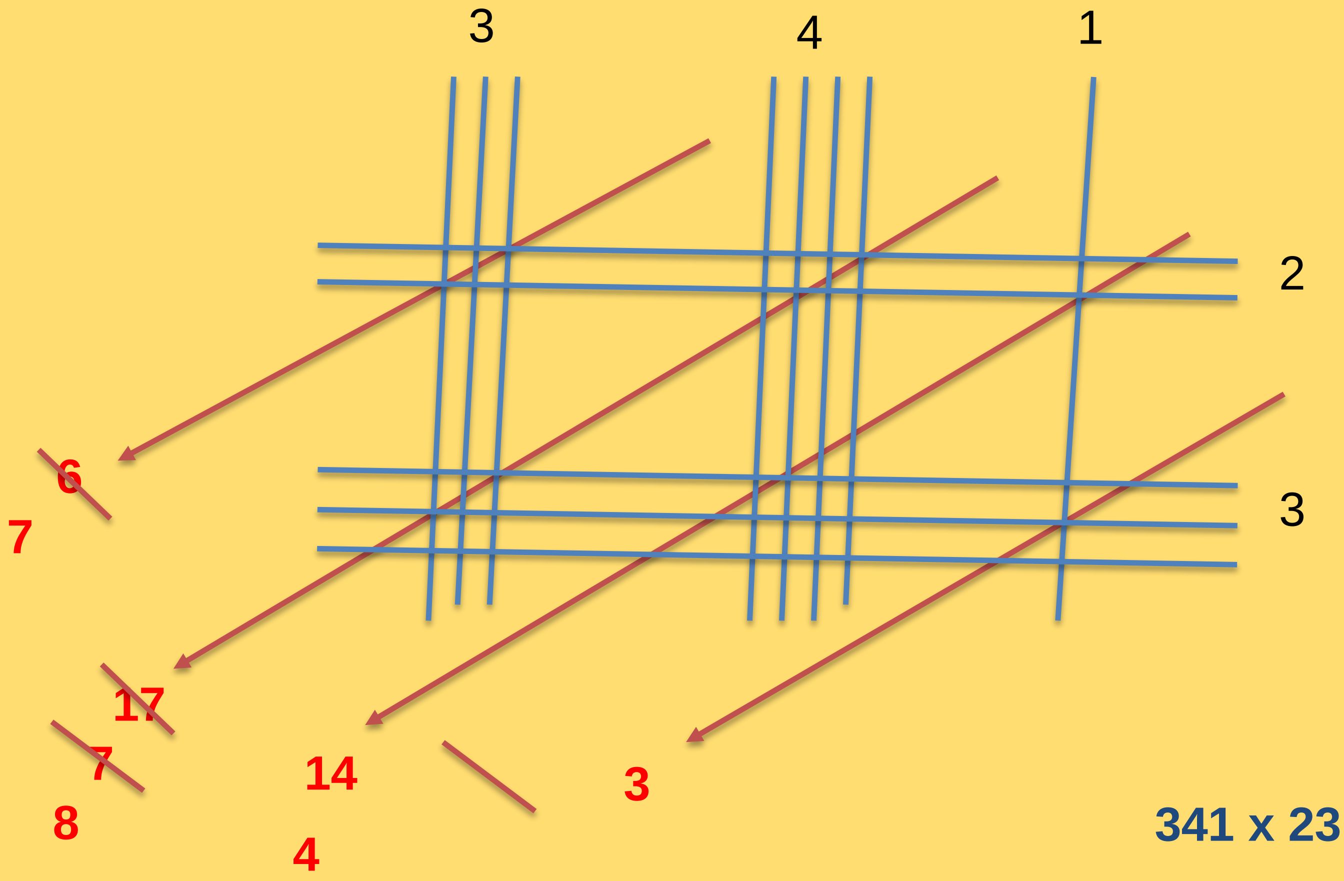
21 x 32 = 672

e.g. 331 x 12



331 x 12 = 3972

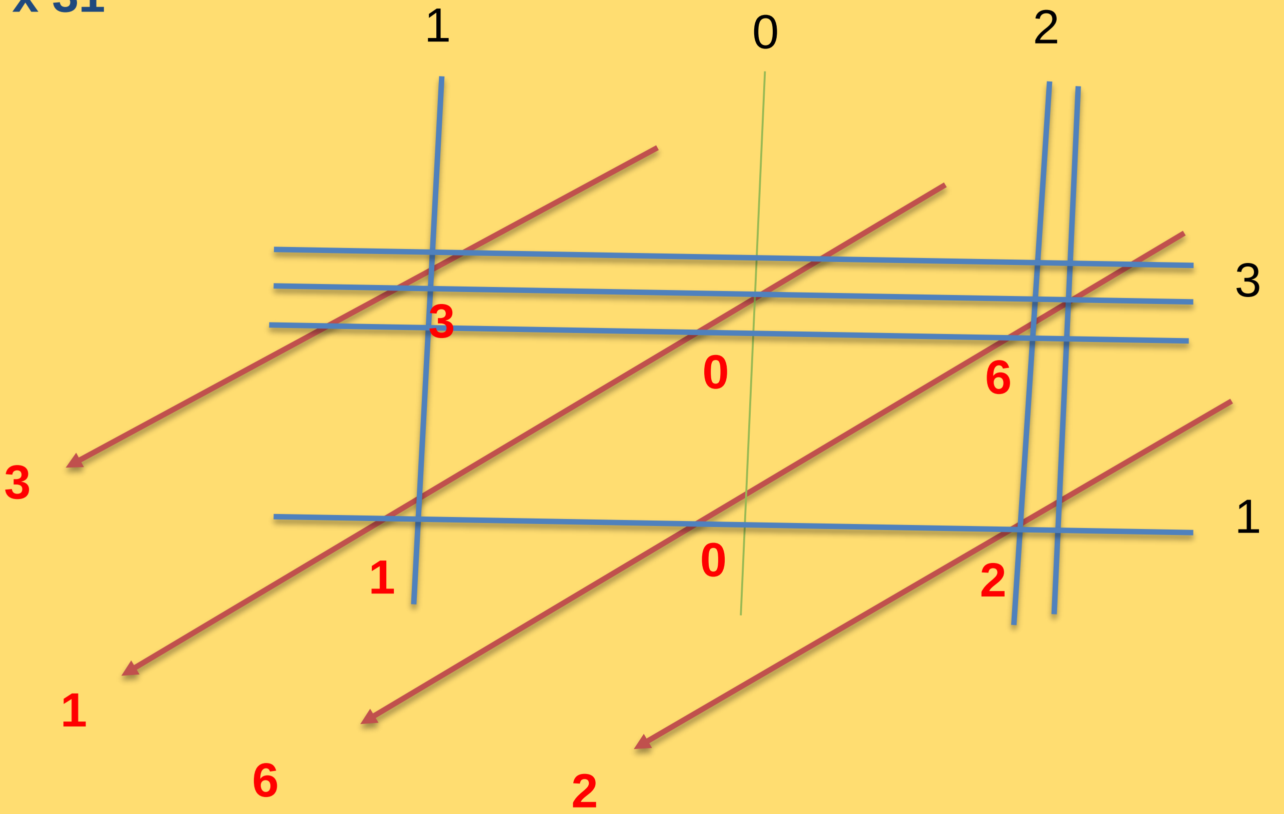
e.g. 341 x 23



341 x 23 = 7843

YOUR TURN …

e.g. 102 x 31

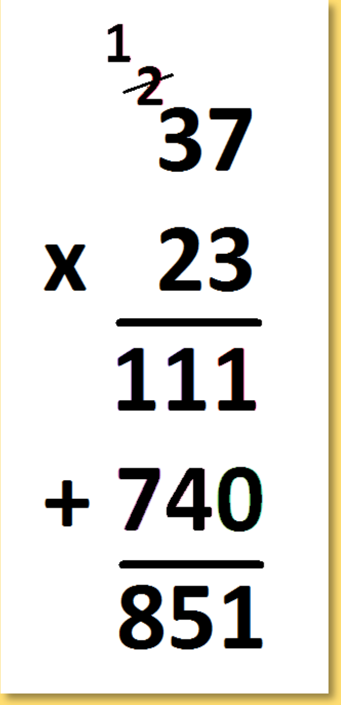


102 x 31 = 3162

Our third weird multiplication:

School Book Multiplication!

e.g. 37 x 23



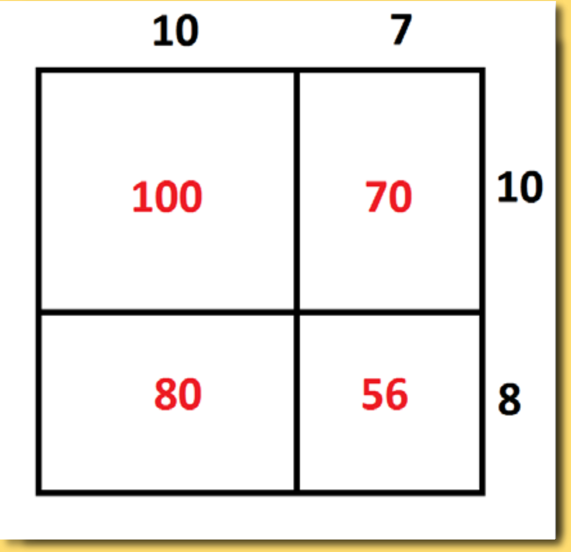
$$\begin{array}{r} {}^{1}{}^{\cancel{2}}37 \\ \times\ \ 23 \\ \hline 111 \\ +\ 740 \\ \hline 851 \end{array}$$

Weird things:

- Right to Left
- Not allowed to write 21
- Are allowed to write 11
- Where 11 is written is weird
- Throw in a zero
- Not allowed to write 14
- Where you write 4 and 7
- End with an **addition** problem!

What is multiplication really?

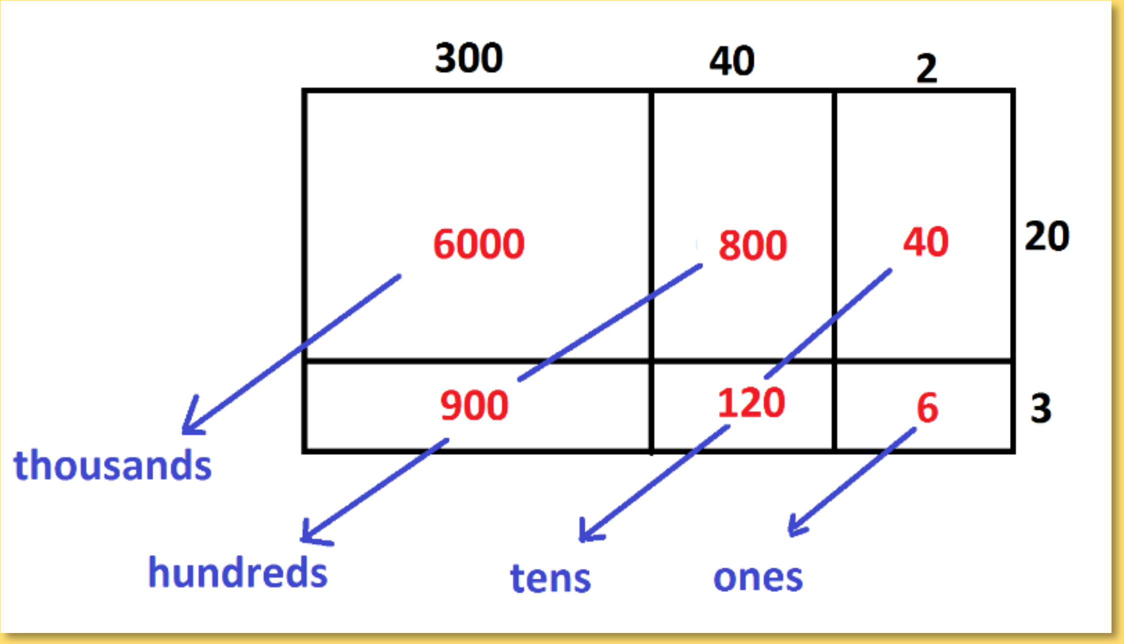
e.g. 17 x 18



10	7	
100	70	10
80	56	8

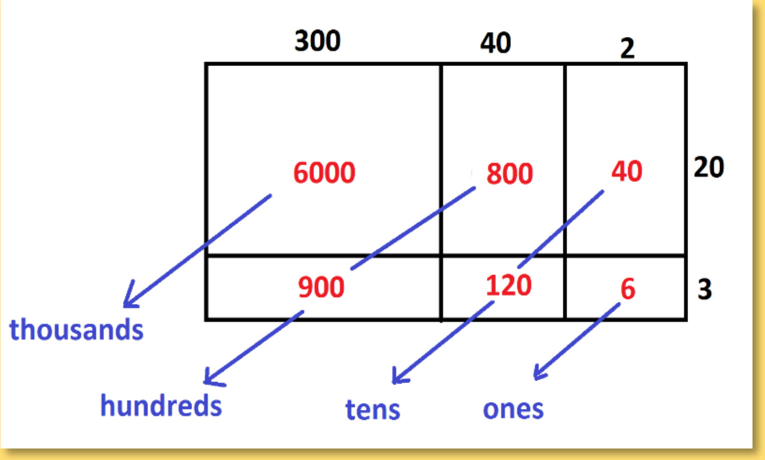
17 x 18 = 100 + 70 + 80 + 56 = 306

e.g. 342 x 23

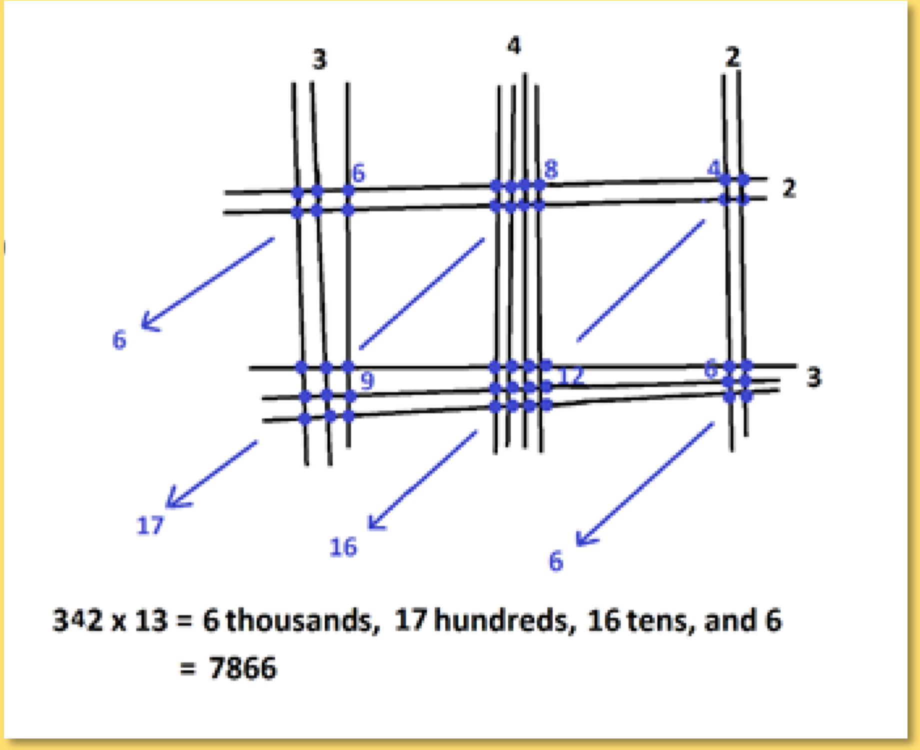


300	40	2	
6000	800	40	20
900	120	6	3

342 x 23 = 6000 + 1700 + 160 + 6 = 7866

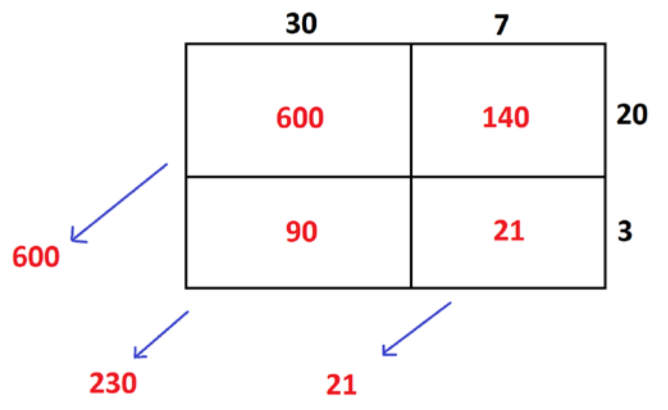


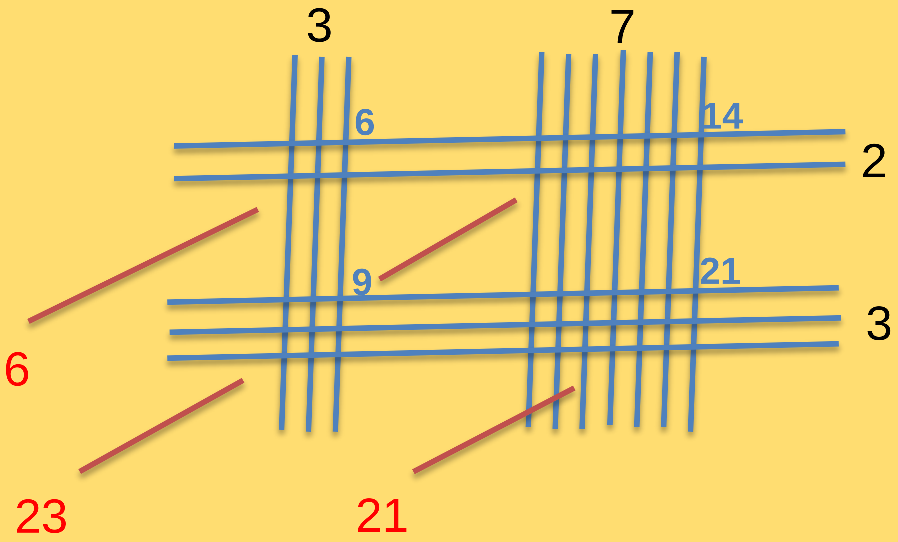
342 x 23 = 6000 + 1700 + 160 + 6
= 7866

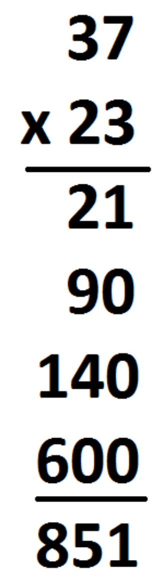


342 x 13 = 6 thousands, 17 hundreds, 16 tens, and 6
= 7866

Our ealier example: 37 x 23.



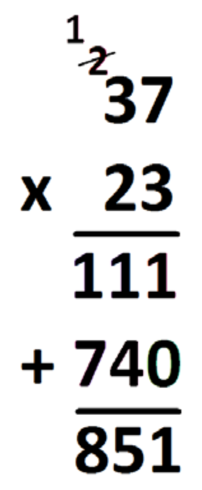




37
x 23

21
90
140
600

851



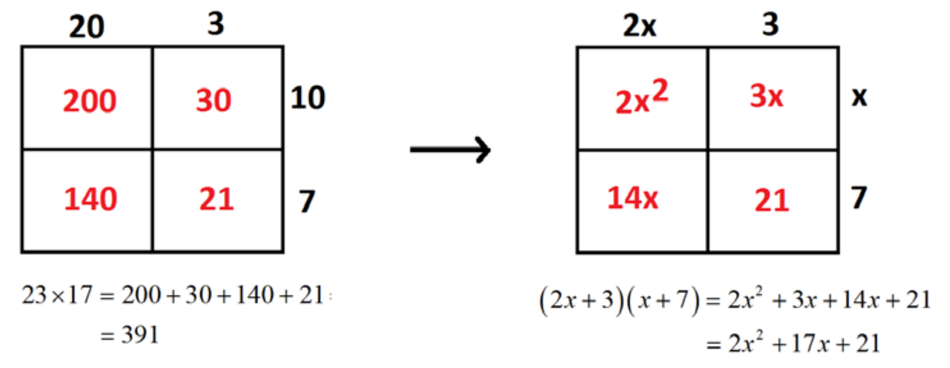
$\overset{1}{\cancel{2}}37$
x 23

111
+ 740

851

All approaches are the same geometry in disguise!

This area model connects to high school mathematics.



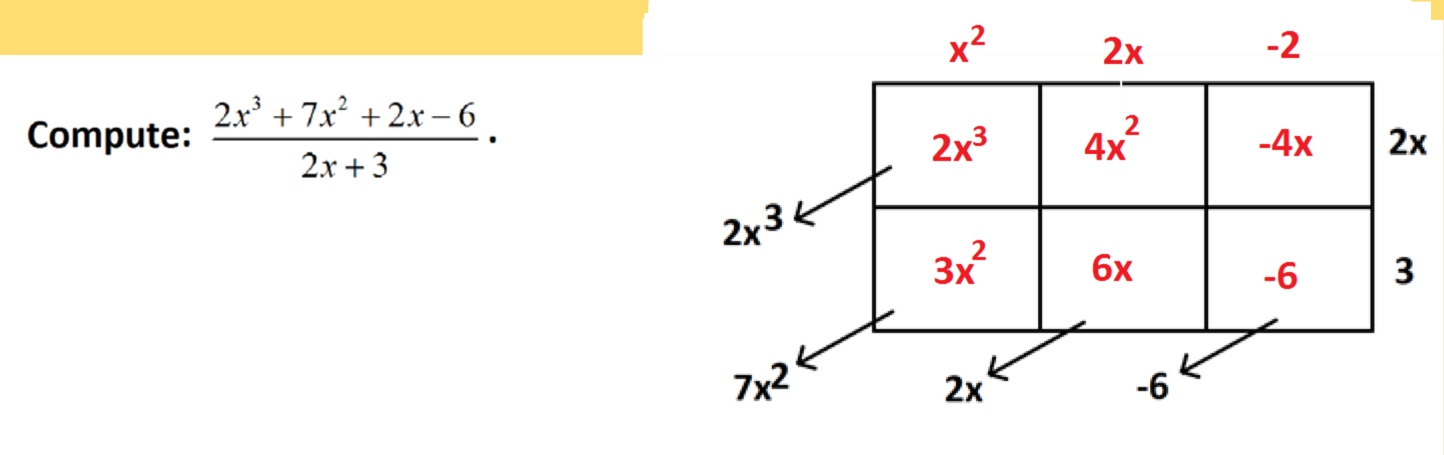
20	3	
200	30	10
140	21	7

$$23\times17=200+30+140+21$$
$$=391$$

$\longrightarrow$

2x	3	
$2x^2$	$3x$	x
$14x$	21	7

$$(2x+3)(x+7)=2x^2+3x+14x+21$$
$$=2x^2+17x+21$$



Compute: $\dfrac{2x^3+7x^2+2x-6}{2x+3}$.

x^2	$2x$	-2	
$2x^3$	$4x^2$	$-4x$	$2x$
$3x^2$	$6x$	-6	3

$2x^3$

$7x^2$

$2x$

-6

Age old question: Why is negative times negative positive?

2 x 3 = 3 + 3 = 6

2 x (-3) = -3 + -3 = -6

(-2) x 3 = 3 x (-2) = -2 + -2 + -2 = -6

(-2) x (-3) = ????

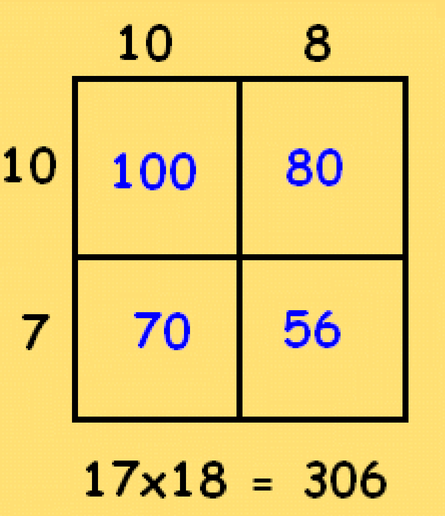
a x b = b x a

Age old question: **Why is negative times negative positive?**

Multiplication = Areas of rectangles.

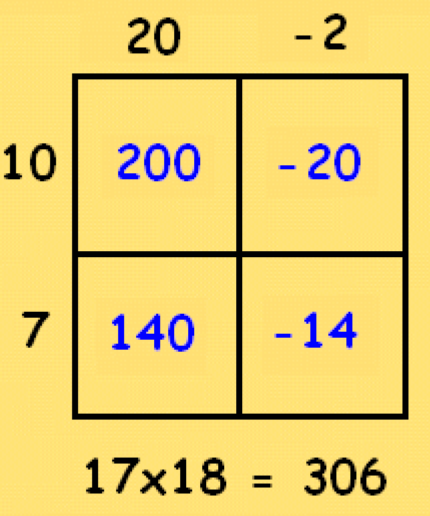
Wild idea … Allow rectangles to have negative side lengths!

Consider 17x18



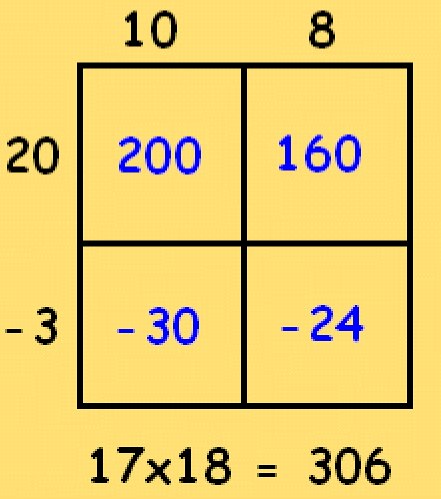
	10	8
10	100	80
7	70	56

17x18 = 306



	20	-2
10	200	-20
7	140	-14

17x18 = 306



	10	8
20	200	160
-3	-30	-24

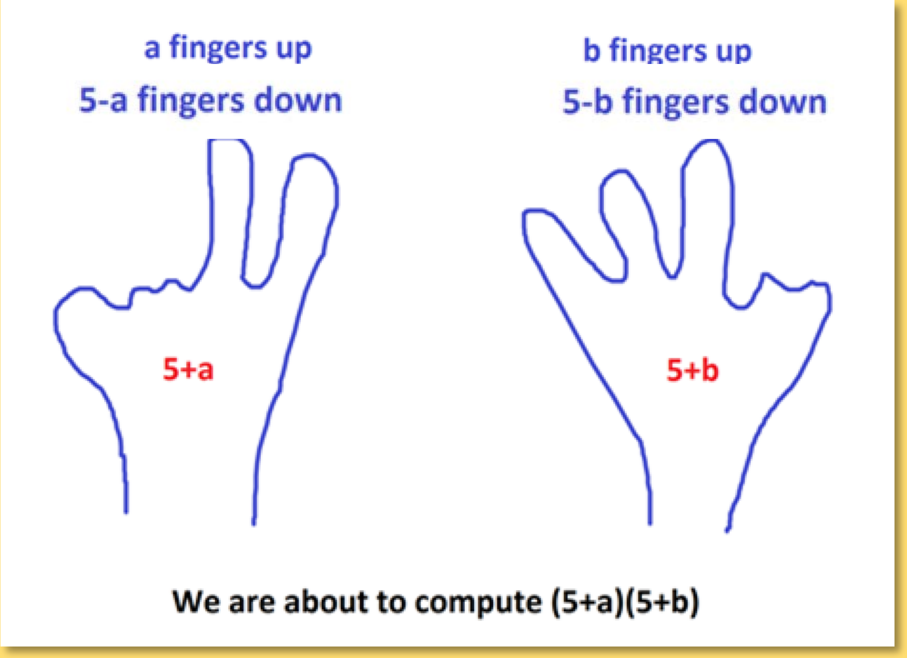
17x18 = 306

	20	-2
20	400	-40
-3	-60	??

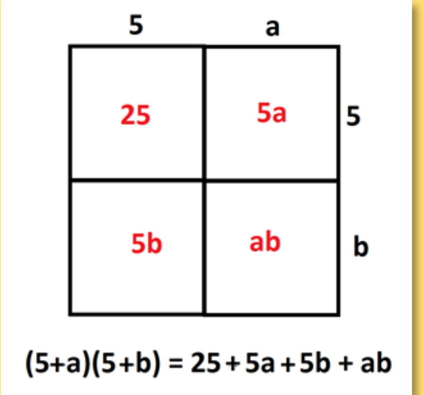
17x18 = 400 - 40 - 60 + ?? = 306

+6

We can now explain finger multiplication.

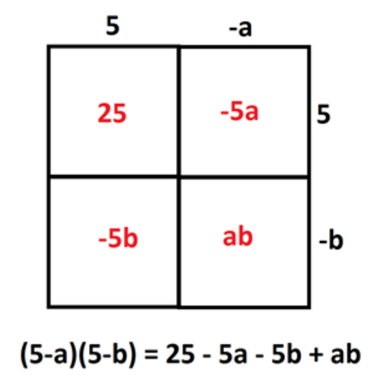


We are about to compute (5+a)(5+b)



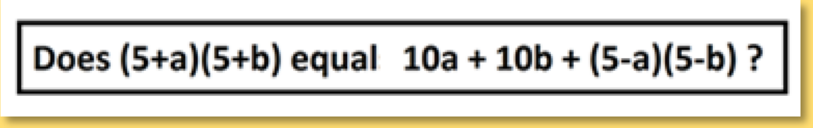
	5	a	
	25	5a	5
	5b	ab	b

(5+a)(5+b) = 25 + 5a + 5b + ab



	5	-a	
	25	-5a	5
	-5b	ab	-b

(5-a)(5-b) = 25 - 5a - 5b + ab



Does (5+a)(5+b) equal 10a + 10b + (5-a)(5-b) ?

10a +10b + (5-a)(5-b)

= 10a + 10b + 25 -5a -5b +25

= 25 + 5a + 5b + 25

= (5 + a)(5 + b) **YES!**

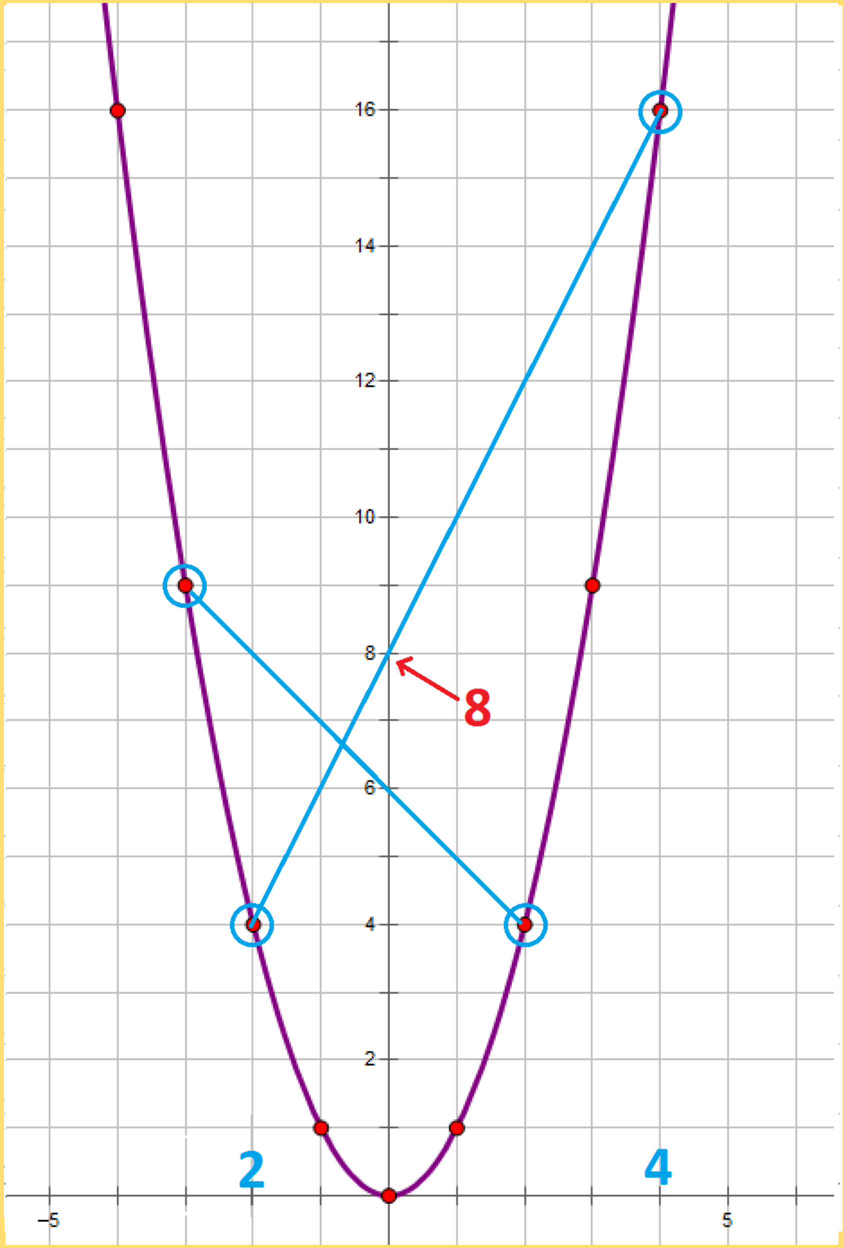
FINGERS AND TOES?

A fourth weird way to multiply.

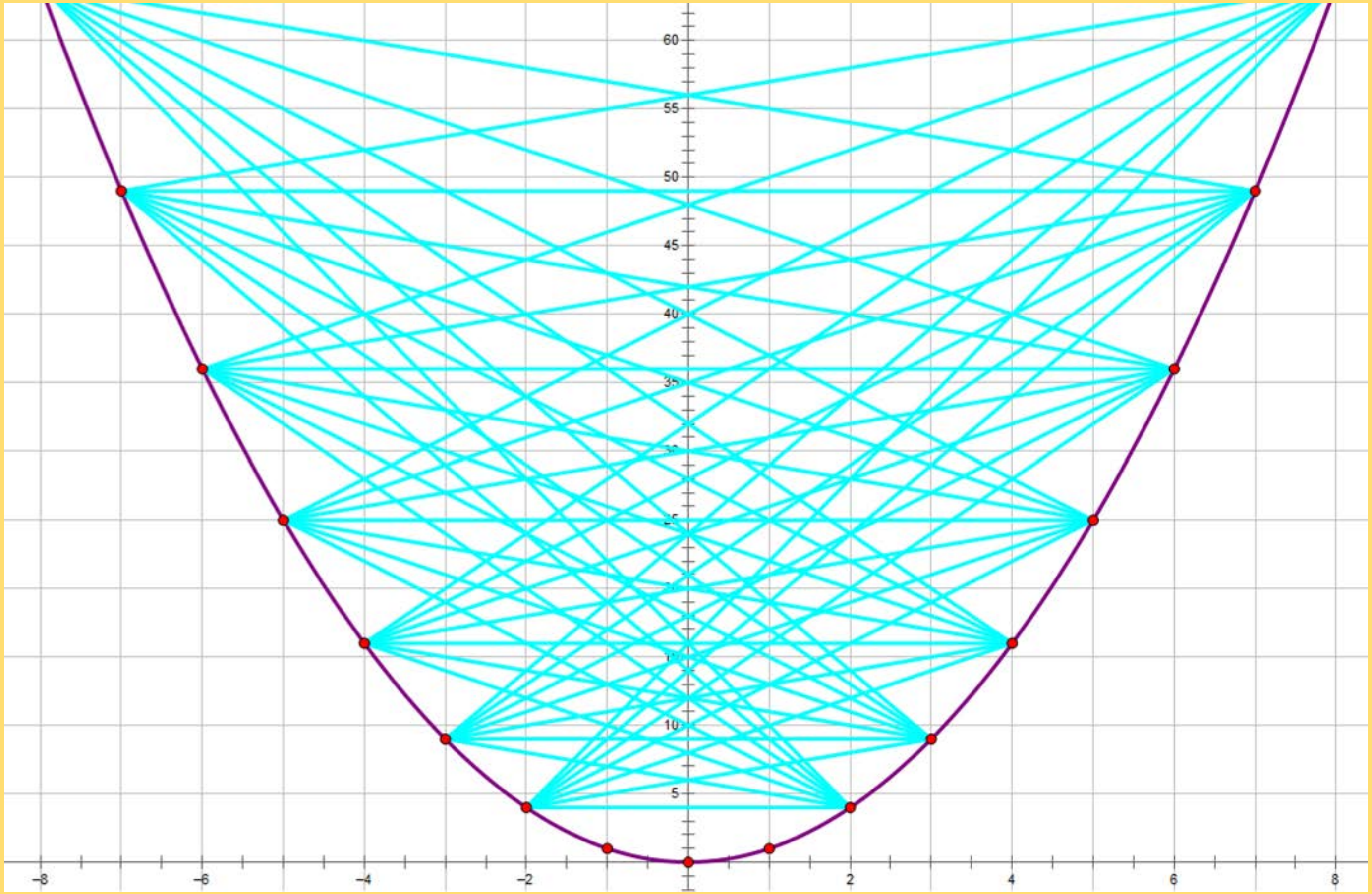
The graph of $y = x^2$.

e.g. 3 x 2

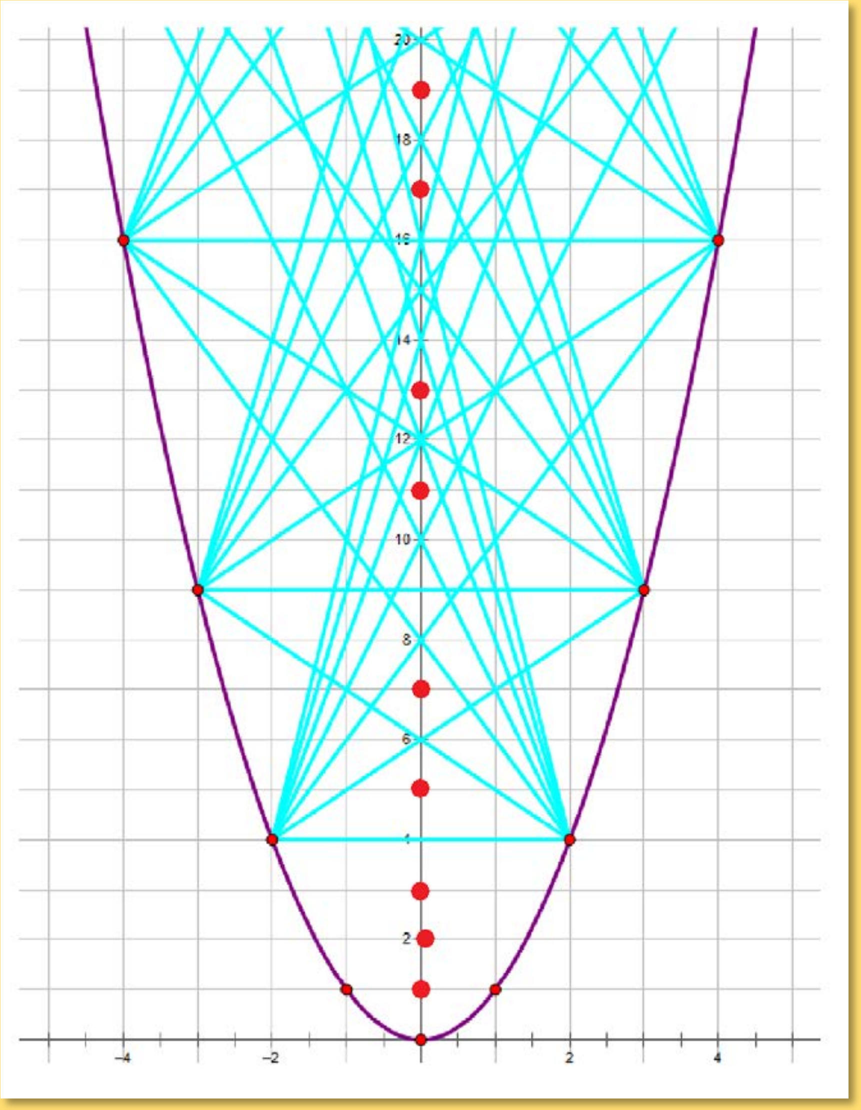
2 x 4



The parabola is a prime number generator!

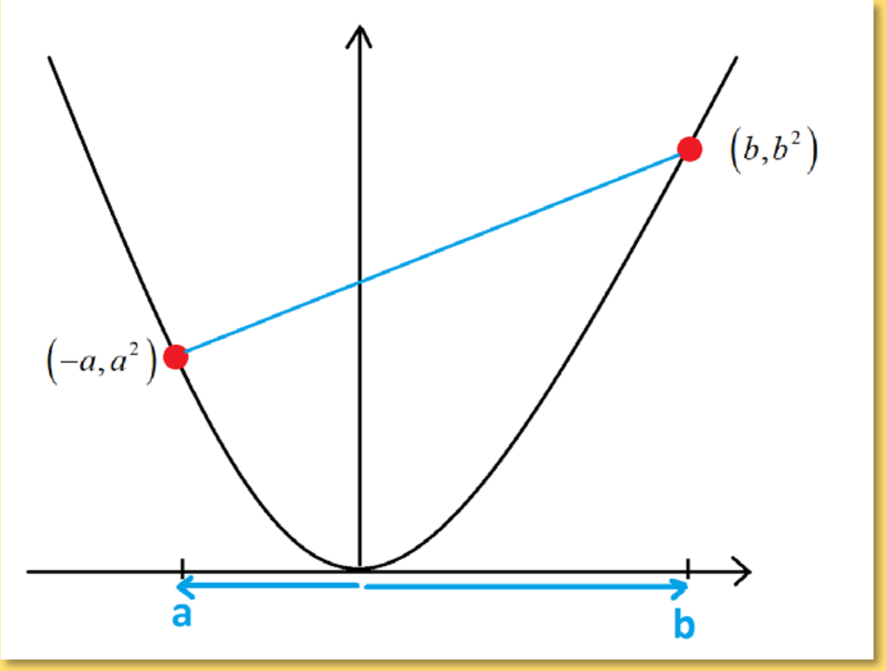


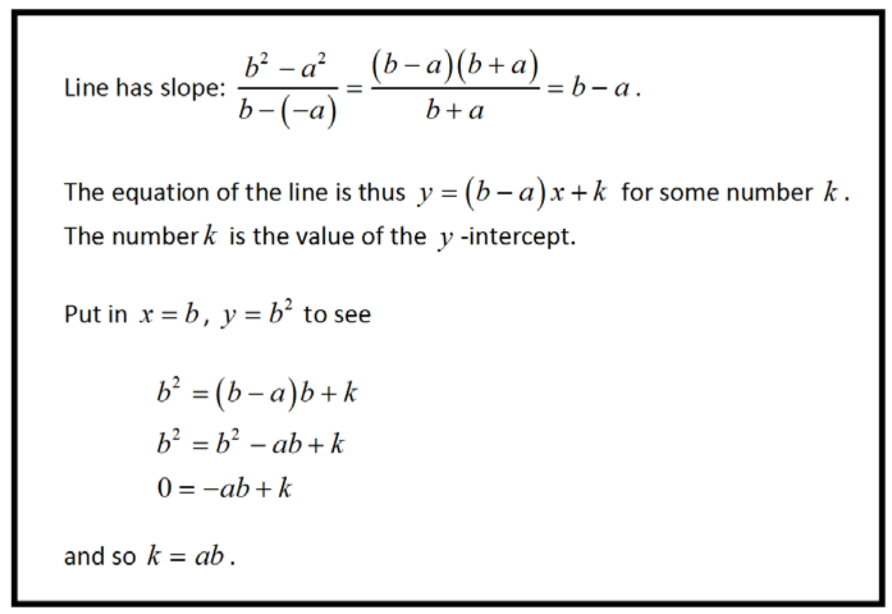
Look at the numbers not crossed on the vertical axis.


20
18
16
14
12
10
8
6
4
2
-4
-2
2
4

Of course, the real question is:

Why does this parabolic multiplication work?





Line has slope: $\dfrac{b^2 - a^2}{b-(-a)} = \dfrac{(b-a)(b+a)}{b+a} = b - a$.

The equation of the line is thus $y = (b-a)x + k$ for some number k.
The number k is the value of the y-intercept.

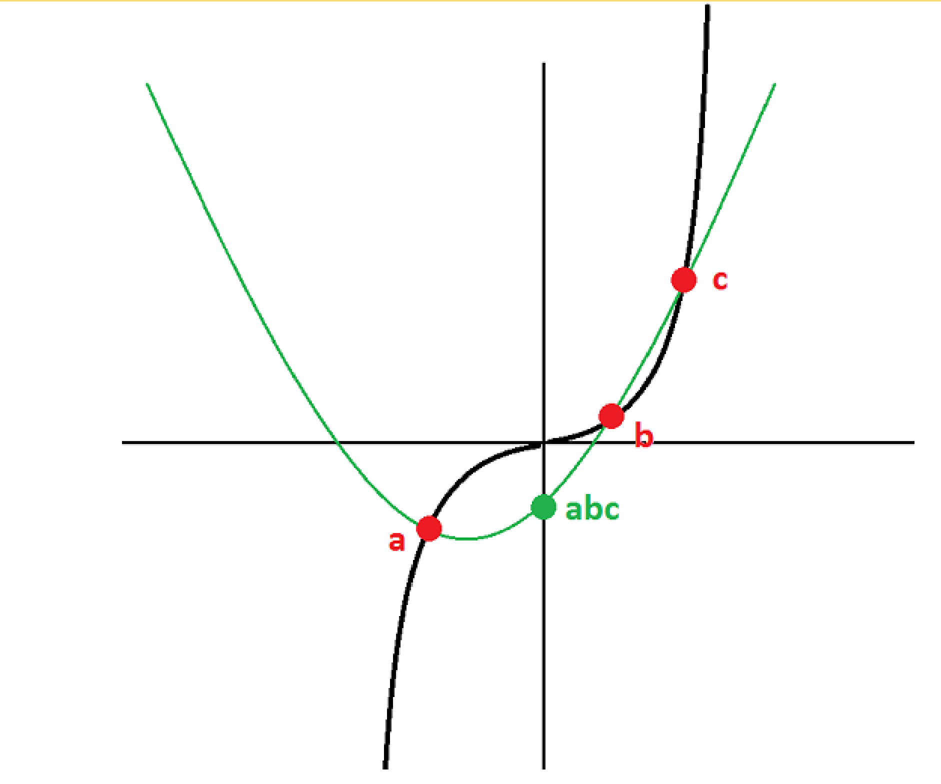
Put in $x = b$, $y = b^2$ to see

$$b^2 = (b-a)b + k$$
$$b^2 = b^2 - ab + k$$
$$0 = -ab + k$$

and so $k = ab$.

Multiplication from other curves? Cubic curves?

The cubic curve y = x^3 allows you to compute the product of three numbers!



On the graph of $y = x^3$ *label points along the curve by their* x *-coordinates, but this time permit negative labels.*

To find the product of three real numbers a *,* b *, and* c *simply(!) draw the unique parabola that passes through the three points with those labels. The location at which this parabola crosses the* y *-axis is the product* abc *.*

THANKS!!

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