Wolves and Sheep

Place 5 wolves on a 5 by 5 chessboard so that 3 sheep are safe. Wolves move like chess queens (any number of squares horizontally, vertically, or diagonally). A sheep is safe if no wolf can reach it in one turn. For instance, the first grid below shows a solution with 6 wolves and room for 2 sheep. The squares marked W are wolves, x marks the unsafe spaces, and S marks the two sheep spaces.

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W x W W x
W x W x x
x x x x W
x S x x x
x S x x x
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Additional Questions

1) Solve a simpler problem: How many sheep can fit with 0 wolves? 1 wolf? 2 wolves?

2) How many different ways are there to place the 5 wolves on the board? (ignoring the sheep)

3) How many different ways are there to place the 3 sheep on the board? (ignoring the wolves)

4) How many different solutions are there to the original problem? Can you prove your answer?

5) Generalize: How many sheep can fit with \( w \) wolves on an \( n \) by \( n \) board? Perhaps some special cases (small values of \( w \) and/or \( n \)) would be easier to answer to help get you started.