

Compositions

There are four ways for 3 to be the total of a list of positive integers:

$$3 = 2 + 1 = 1 + 2 = 1 + 1 + 1.$$

Some people prefer to write the same thing with tally marks instead of numbers:

$$| \quad | \quad | = | \quad | + | = | + | \quad | = | + | + |.$$

We say there are four *compositions* of 3. The numbers making up each composition are called the *parts*, so for example one of the compositions of 3 has first part 2 and second part 1. We'll find some patterns by organizing these lists of compositions in different ways.

1. List all the compositions of 1. Now try 2. Now try 3 (which you can just copy from the introduction up there!). Now try 4. How many compositions does it have?
2. Do you see a pattern here? Can you see why the pattern will continue that way? How many compositions will 10 have?
3. Now we're going to organize our compositions by the size of the first part. For 3, we see that two of the compositions begin with 1, one composition begins with 2, and one with 3. Can you finish filling in the chart below? What patterns do you notice? Why do they occur?

		First part of composition							
		1	2	3	4	5	6	7	8
1									
2									
3	2	1	1						
4									
5									
6									
7									
8									

4. If you look at all the compositions of a number, how many of them are made of only odd parts? For instance, some of the compositions of 5 that would work are 5, 1 + 3 + 1, and 1 + 1 + 1 + 1 + 1, but there are others. How many?

