A Grid of Liars

Ryan Morrill
rmorrill@ualberta.ca
University of Alberta

Say you have a row of 15 people, each can be either a knight or a knave. Knights always tell the truth, while Knaves always lie.

Each person claims that there is exactly one knave adjacent to them.

What are the possible distributions of the knights and knaves?

If the first person is a knight,

If the first person is a knight,

If the first person is a knight,

If the first person is a knave,

If the first person is a knight,

If the first person is a knave,

then we get a contradiction!

From here we are in a good position to write out the solution(s) for a row of n people, for any n.

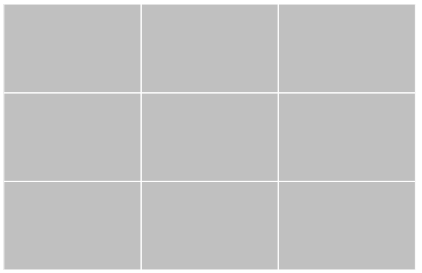
We can also look at an infinite row of people, or a row that's finite on one end and infinite on the other.

Where to next?

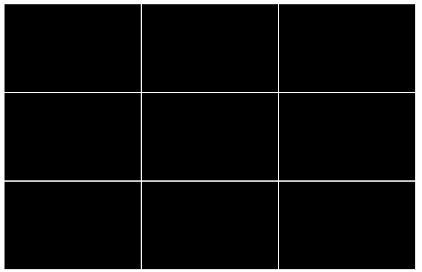
Consider a 3x3 grid of people, each claiming that one of the adjacent(North, West, East or South) people is a knave.

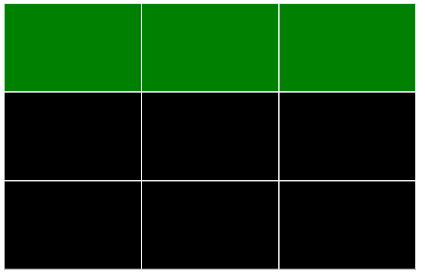
What solutions are there?

The 3x3 grid:



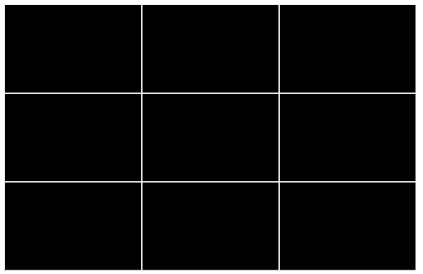
There are two solutions, up to rotation/reflection.

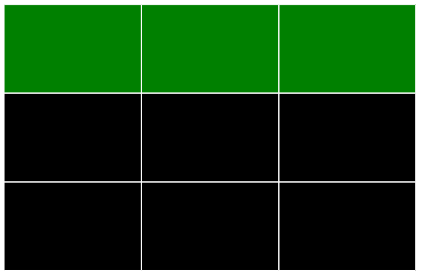




It is not too hard to show these are the only two, you just need to break up your analysis into cases. I suggest one case to be “one corner is a knight” and “all corners are knaves”.

The 3x3 grid:

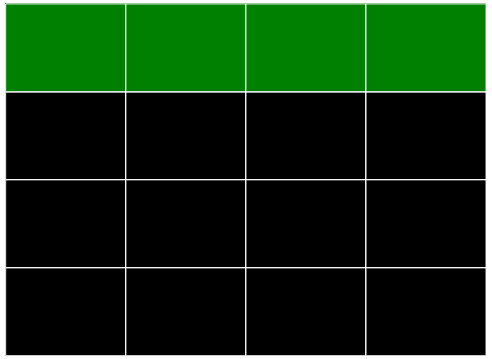


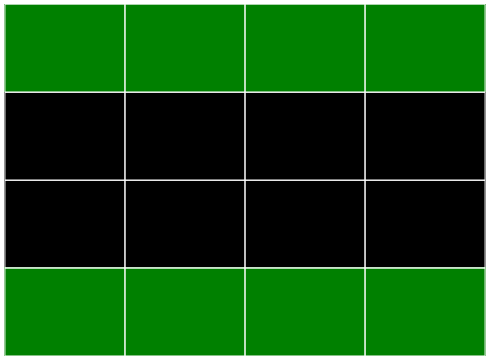


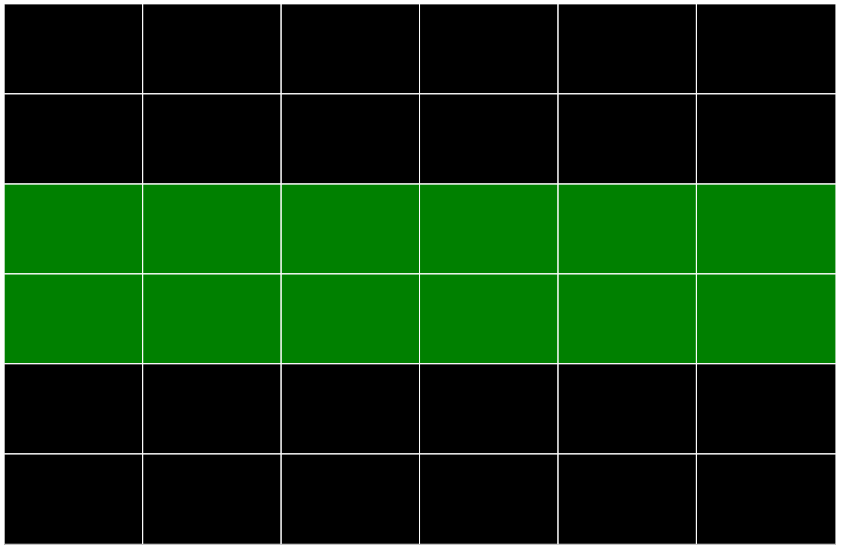
The solution on the left is what we call the “trivial solution”. This solution of making every person a knave works for any nxn board.

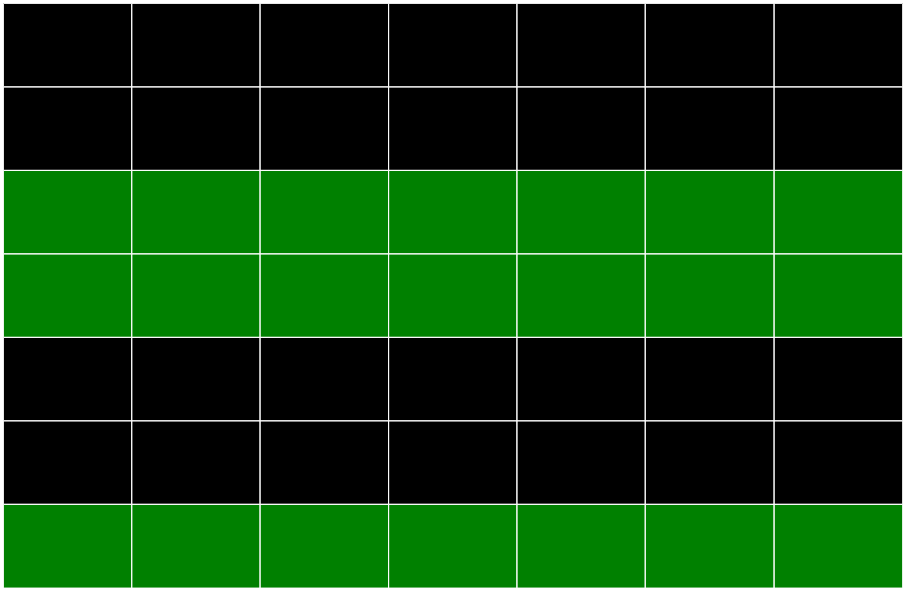
The solution on the right is what we call a “regular solution”. The idea behind this solution is also fairly simple to generalize to any nxn board.

Regular solutions







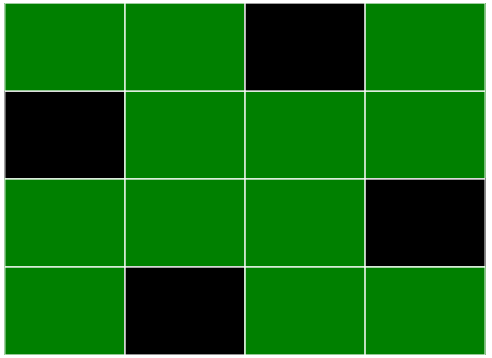


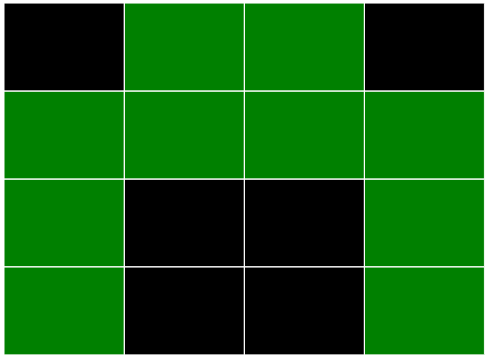
While organizing the regular solutions in a nice way is a nice exercise, the really interesting question is what kind of irregular solutions can we find?

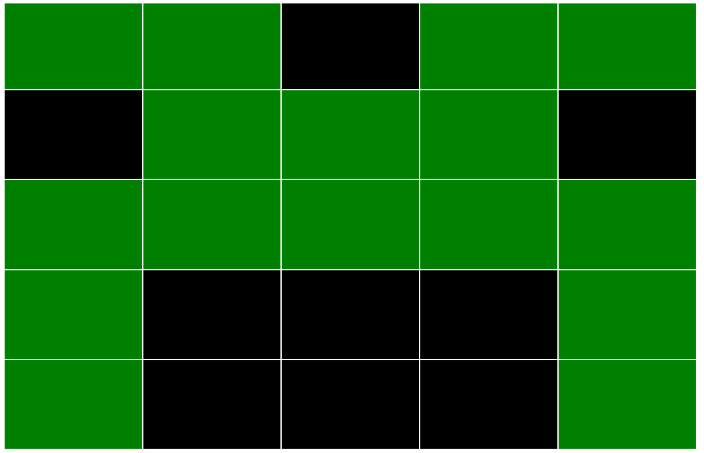
It turns out that

- for the 4x4 there is one irregular solution,
- for the 5x5 there is two irregular solutions,
- for the 6x6 there are no irregular solutions,
- for the 7x7 there are many irregular solutions

Irregular solutions



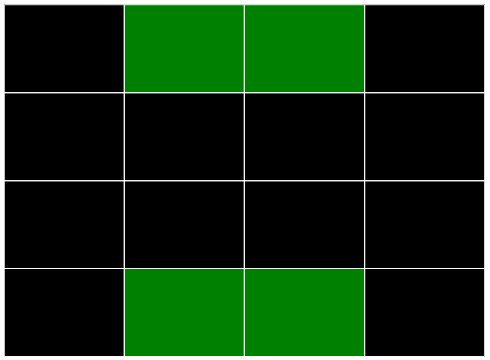


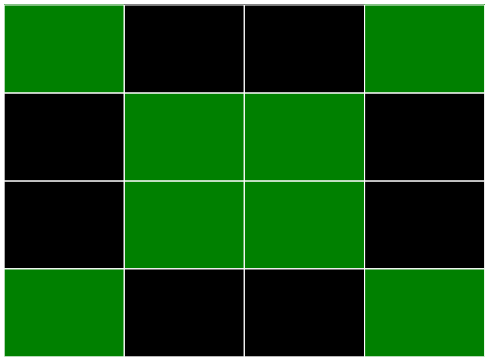


Where to next?

- What if everyone claims two knaves are adjacent?
- What if not everyone claims the same thing?
- What about an infinite plane?
- What about 3 dimensions?
- What about an arbitrary graph with vertices as people and edges indicating who are “adjacent”?

What if everyone claims two knaves are adjacent?





What if not everyone claims the same thing?

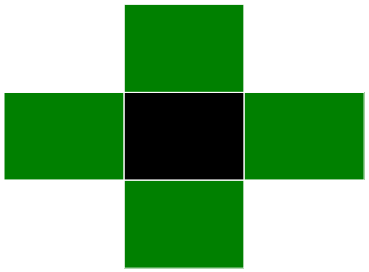
Some examples:

1	2	2
2	1	2
2	2	1

2	2	0
2	1	0
1	2	2

What about an infinite plane?
(everyone claiming one knave to be adjacent)

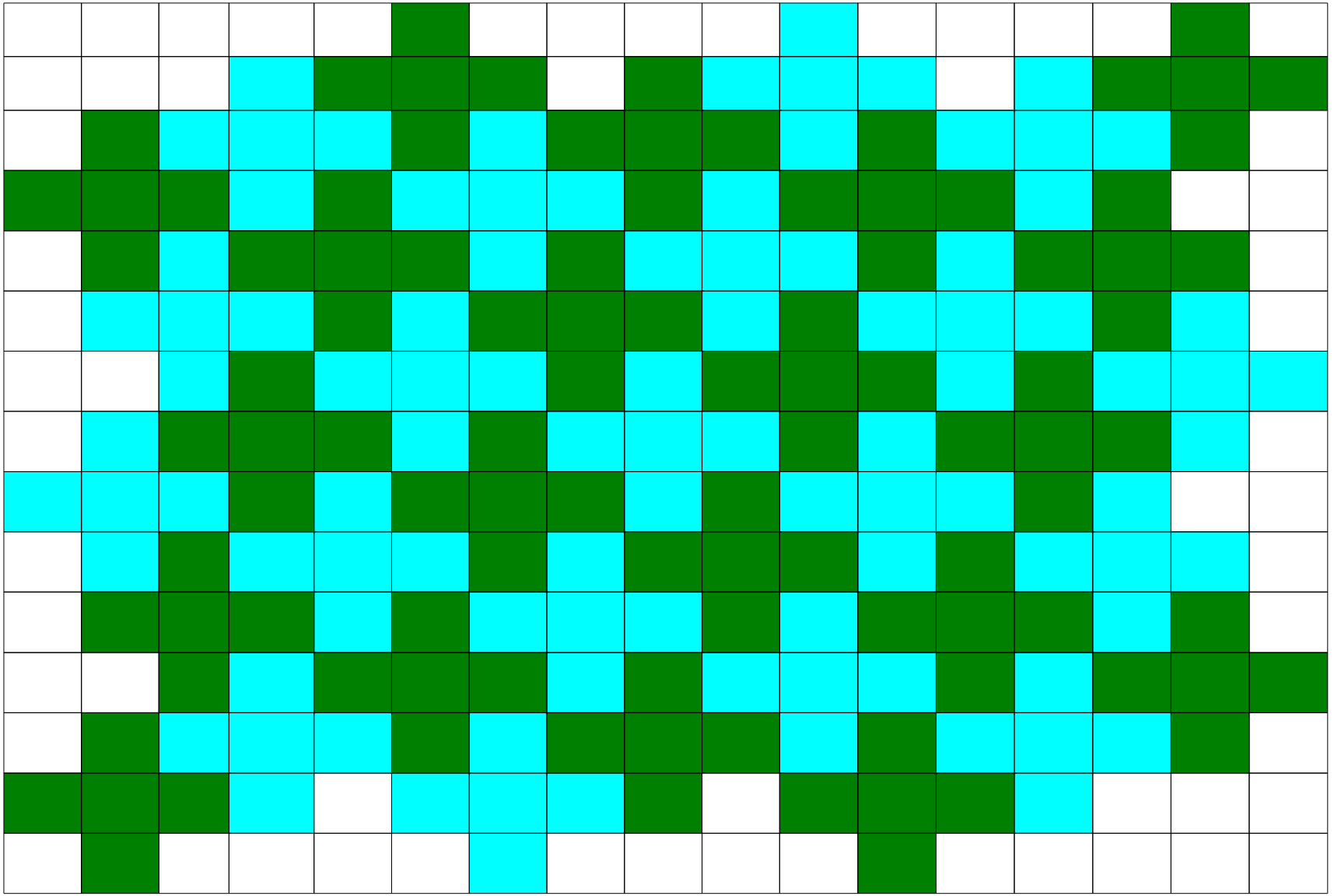
- The regular solutions still exist.
- New patterns turn up!



We can use this to tile the plane!

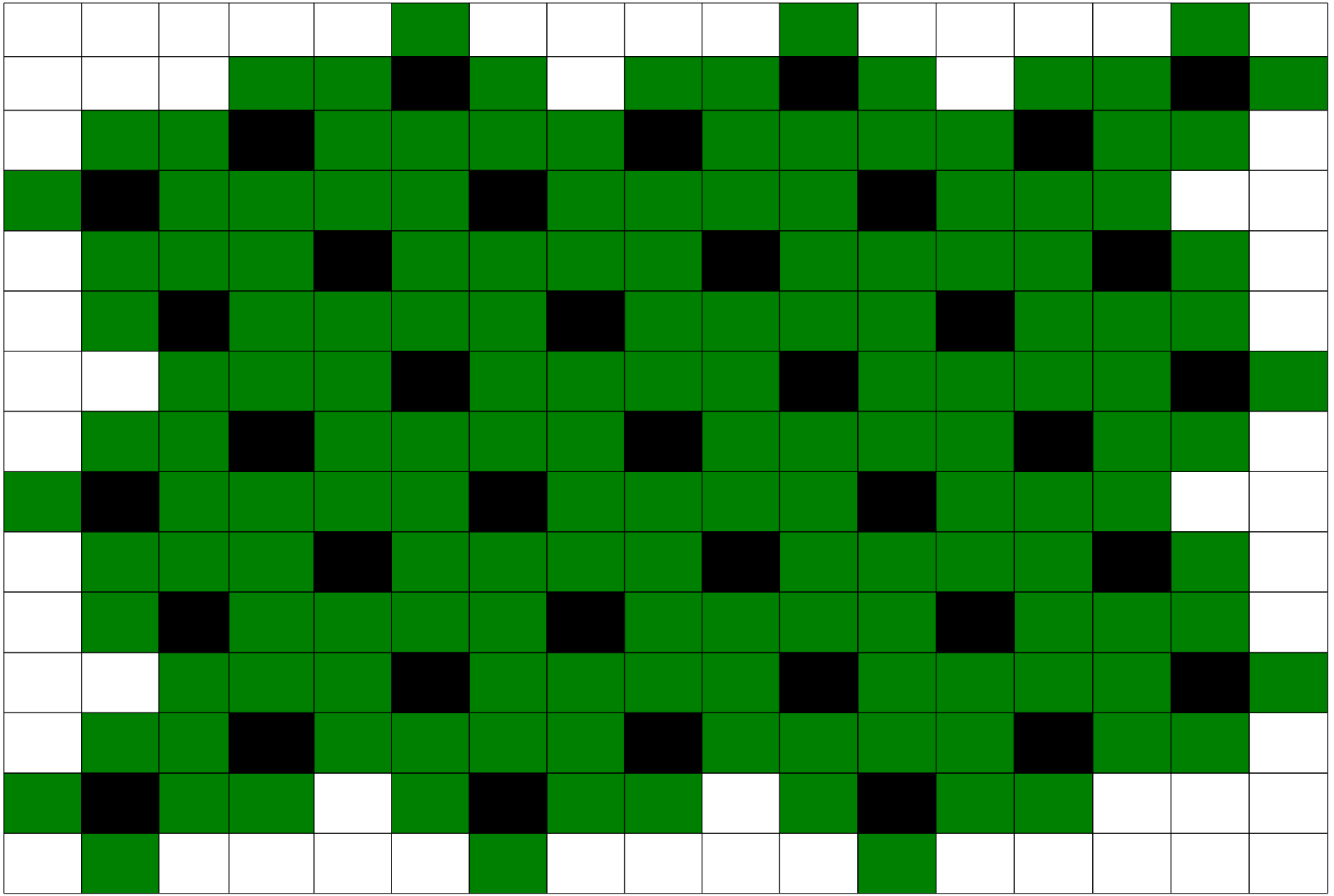
What about an infinite plane?

(everyone claiming one knave to be adjacent)



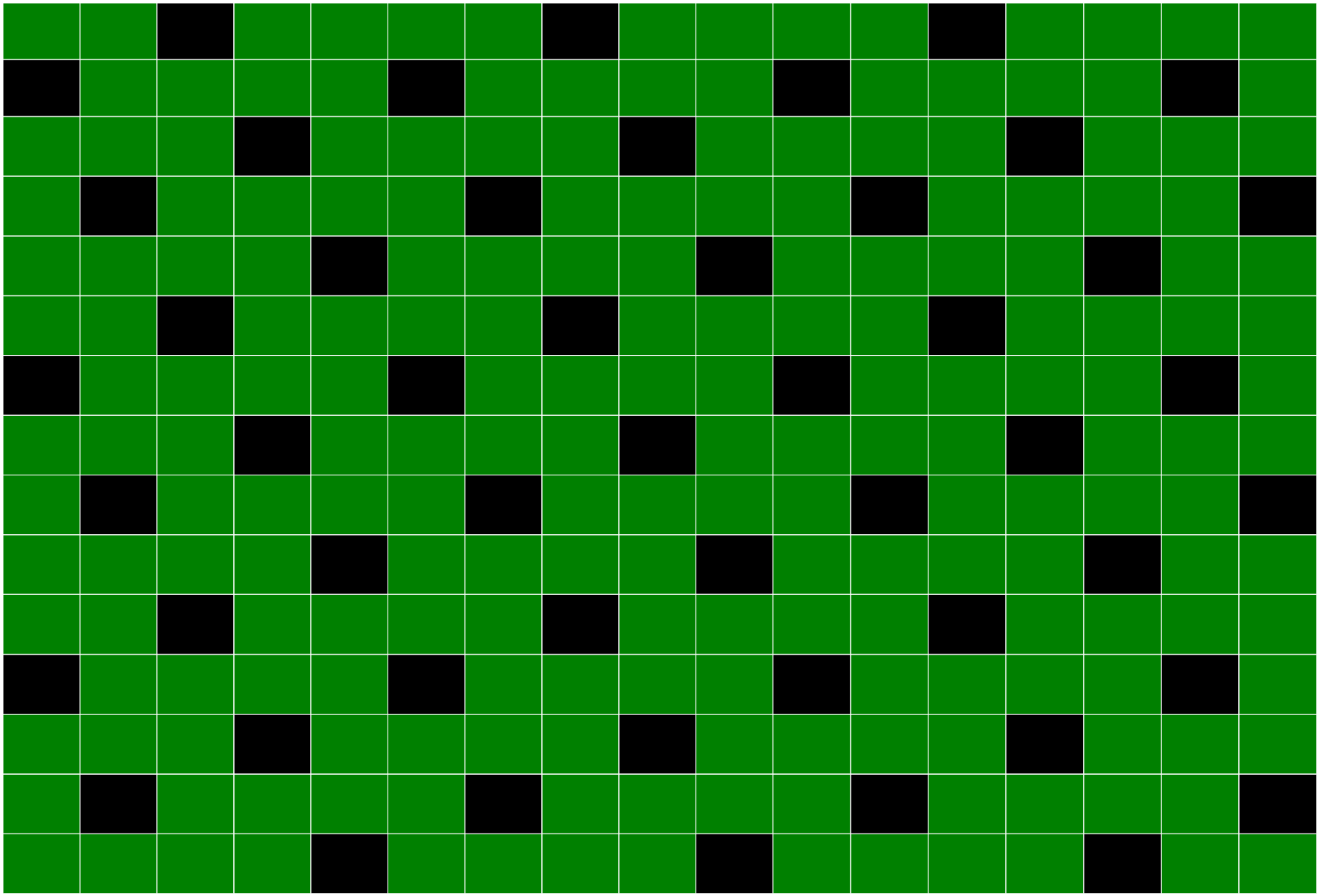
What about an infinite plane?

(everyone claiming one knave to be adjacent)



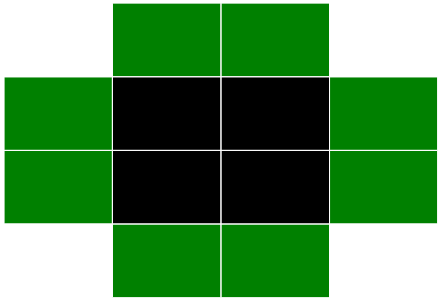
What about an infinite plane?

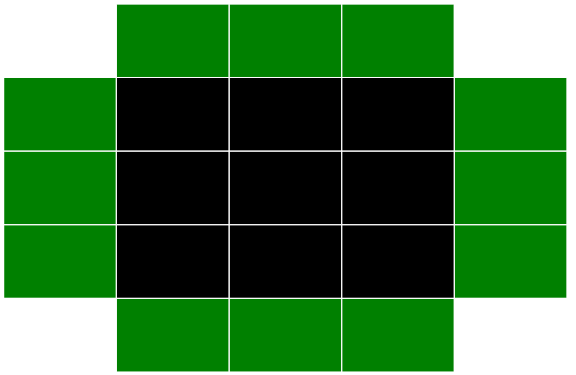
(everyone claiming one knave to be adjacent)



What about an infinite plane?

(everyone claiming one knave to be adjacent)



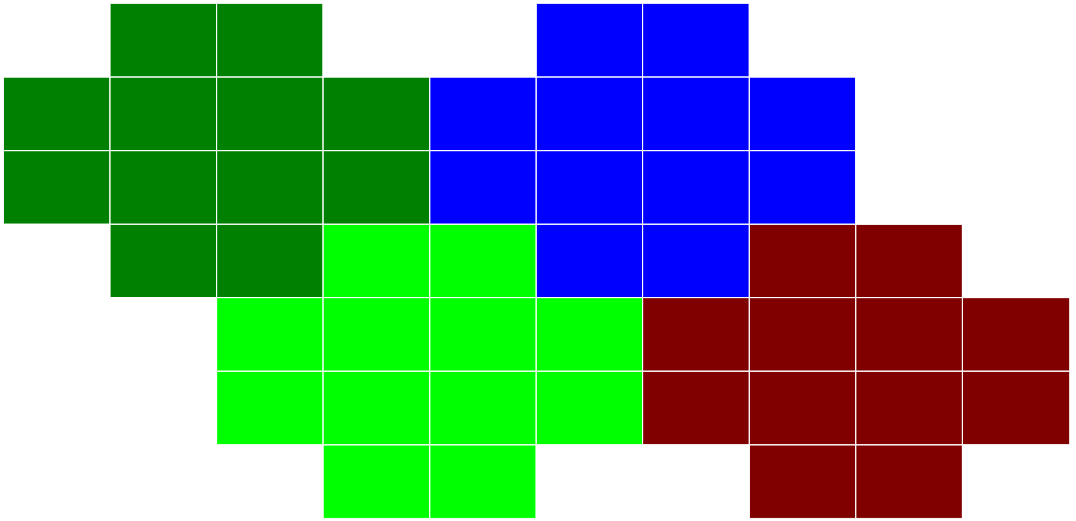


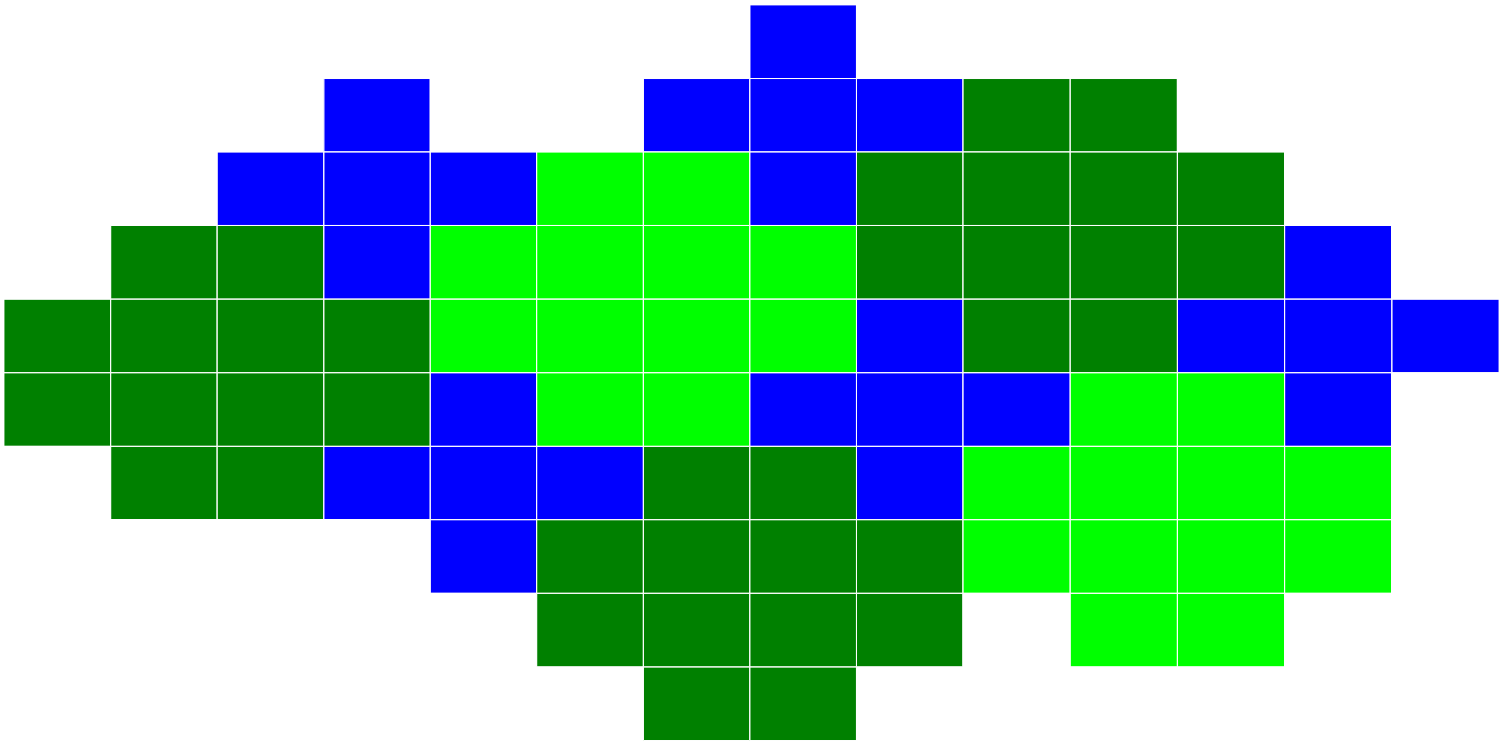
etc...

These can also be used to tile the plane!

What about an infinite plane?

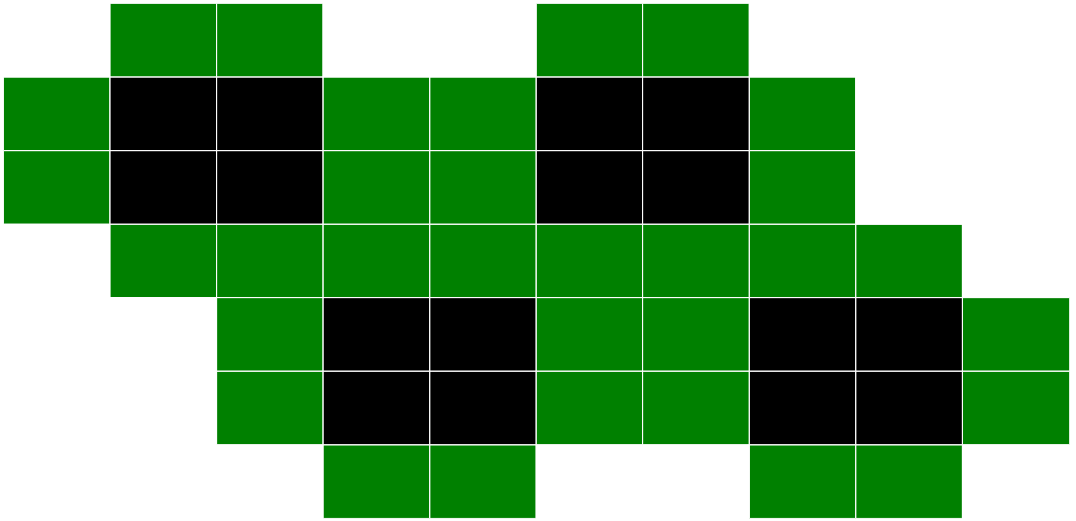
(everyone claiming one knave to be adjacent)

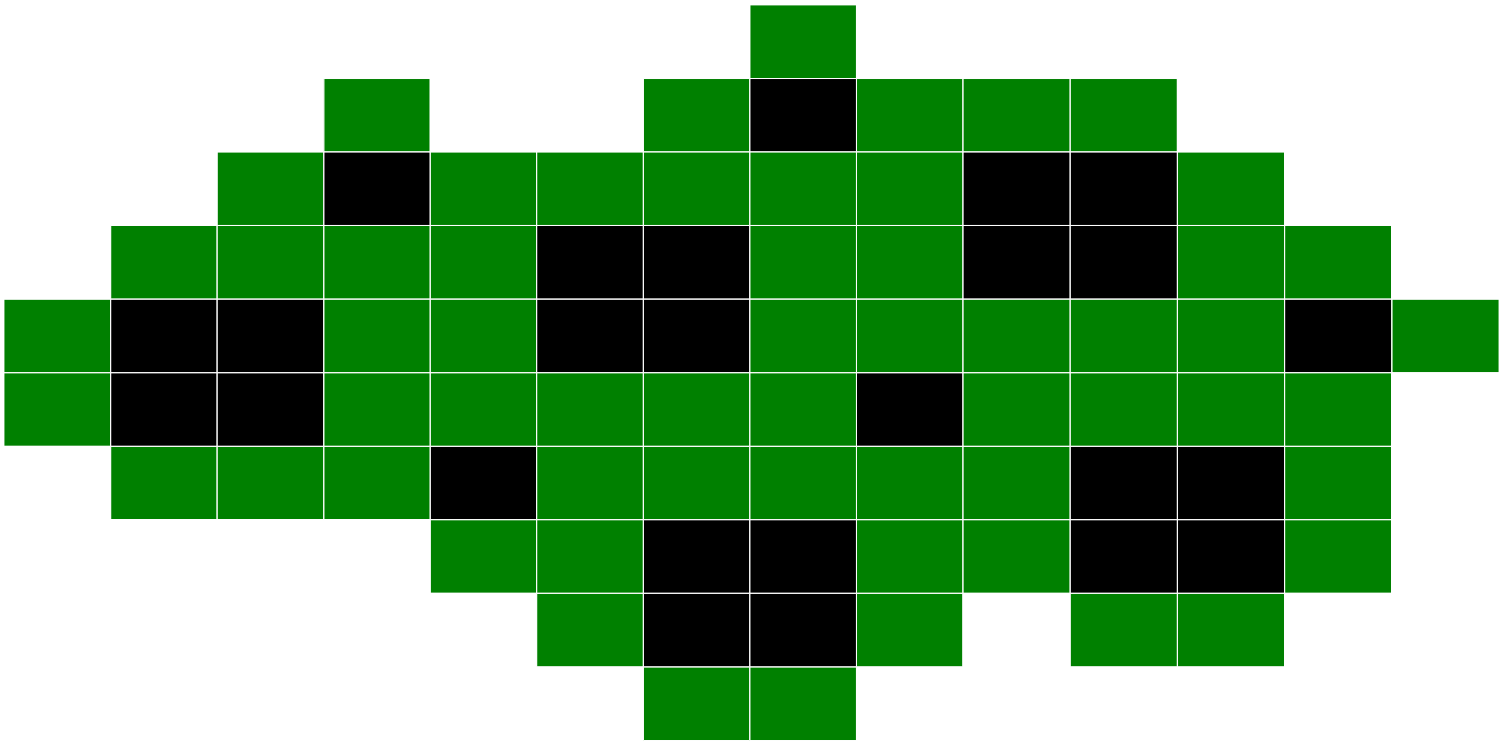




What about an infinite plane?

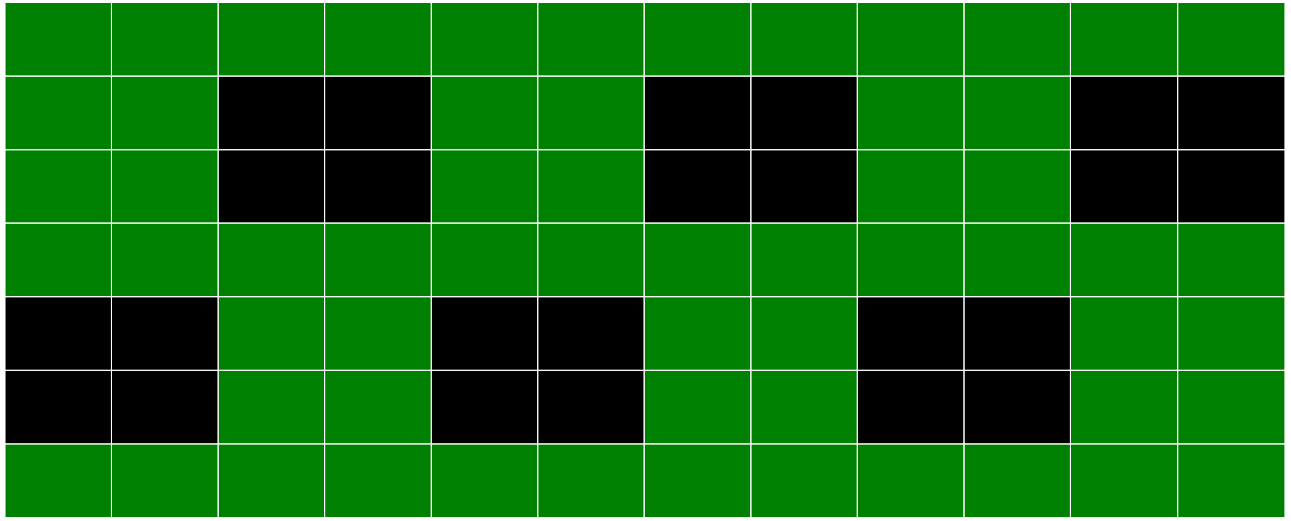
(everyone claiming one knave to be adjacent)

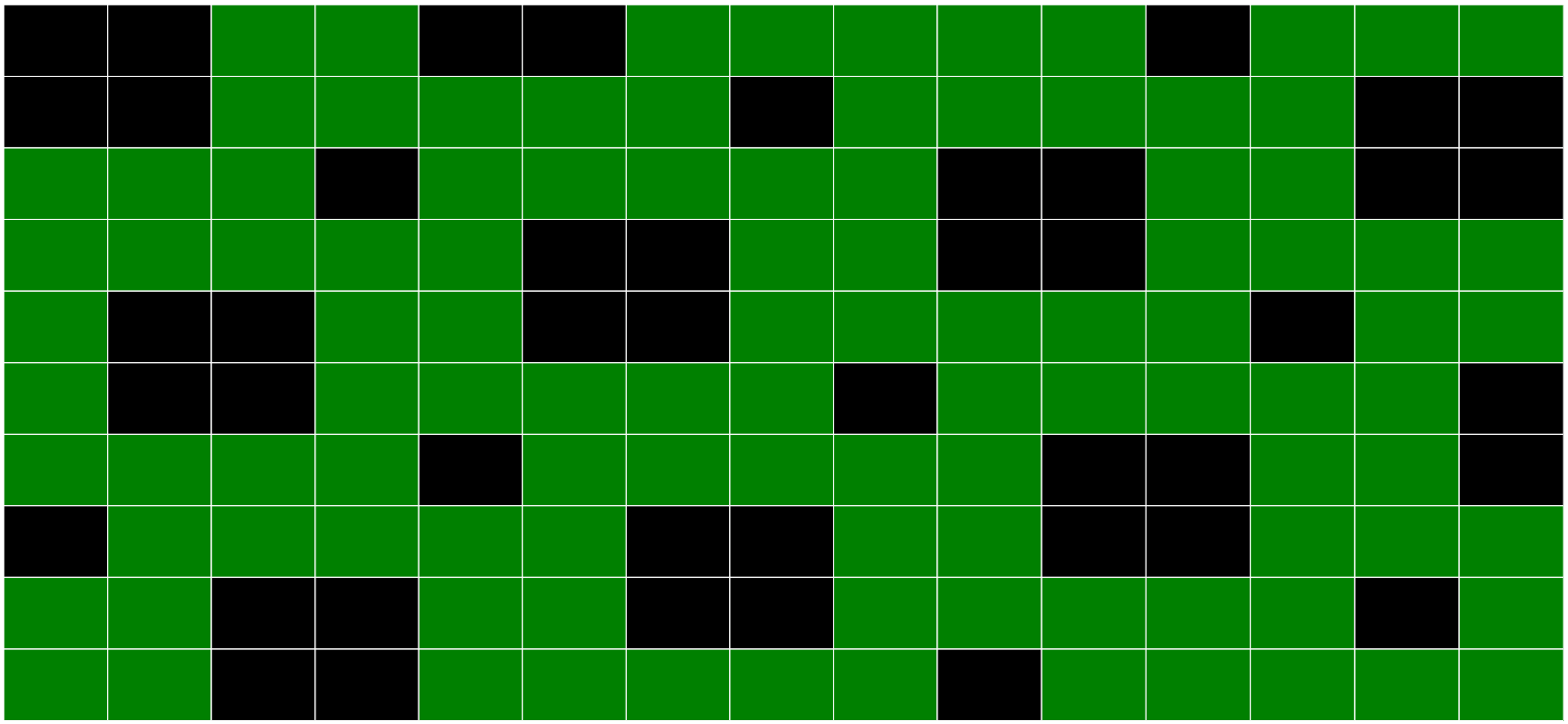




What about an infinite plane?

(everyone claiming one knave to be adjacent)

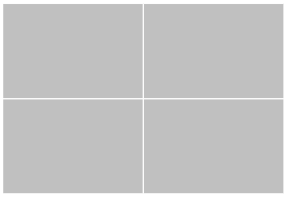




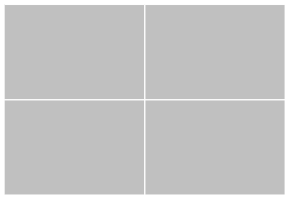
What about 3 dimensions?
(everyone claiming one knave to be adjacent)

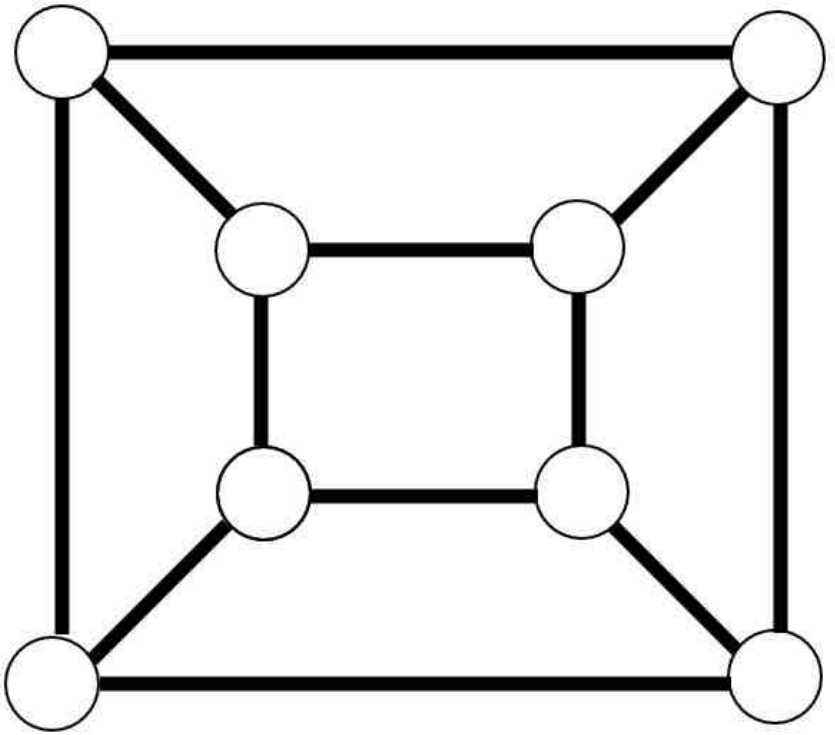
- There is a 2x2x2 apartment complex with 8 units.
- There is a knight or a knave in each unit.
- Each resident claims that there is one knave living next to or above/below them, but not diagonal.

bottom floor



top floor

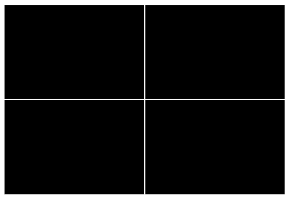




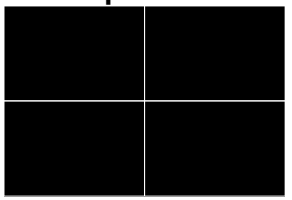
What about 3 dimensions?

(everyone claiming one knave to be adjacent)

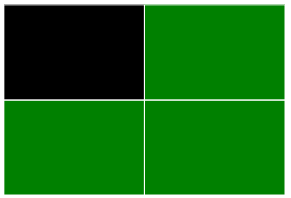
bottom floor



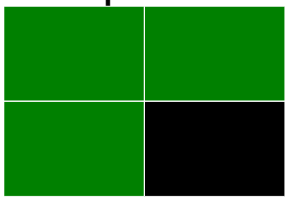
top floor



bottom floor



top floor



Thank you!