Math Circles for Integrated STEM Learning Communities

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Abstract

Aligning with the Vision 2025 for mathematical sciences and following the legacy of Felix Klein, these novel math circle activities bring together mathematics and science teachers, mathematicians, and undergraduate peer mentors to study the dynamics of an invasive lionfish population that is threatening the local ecosystem. We consider alternative harvesting regimes and study their mathematical and scientific models using both continuous and discrete data. This approach allow teachers to practice mathematical modeling for an integrated STEM learning using population growth as a theme.

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Aligning with the Vision 2025 for mathematical sciences and following the legacy of Felix Klein, these novel math circle activities bring together mathematics and science teachers, mathematicians, and undergraduate peer mentors to study the dynamics of an invasive lionfish population that is threatening the local ecosystem. We consider alternative harvesting regimes and study their mathematical and scientific models using both continuous and discrete data. This approach allow teachers to practice mathematical modeling for an integrated STEM learning using population growth as a theme. Exemplified math circle activities provide support for teachers and students looking to develop interdisciplinary connections between discrete mathematics, calculus, statistics, and biology which have an impact on the local ecology and fishing industry.

Keywords: Mathematics Circles, STEM Teacher Circles, Population Dynamics, Recursive Models, Cross-curricular projects, Connecting Discrete and Continuous Models

Overview

Background for Interdisciplinary Math Circles

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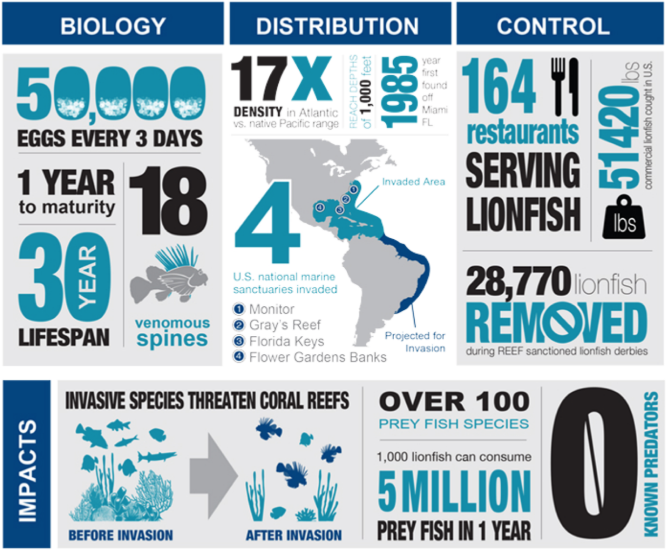
- **Our stance:** Integrated STEM with modeling & PBL

Interdisciplinary Math Circle for Learning M in STEM

- Math emphasized: discrete and continuous modeling of population dynamics.
- Share disciplinary advanced perspectives and most up-to-date research on dynamics and control of local populations such as lionfish, sea turtles, deer and conch.
- Provide needed fundamental, diverse, and rich mathematical perspectives on population dynamics for math and science teachers:
  - Fundamental - Math for collaborating science teachers.
  - Diverse - Mathematical perspectives for math teachers: continuous, discrete and statistical methods for population dynamics.
  - Rich - inclusive perspectives with mathematicians, scientists, community experts and partners such as CORE, environmental protection agencies.

- **When: Summer++**: Summer institute activities on integrative math & science projects.
- **Yearlong Followup** providing focused math content training with interdisciplinary math circles.
- Circles on population dynamics and control were June 29, 30, Nov 4 and Nov 18.
- Building on Klein's Legacy: Advanced Perspectives on Elementary Mathematics
- **Support:** NSF VI EPSCoR Mare Nostrum Caribbean

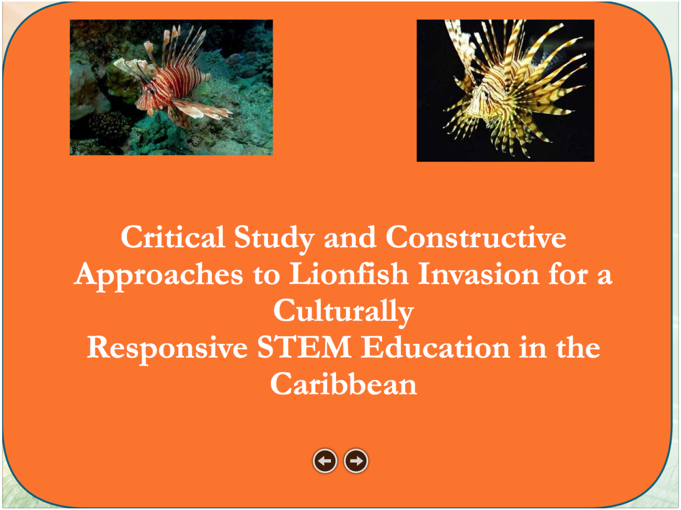
Lionfish Problem for the Caribbean



Driving Question for our Interdisciplinary Math Circles?

Can You Eat to Beat the Lionfish Problem?

Ongoing STEM Teacher Project on Lionfish since Fall of 2015



AFT-Lionfish PowerPointAFT.pptx

Ongoing STEM Teacher Project with Lionfish

AFT-Lionfish PowerPointAFT.pptx

Mathematical Descriptions by STEM Teachers in the Lionfish team

Are there lots of lionfish on our reefs?

The first assessment of lionfish densities reported an average of 2 lionfish per acre across 4 locations in 2008. Presently this number has grown exponentially. They will continue to grow at this rate.

Unknown in the Americas 30 years ago, lionfish have multiplied at a rate that is almost unheard of in marine history. (Visuals Unlimited / Corbis)

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Launching Question

1. How do you describe the population growth mathematically?

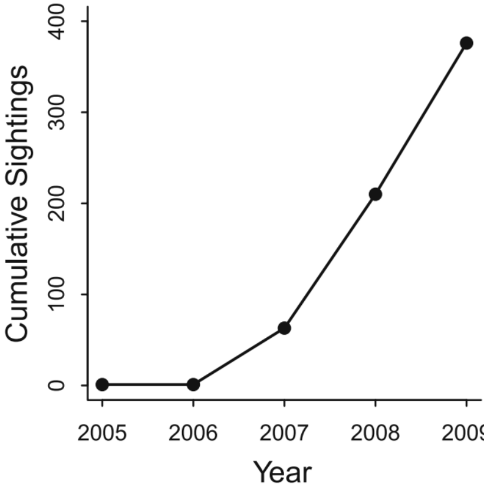


Fig. 1 Cumulative number of lionfish sightings at 7 co

Mathematical Models for Populations:

- Continuous Models as analytic solutions to differential equations $P'(t) = f(P, t)$.
- Discrete Models with difference equations using recurrences for numeric approximations.
  $\Delta P = f(P_n)$
- Statistical Models: population parameter estimations

Different Mathematical Models

Each discrete and continuous model can be examined in linear, exponential, and logistic growth scenarios.

Analysis of the Behavior - Local and Global

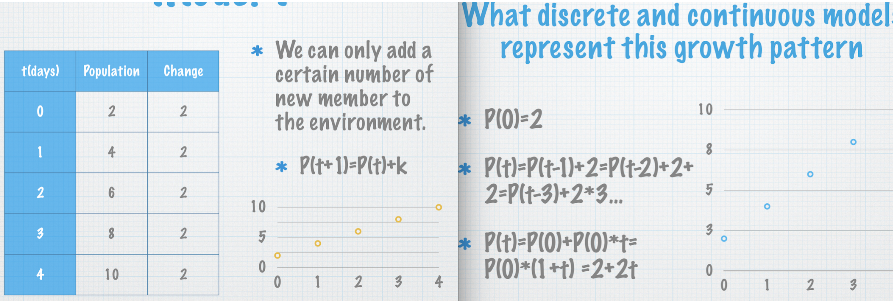
- numerical
- graphical
- algebraic
- sensitivity to P_o, r
- verbal/interpretive- changes of assumptions-scenarios

Representations of Three Growth Scenarios

Verbal	Analytic	Difference	Differential Equations
Constant change	Linear	$\Delta P = P_{n+1} - P_n = r$	$P'(t) = (1 + r)$
Unbounded	Exponential	$\Delta P = rP_n$	$P'(t) = (1 + r)P$
Limited	Logistic	$\Delta P = rP_n(1 - \frac{P_n}{C})$	$P'(t) = (1 + r)P(1 - \frac{P}{C})$

Table: Summary for the Models for Population Dynamics

Linear Difference and Differential Equations



t(days)	Population	Change
0	2	2
1	4	2
2	6	2
3	8	2
4	10	2

Figure: Representations of Discrete and Continuous Linear Models.

Logistic Models

1. Discrete:
   $P_{n+1} = (r+1)P_n - \frac{r}{K}P_n^2$
2. Continuous $P(x) = \frac{K}{1+(K/P_0 - 1)e^{-rt}}$,
   $P_0 = 2,\ K = 350,\ r = 0.24$

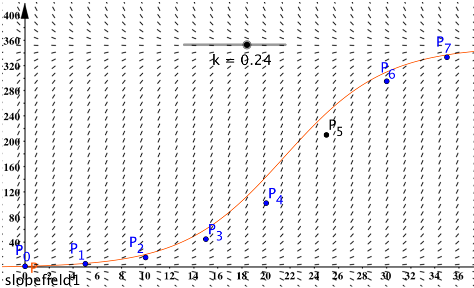
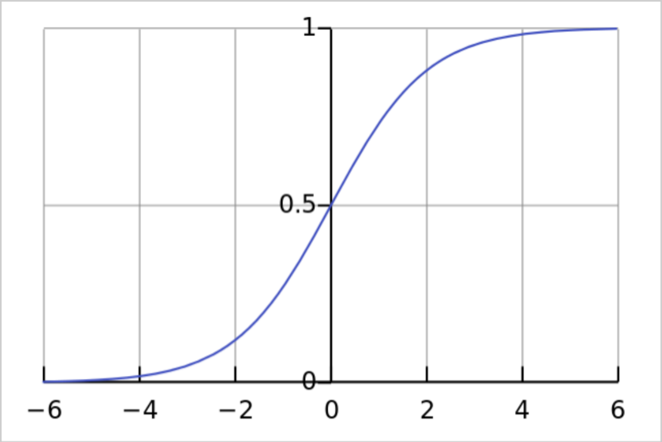


Figure: Graphical Reps of Discrete and Continuous Models.

	Discrete Solns	**Continuous Solns**	Error
P_1: t= 5	6	7	-1
P_3: t= 15	45	61	-16
P_5: t= 25	210	245	-35
P_7: t= 35	333	337	-4
P_9:t= 45	349	349	0

Table: Model Comparison

Model fitting for given s-shaped data with parent functions



Standard logistic sigmoid function

Figure: Transforming sigmoid into a given data, $a\tanh(b(x-c))+d$

How much is enough? harvesting regimes and control

Derbies Remove Lionfish by the Thousands

The Green Turtle Cay Lionfish Derby held in June 2009 in Abaco, Bahamas, was the first lionfish derby held in the invaded region. In a single day, 1,408 lionfish were removed from local waters by 18 teams competing for prize money and bragging rights. Prizes are offered for most, largest, and smallest lionfish. Since that time, numerous derbies have been organized around the region, serving to educate divers, raise awareness, and remove lionfish. The Florida Keys hosts a derby series (Figure 4.21) and some recent derbies in Mexico and the Bahamas have removed well over 2,000 lionfish in a single day. Monthly ongoing removal contests are also a successful way both to encourage ongoing removals and offer small prizes and recognition to divers and dive operators.

Visit http://www.lionfishderby.com for derby examples.

Figure 4.20 Lionfish derbies can provide intensive local removal success as well as excellent opportunities for outreach and the chance for participants to feel a sense of community and purpose.

Harvesting Impact Graphically

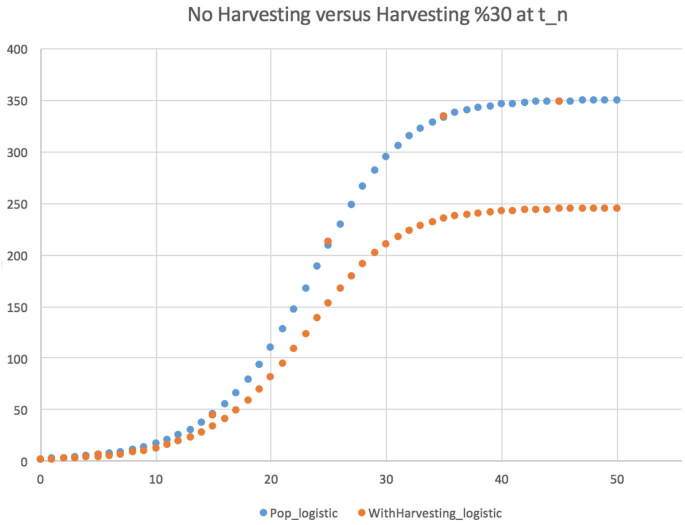


Figure: 30% is harvested at time t_n

As a Dynamical System - Fish Population Dynamics

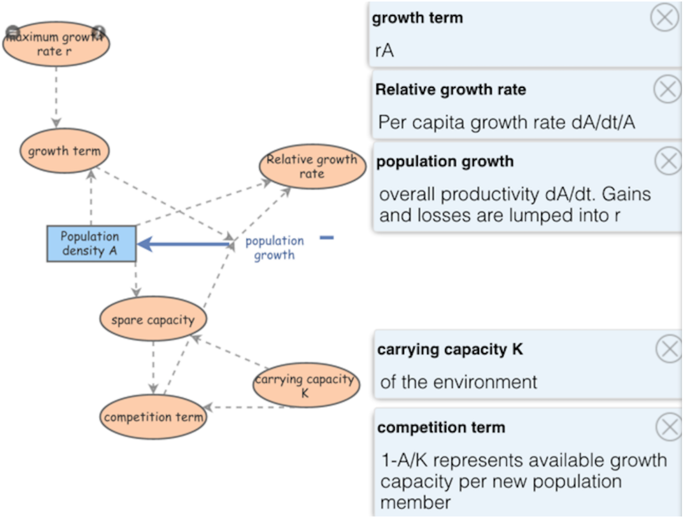


Figure: Dynamical System modeling with Insightmaker

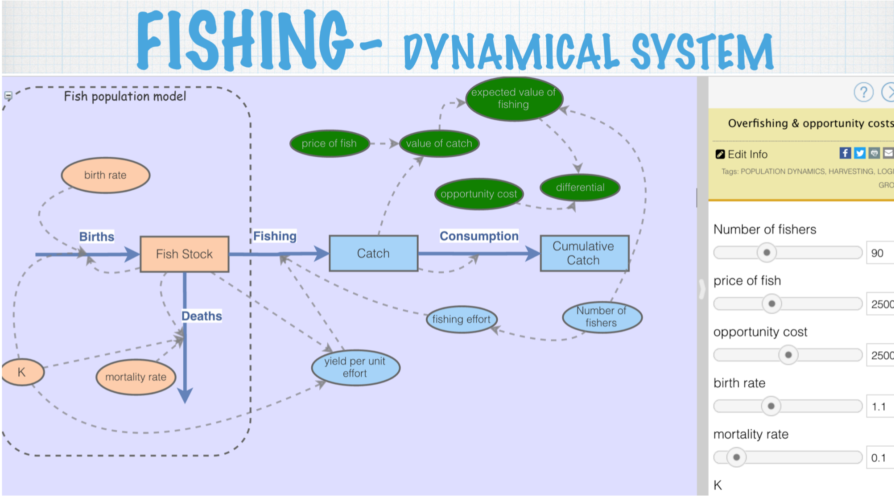


Figure: Dynamical System Detailed

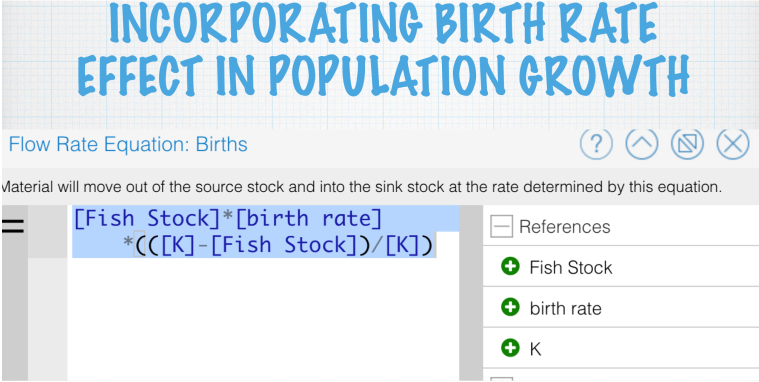


Figure: Dynamical System modeling

Conclusions

- Discrete approach made advanced math more approachable for high school
- Teachers reported that students are deeply **engaged** with a locally relevant problem and took more **ownership** of their math and science.
- Provided more **equitable** access to deeper math and science for historically underprivileged students.
- **Progress is slow** with an interdisciplinary and inclusive approach but highly promising.
- Now we have a cohort of mentor teachers providing local training and support
- We are still working on improving teachers knowledge of mathematical content and modeling for population dynamics in high schools.
- Teachers are currently conducting action research to measure its impact in developing STEM identity and orientation.

Thank you for your attention!

etropoulou, E. N., P. D. Siafarikas, and E. E. Tzirtzilakis, A "discretization" technique for the solution of ODEs, Journal of Mathematical Analysis and Applications, vol. 331, no. 1, pp. 279-296, 2007.

asmussen, C. 2001. New directions in differential equations : A framework for interpreting students' understandings and difficulties. The Journal of Mathematical Behavior 20(1):55-87. DOI: 10.1016/S0732-3123(01)00062-1

Rasmussen, C., and O. Kwon. 2007. An inquiry oriented approach to undergraduate mathematics. *Journal of Mathematical Behavior, 26*, 189-194. DOI:10.1016/j.jmathb.2007.10.001

Johnson and Swenarton (2016), Age, growth and population structure of invasive lionfish (Pterois volitans/miles) in northeast Florida using a length-based, age-structured population model. PeerJ 4:e2730; DOI 10.7717/peerj.2730