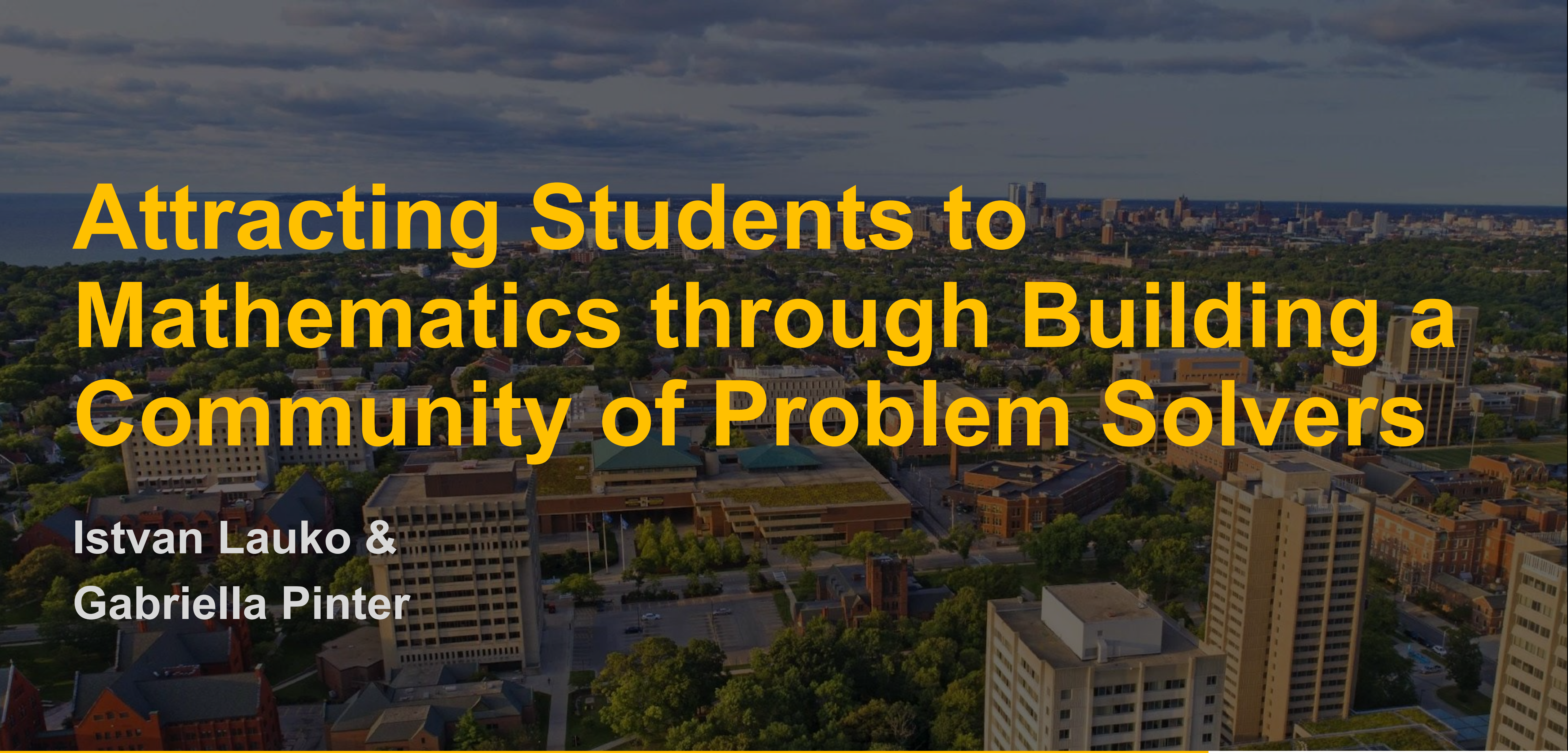

Attracting Students to Mathematics through Building a Community of Problem Solvers

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UWMILWAUKEE

Engagement in Mathematics

- Math Circles
- Math Festivals
- Public Math Projects
- Math Camps, Math courses
- Math Competitions
- Math Outreach
- Math Journals for K-12 Students

квант для младших школьников

Задачи

1. Девочка заменила в своем имени каждую букву ее номером в русском алфавите и получила число 2011533. Как ее звали?

2. Три школьных товарища купили 14 пирожков, причем Коля купил в 2 раза меньше Вити, а Женя больше Коли, но меньше Вити. Сколько пирожков купил каждый из товарищей?

3. Однажды я раскладывал на столе косточки домино, пытаясь сложить какую-нибудь интересную фигуру. После того как получилась фигура, изображенная на рисунке, на руках у меня оставался дупель пусто-пусто. Тут я обратил внимание, что в каждом горизонтальном ряду и каждом вертикальном ряду сумма очков одна и та же. Я перерисовал картинку, поставив вместо очков условные значки. Недавно я нашел эту картинку среди бумаг, но забыл, что означает каждый значок. Помогите мне восстановить их значения.

4. Найдите все числа, которые в 13 раз больше суммы своих цифр.

5. Известно, что убывающий месяц повернут выпуклостью влево, как буква *С* — первая буква слова «старый». Прибывающий же месяц повернут выпуклостью вправо, как закругление буквы *Р* — первой буквы слова «растущий». Везде ли верно это правило?

Эти задачи нам предложили: *В. Кудряшов — ученик 10 кл., г. Пенза, В. Бялко — ученица 7 кл., г. Москва, М. В. Варга, Н. К. Антонович, А. Л. Тоом.*

35

Student journals in different countries

A few examples:

- Kvant (Soviet Union, Russia)
- Gazeta Mathematica (Romania)
- Crux Mathematicorum (Canada)
- KöMaL and Abacus (Hungary)

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Content

- Articles, news
- Ideas for problem solving
- Problem solving contest problems
- Solutions to problems by students
- Chess, logic problems, puzzles
- Coding
- Physics

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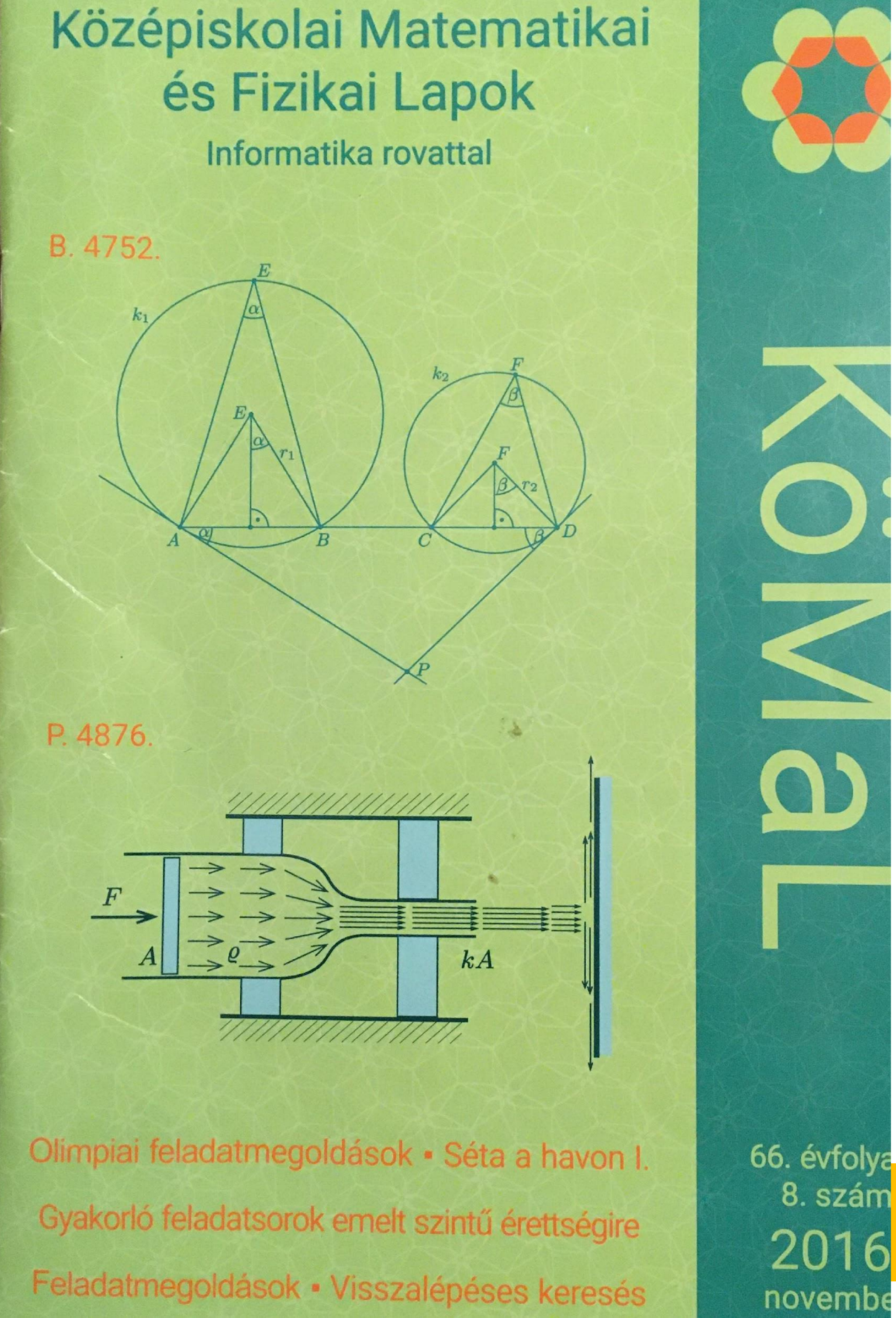


KöMaL – for high school students

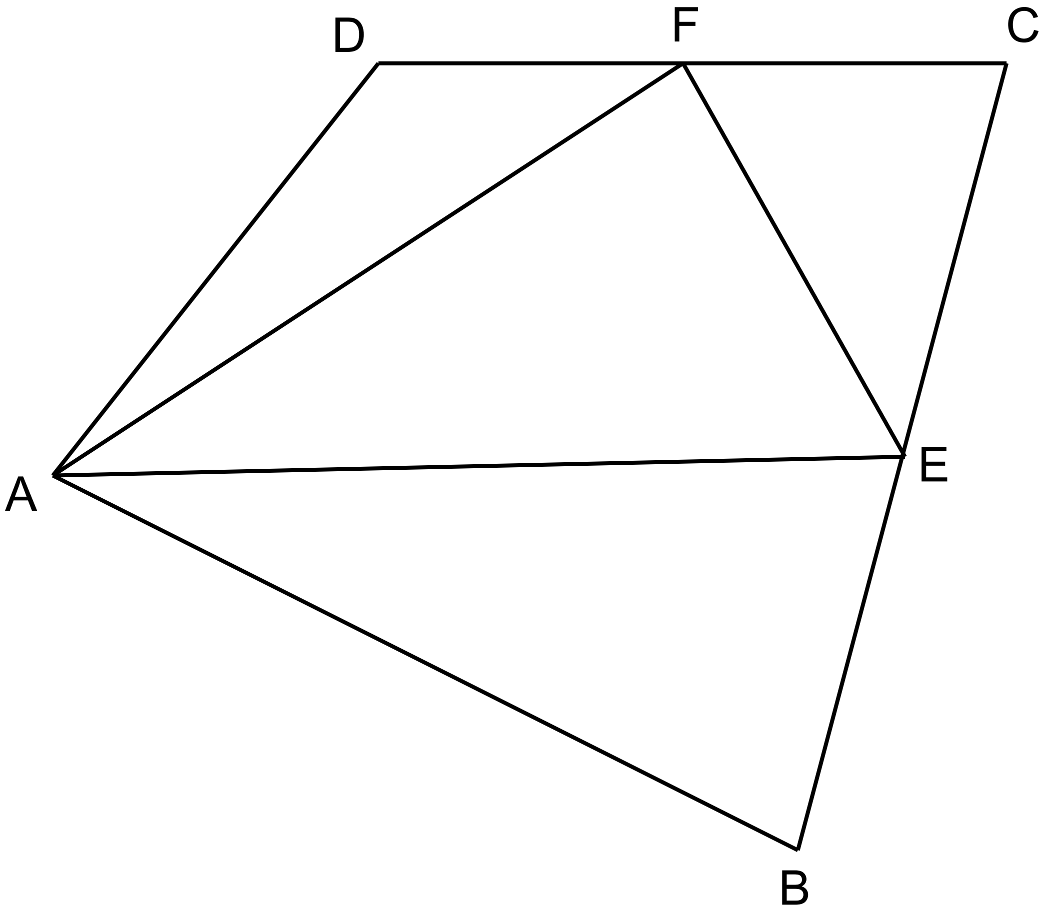
- Founded in 1894 by Dániel Arany
  - appeared 10 times a year (16 or more pages)
  - 239 problems, 1055 solutions submitted by 151 students
  - 132 subscribers in its first year
- 1959 Physics section
- 1976 Informatics section
- Problem solving competitions at several levels

Problem sets

- K - 6 problems for grade 9 or lower
- C - 2+3+2 – for grades up to 10, everyone and grades 11-12
- B - 8 problems (can send solutions for up to 6)
- A - 2-3 problems, very hard

A few favorite problems

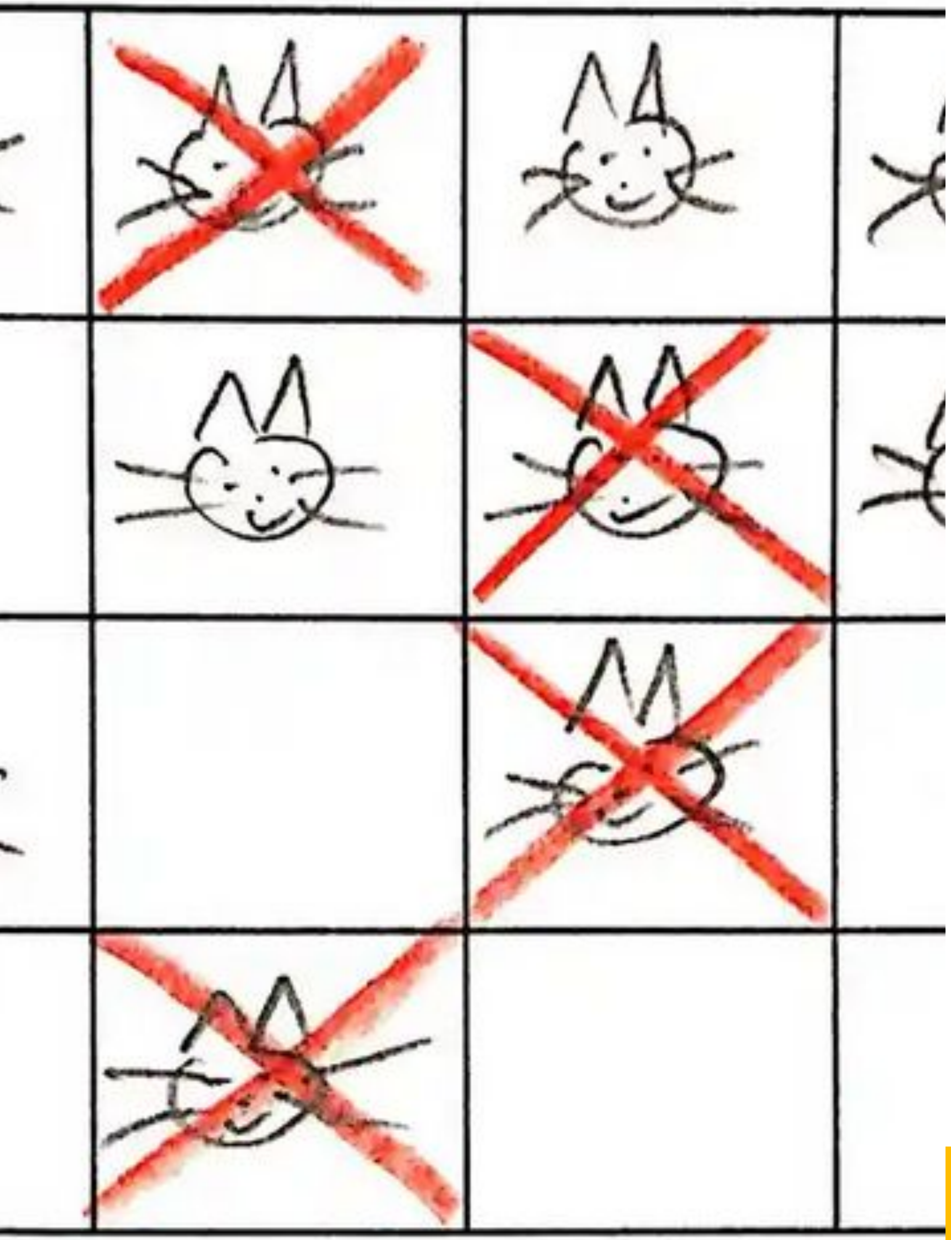
- Let ABCD be a convex polygon, E the midpoint of BC, F the midpoint of CD and draw the line segments AF, FE and AE. These line segments divide the polygon into four triangles whose areas are consecutive positive integers. What is the maximum possible area of the triangle ABD?



Find my number

Al and Bill are playing the following game. They agree on a fixed number $n \geq 3$, and then Al thinks of a number from the set $\{1,2,\ldots,n\}$. Now Bill can guess the number. He will only get yes or no answers. If the answer is yes, the game terminates.
If the answer is no, Al will change the number: either increases or reduces it by 1, but the number must remain positive (it is allowed to go beyond n though). Then Bill can guess again, trying to hit the new number. The procedure is repeated until finally Bill gets the number.

Prove that Bill has a strategy to end the game with at most $(3n-5)$ guesses.

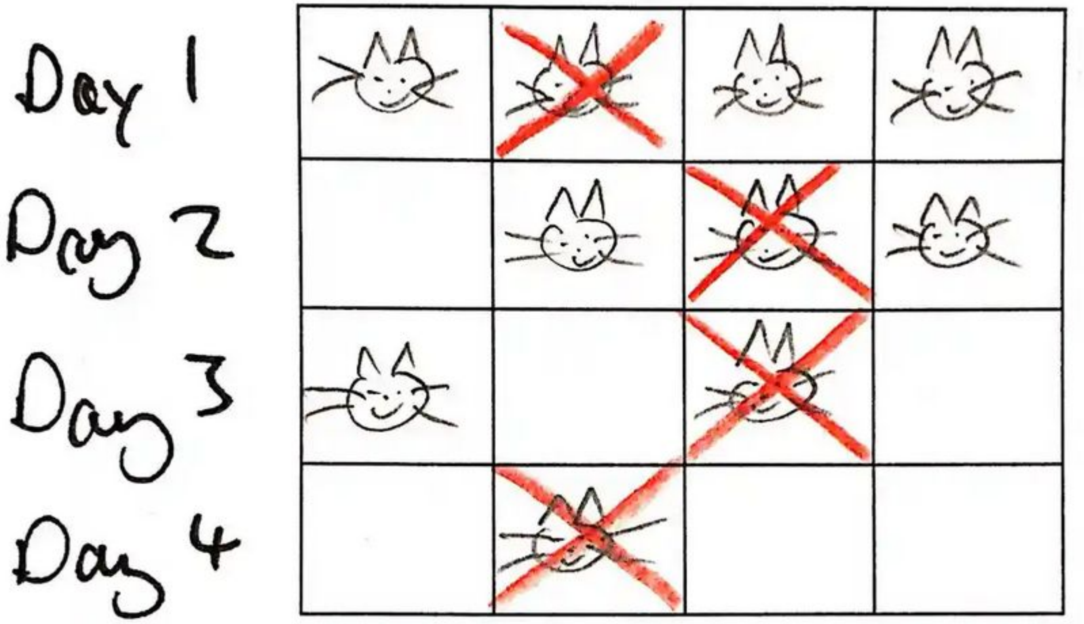


Are you smarter than a cat?

- Alex Bellos (Monday Puzzles in The Guardian)
- *A straight corridor has 7 doors along one side. Behind one of the doors sits a cat. Your mission is to find the cat by opening the correct door. Each day you can open only one door. If the cat is there, you win. You are officially smarter than a cat. If the cat is not there, the door closes, and you must wait until the next day before you can open a door again. However, every night the cat moves one door either to the left or to the right.*

How many days do you now need to make sure you can catch the cat?

Are you smarter than a cat?



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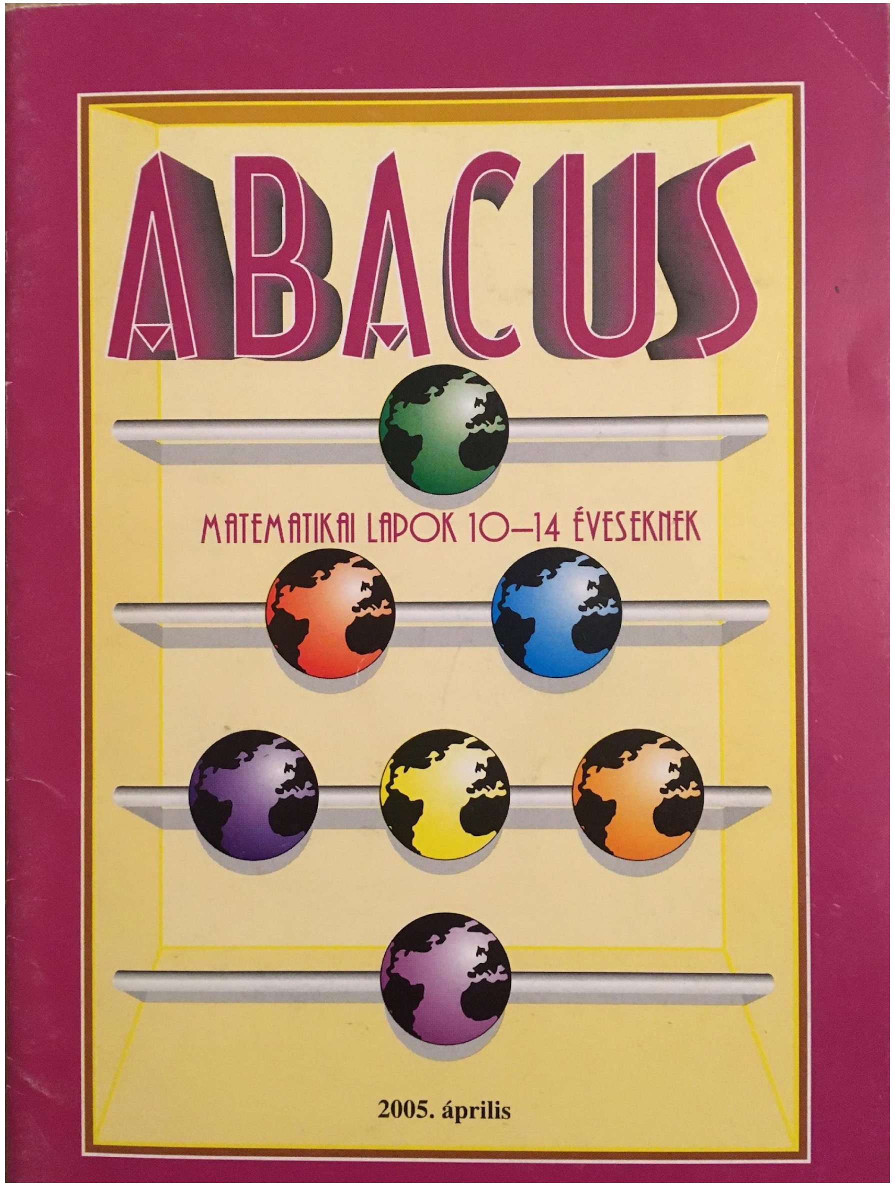
Abacus for grades 3-8

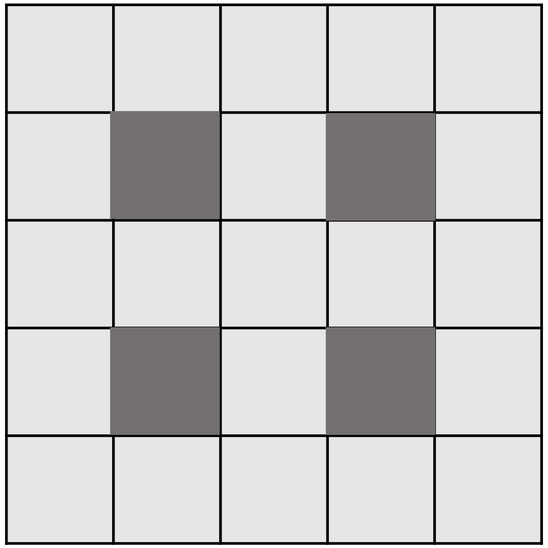
- Problems and solutions
- Logic problems
- Nonograms, Sudoku
- Problems in English and German
- Info-derby
- Chess
- Astronomy
- Book reviews



Ant Maze

A cube is made from 125 1cm x 1cm x 1cm unit cubes. To construct a maze, holes with square cross-sections are made through the cube perpendicular to each face. Grey squares on a face indicate the locations of the holes as shown in the figure. The ant maze is immersed in red paint.

- How many unit cubes make up the maze?
- How many cm^2-s got painted red?



Student journals in the US

- The Mathematics Student – NCTM (1952-1981)
- Quantum Magazine (Math and Science) – NSTA (1990-2001)
- Science News for Students (online)

- Journals for undergraduate students

COMPETITION CORNER

CONTEST ANNOUNCEMENT

Beginning with the present issue, The MATHEMATICS STUDENT will feature its third year-round problem solving contest. Students are strongly encouraged to take advantage of this unique opportunity in sharpening their problem solving skills by submitting their solutions to the problems posed. Each correct solution will be credited by 2-4 points depending on the difficulty of the problem; extra credit will be available for the problems marked by a star (*). Students should try to provide alternative solutions, generalizations, insightful remarks and reasonable conjectures along with their solutions for these problems. On the basis of the points gathered during the school-year, valuable prizes will be awarded to the winners. The contest should provide excellent opportunities for many of the non-winners as well in allowing them to gain confidence and satisfaction from their progress, achieve personal goals and outdo even the best problemists on individual problems.

Students who find the Contest Problems beyond their strength are encouraged to peruse past issues of The MATHEMATICS STUDENT and to work on the Warm-Up Problems each month. They are also advised to make use of the many excellent references annotated in the Bookshelf column of The MATHEMATICS STUDENT (November 1978 through May 1979.)

CONTEST RULES

(1) The deadline for submitting solutions is the 20th of the month following publication. Thus the October problems are due on the 20th of November. However, students wishing to have their solutions selected for publication should mail them in **much sooner.**

(2) Each solution should be on a separate sheet; problems need not be restated, but their numbers should be clearly indicated. Legible, neat work is

WARM-UP PROBLEMS

1. Show that $a^2+b^2+c^2+d^2+ab+ac+ad+bc+bd+cd > 14$ if a, b, c and d are positive real numbers whose product is equal to 2.
2. Prove "cleverly" (i.e. no tables, no calculators) that $2^{1/3} + 2^{2/3} < 3$. (Proposer: **Mr. F. David Hammer**)
3. Discover the rule governing the following table of pairings of partitions of 12.

(5,4,3)	→	(3,3,3,2,1)
(4,4,4)	→	(3,3,3,3)
(5,3,2,1,1)	→	(5,3,2,1,1)
(6,6)	→	(2,2,2,2,2,2)
(6,3,3)	→	(3,3,3,1,1,1)
(4,3,3,2)	→	(4,4,3,1)

(Proposer: **Professor Chris Harman**)

NOTE: You are encouraged to discuss the above problems with your teacher as well as with other students, but **do not** submit your solutions to the Problems Editor. Hints and comments concerning them will appear in next month's issue.

CONTEST PROBLEMS

Problem 536: Prove that if an arithmetic progression of positive integers contains a perfect cube, then it contains infinitely many perfect cubes. (Proposer: **Mr. F. David Hammer**)

Problem 537: Find all integers x for which $x^4 + 4x^3 + 8x^2 + 4x + 16$ is a perfect square.

***Problem 538:** Show that a square can be dissected into six obtuse triangles. (Proposer: **Mr. Velt Elser**)

Problem 539: Show that if a $1 \times k$ rectangle tesselates (i.e. duplicates of it cover without overlapping) an $m \times n$ rectangle, then either $k|m$ or $k|n$. (Proposer: **Professor Andrew Liu**)

Problem 540: Show that the longest angle bisector of a triangle is a least as long as its shortest median. (Proposer: **Professors Murray S. Klamkin** and **A. Meir**)

SOLUTIONS

Problem 513: For each $n = 1,2,3, \ldots$, let $x_n = 9n + 7$. How many perfect squares are there among the numbers $x_1, x_2, x_3, \ldots, x_{1000}$?

Solution: Steve Carrow, Braxton County HS, Sutton, WV.
First note that if $9n+7=m^2$, then $m^2 \equiv 7 \pmod 9$. For $0 \le m \le 8$ this holds only if $m = 4$ or 5; hence m must be of the form $9k+4$ or $9k+5$, $k = 0, 1, 2, .$ Since $|9(10)+4|^2 = 8836 < x_{1000} = 9007 < 9025 = |9(10)+5|^2$, we must have $m \le 94$. This holds for

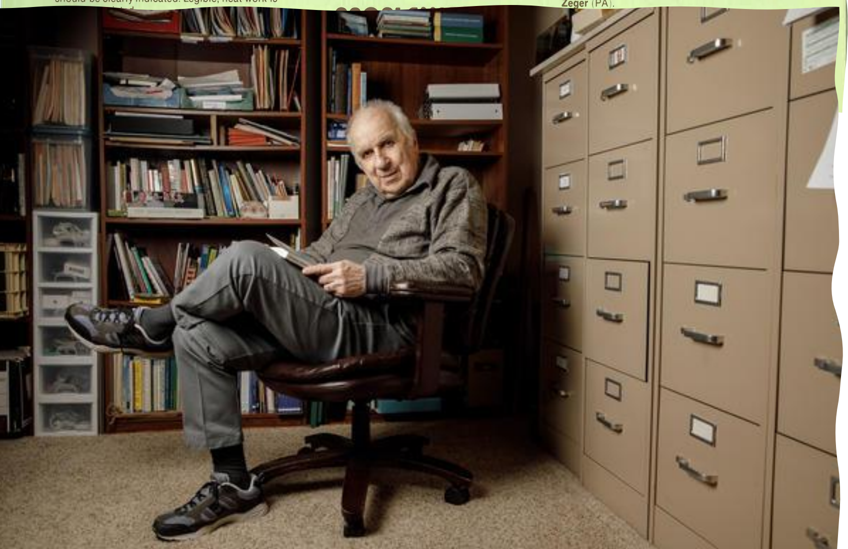
$9k+4$ if $0 \le k \le 10$, and for $9k+5$ if $0 \le k \le 9$, resulting in $11+10$ or 21 perfect squares among the first 1000 terms of the given sequence.

Comments: Several solvers benefited from their knowledge of the theory of quadratic residues -- a most interesting area of study in number theory. **Jay Siegel** (NJ), **Cathy Reader** (Ont., CAN), **Jeremy Resnick** (PA) and others observed that the values of n for which x_n is a perfect square may be obtained as partial sums in the series $1 + 1 + 16 + 3 + 32 + 5 + 48 + \cdots$, where consecutive odd numbers alternate with multiples of 16. **Eric Wear** (AR) found the following pattern for the 1st, 3rd, 5th, ... squares in the sequence (with a similar one holding for the 2nd, 4th, 6th, ... squares also):

$$16 = 0(153) + 0(162) + 16$$
$$169 = 1(153) + 0(162) + 16$$
$$484 = 2(153) + 1(162) + 16$$
$$961 = 3(153) + 3(162) + 16$$
$$1600 = 4(153) + 6(162) + 16$$
$$\cdots \quad \cdots \quad \cdots$$

It may be interesting to explore similar patterns for squares in other arithmetic sequences. Several solvers expressed the answer in closed form, i.e. they found that among the numbers $x_1, x_2, \cdots, x_k$, there are $[(\sqrt{9k+7}-4)/9] = [(\sqrt{9k+7}-5)/9]+2$ perfect squares; others extended the search to perfect r th powers for $r = 3,4, \ldots$.

Other Commended Solvers: Bruce Brandt (MN), **David Compton** (CA), **John Im** (Ont., CAN), **Chris Jantzen** (IL), **Grant Liu** (PA), **Fred MacKintosh** (WA), **Jonathan Miller** (TN), **Allan Murray** (CA), **Gina Roberts** (B.C., CAN), **Hilarie Shickman** (MO), **Doug Shors** (MO), **Mike Stelman** (CA), **Kazuko Suzuki** (IL), **Jonathan Tanner** (IN), **Wayne Wheeler** (AZ), **David Yuen** (IL) and **Ken Zeger** (PA).



The Mathematics Student Competition Corner

- Edited by Dr George Berzsenyi
- Three years of existence (1978-1981) when publication was defunded
- 20 problem sets (Rounds), a total of 103 problems

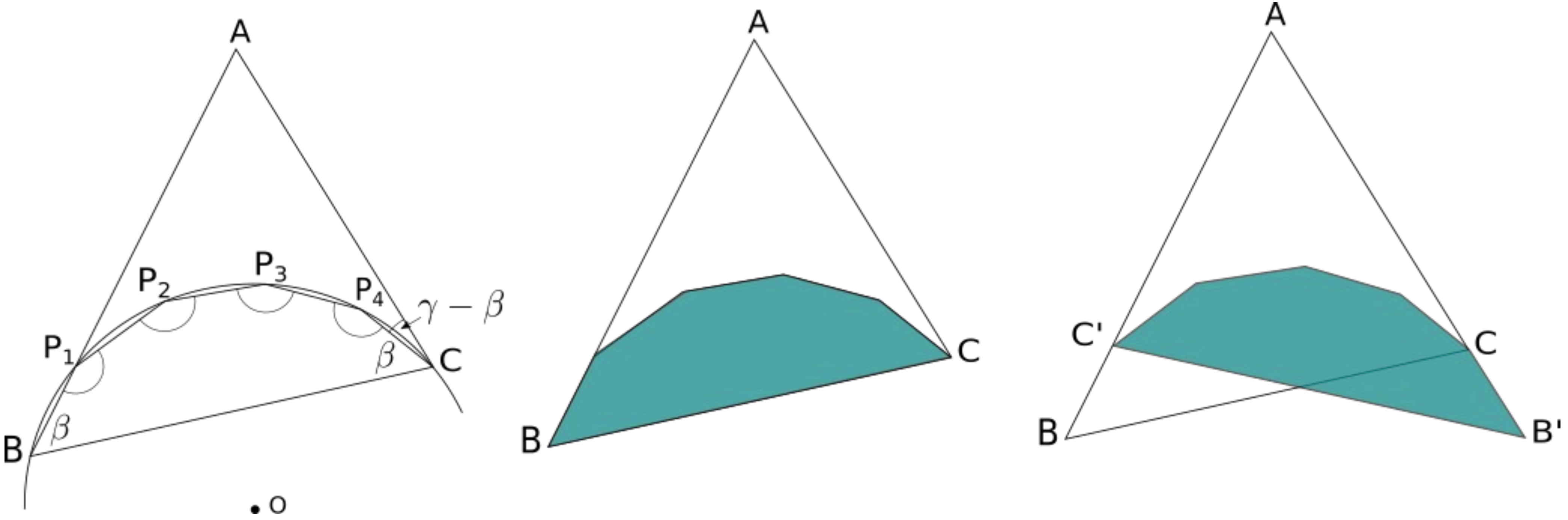
MS Competition Corner features

- Approx. 450 total High School student contestants from US and Canada
- Approx. 3000 different submissions to problem sets (averaging 69 submissions for each problem)
- 32 different problem proposers
- A variety of challenging pre-calculus problem types
- These problems, along with more than 200 student solutions, with 125 figures and contestant profiles, appeared in a problem book in 2021; https://www.ms-competitioncorner.com/

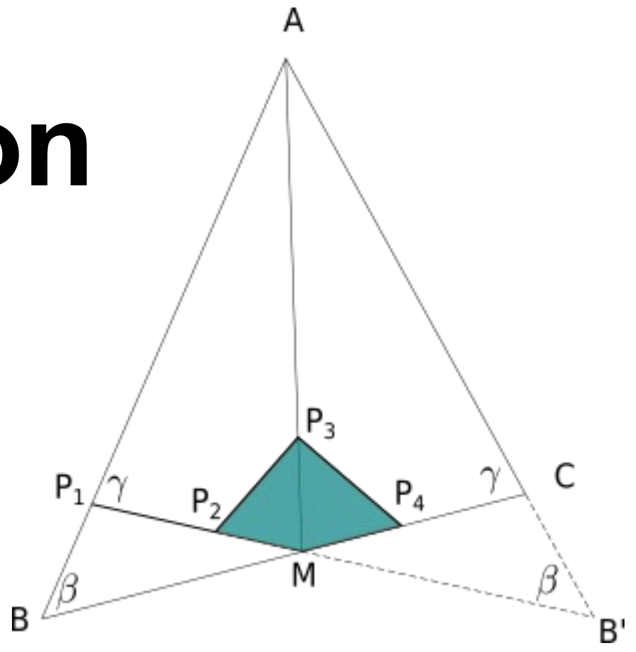
Flatland

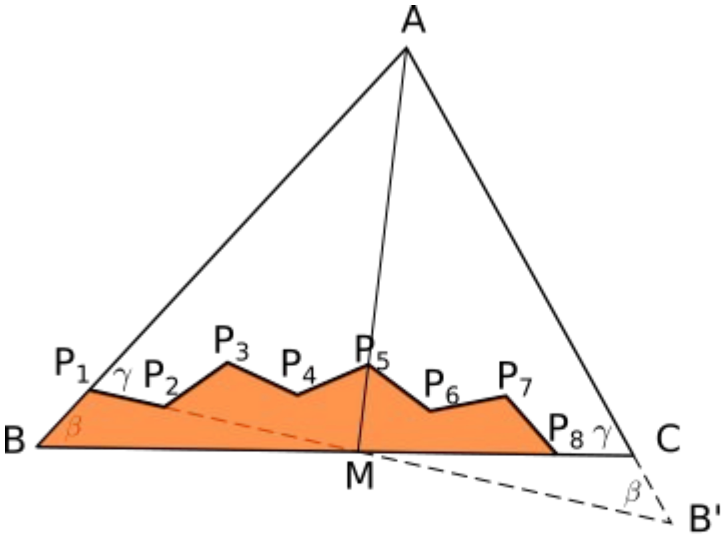
The people of Flatland sometimes find it necessary to produce the mirror images of various polygons. As they cannot simply *turn things over* as we can, they cut the polygons into a number of pieces and then slide the pieces around in their plane to form the mirror images. Triangles, in general, can be dealt with by cutting them into three pieces, but for certain triangles, this can be improved upon. Show, for example, how a Flatlander would cut a 75 - 55 - 50 triangle into two pieces along a broken line so that the pieces can be rearranged to form the mirror triangle.

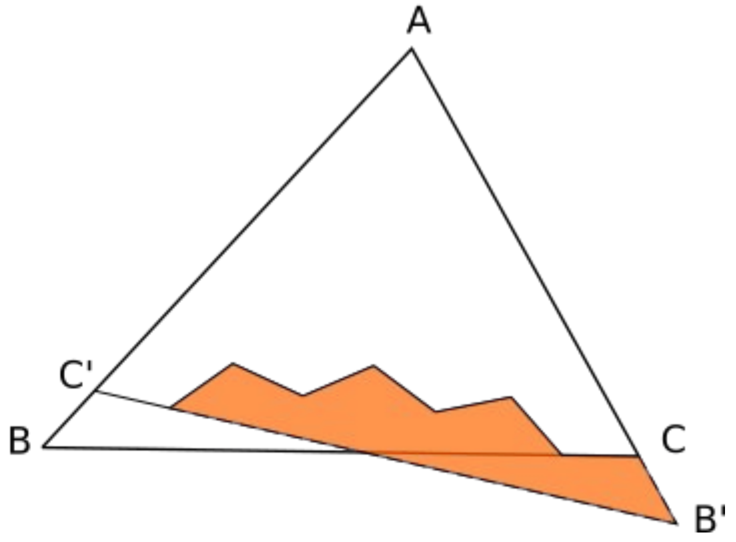
Sliding construction



Zig-zag construction







Why a journal?

- Published online - could reach a wide audience
- Provide a regular resource for teachers and parents and get the word out about other opportunities
- Get kids hooked on puzzles and problems early
- Form a community by solving problems, do activities together
- Problem solving contests – write mathematics
- Explorations, problem posing

Logistics, thoughts?

- Sponsors
- Content
- Design
- Publicity

Thank you!