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What Mathematicians Need to Know to Understand Philosophers of Mathematics

I. Introduction

II. Clearing Up Some Confusions About the Philosophy of Mathematics

1. What is the relationship between mathematics and philosophy of mathematics?

2. What is the core thing that philosophers of mathematics are doing? In other words, what kinds of theories are they putting forward?

Definitions:
1. An abstract object, or a platonic object, is a non-physical, non-mental, non-spatiotemporal object.
2. Platonism is the view that there are abstract objects.
3. Anti-platonism is the view that there are no such things as abstract objects.
4. An ontological theory is a theory about what sorts of things really exist. Thus, e.g., the claim that there are mermaids is a false ontological theory, and the claim that there are Tasmanian devils is a true ontological theory. Platonism, then, as I defined it above, is an ontological theory. It is the theory that abstract objects (i.e., non-physical, non-mental, non-spatiotemporal objects) really exist.
5. A semantic theory is a theory about what certain expressions mean (or refer to) in a specific language. So, for instance, the claim that the term 'Mars' refers (in English) to the Empire State Building is a false semantic theory, and the claim that 'Mars' refers (in English) to the fourth planet from the sun is a true semantic theory. (It's important to note that a semantic theory for a natural language--a language that's actually spoken by real people--is an empirical theory.)

What philosophers of mathematics are really doing: To an outsider, it might seem that philosophers of mathematics are primarily interested in ontology. Well, they are interested in ontology, and for some, this is the ultimate point of their work. But semantics is more central to what philosophers of mathematics are doing. In particular, the most central goal of the philosophy of mathematics is probably to develop an (empirical) semantic theory of the language of ordinary mathematical discourse--or as philosophers sometimes call it, mathematese. The reason semantics can be seen as more central than ontology to what philosophers of mathematics are doing is that while ontological theories are often the conclusions of philosophers' arguments, the arguments themselves are often completely driven by semantic theories.

III. Doing Some Empirical Semantics

Semantic Platonism: Ordinary mathematical sentences like '2 + 2 = 4' and '3 is prime' are straightforward claims about abstract objects (or at any rate, they purport to be about abstract objects).

Semantic Theism: The term 'God' refers (in English) to an omniscient, omnipotent, benevolent Being who created the world (or at any rate, the term 'God' purports to refer to such a Being).

(1) Ordinary mathematical sentences like '2 + 2 = 4' and '3 is prime' should be interpreted at face value; i.e., they should be interpreted literally. For instance, '3 is prime' should be interpreted as having the following logical form: Object O has property P; or as philosophers would put it, it has the form Fa. Thus, what '3 is prime' says is that a certain object (namely, the number 3) has a certain property (namely, the property of being prime).

(M) Mars is round.
(O) Obama is a politician.
(E) The Eiffel Tower is made of metal.
Object O has property P.

(A) The average accountant has 2.4 children.
On average, accountants have 2.4 children.

(2) Given that ordinary mathematical sentences like '2 + 2 = 4' and '3 is prime' should be interpreted at face value--i.e., as making straightforward claims about certain objects (namely, numbers)--we can't interpret them as being about physical or mental objects, and so we have to interpret them as being about abstract objects, i.e., non-physical, non-mental, non-spatiotemporal platonic objects (or more precisely, we have to interpret them as purporting to be about abstract objects).

Semantic Physicalism: Ordinary mathematical sentences like '2 + 2 = 4' and '3 is prime' are best interpreted as straightforward claims about ordinary physical objects.

Semantic Psychologism: Ordinary mathematical sentences like '2 + 2 = 4' and '3 is prime' are best interpreted as straightforward claims about ordinary mental objects--i.e., things like ideas that actually exist inside of our heads.

Arguments against semantic physicalism: (i) If this theory were right, then it would be reasonable to worry that there just aren't enough objects in the world to make our mathematical theories true. (ii) Semantic physicalism implies that our mathematical theories are empirical theories and that the right methodology for determining whether there are, say, infinitely many primes would involve an empirical investigation into the number of physical objects in the universe. (iii) When we apply semantic physicalism to set theory, we get the conclusion that expressions that are supposed to refer to sets are supposed to refer to piles of physical stuff, and this can't be right, because corresponding to every pile of physical stuff—indeed, every individual physical object—there are infinitely many sets.

Arguments against semantic psychologism: (i) If this view were right, then it would be reasonable to worry that there just aren't enough mental objects in the world to make our mathematical theories true. (ii) Semantic psychologism implies that our mathematical theories are empirical theories and that the right methodology for determining whether there are, say, infinitely many primes would involve an empirical investigation into the number of actual number-ideas that exist in the universe.

Possible psychologistic response: Look, the psychologistic view isn't that mathematics is about actual ideas that really exist inside of human heads. We can take the view to be about what it's possible to do in our heads. For instance, to say that there are infinitely many prime numbers is not to say that there really exists an actual infinity of prime-number ideas inside of human heads; it's to say that it's possible to construct infinitely many prime numbers in our heads.

Rejoinder: The view described here isn't a psychologistic view at all, and so it's no defense against the above objections.

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(2) Given that ordinary mathematical sentences like '2 + 2 = 4' and '3 is prime' should be interpreted at face value--i.e., as making straightforward claims about certain objects (namely, numbers)--we can't interpret them as being about physical or mental objects, and so we have to interpret them as being about abstract objects, i.e., non-physical, non-mental, non-spatiotemporal platonic objects (or more precisely, we have to interpret them as purporting to be about abstract objects). Therefore,

(3) Semantic platonism is true. In other words, ordinary mathematical sentences like '2 + 2 = 4' and '3 is prime' are straightforward claims about abstract objects (or at any rate, they purport to be about abstract objects).

IV. From Level-Headed Empirical Semantics to Crazy Ontology

(i) Semantic platonism is true--i.e., ordinary mathematical sentences like '2 + 2 = 4' and '3 is prime' are straightforward claims about abstract objects (or at any rate, they purport to be about abstract objects). Therefore,
(ii) Mathematical sentences like '2 + 2 = 4' and '3 is prime' could be true only if platonism were true--i.e., only if abstract objects existed. But
(iii) Mathematical sentences like '2 + 2 = 4' and '3 is prime' are true. Therefore,
(iv) Platonism is true.

(i) Semantic platonism is true--i.e., ordinary mathematical sentences like '2 + 2 = 4' and '3 is prime' are straightforward claims about abstract objects (or at any rate, they purport to be about abstract objects). Therefore,
(ii) Mathematical sentences like '2 + 2 = 4' and '3 is prime' could be true only if platonism were true--i.e., only if abstract objects existed. But
(iii*) Platonism isn't true--i.e., there's no such thing as platonic heaven, and there are no such things as non-physical, non-mental, non-spatiotemporal abstract objects. Therefore,
(iv*) Mathematical sentences like '2 + 2 = 4' and '3 is prime' are not true.

Fictionalism: The view that mathematical sentences aren't literally true because (a) they're supposed to be about abstract objects (i.e., semantic platonism is true), and (b) there are no such things as abstract objects. (Note that fictionalists do not say that mathematics is analogous to novel writing, or anything like that. So a better name for the view might be not-literally-true-ism.)

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(ii) Mathematical sentences like '2 + 2 = 4' and '3 is prime' could be true only if platonism were true--i.e., only if abstract objects existed. Therefore,
(iii**) Either platonism or fictionalism is true.

V. After the Background

Objection to platonism: Platonism is inconsistent with the fact that human beings have mathematical knowledge.

Objections to fictionalism: (a) Fictionalism is incompatible with the fact that mathematics is objective and factual. (E.g., there's an obvious sense in which '2 + 2 = 4' is right, and '2 + 2 = 5' is wrong.) (b) Fictionalism is incompatible with the fact that mathematics is applicable in empirical science.