Some ways of reasoning productive for the logic of mathematical reasoning

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A different approach to logic

Theorem 1. "For every integer x, if x is a multiple of 6, then x is a multiple of 3."

Theorem 2. "For any integer x, if x is a multiple of 2 and a multiple of 7, then x is a multiple of 14."

Theorem 4. "For any quadrilateral ■$ABCD$, if ■$ABCD$ is a rhombus, then the diagonal $\overline{AC}$ forms two congruent, isosceles triangles ΔABC and ΔCDA."

Theorem 9. "Given any functions f, g that are continuous on the domain $[a, b]$, if $f(a) = g(b)$ and $f(b) = g(a)$, then there exists some c in $[a, b]$ such that $f(c) = g(c)$."

Building logic from students' own reasoning and language use

A different approach to logic

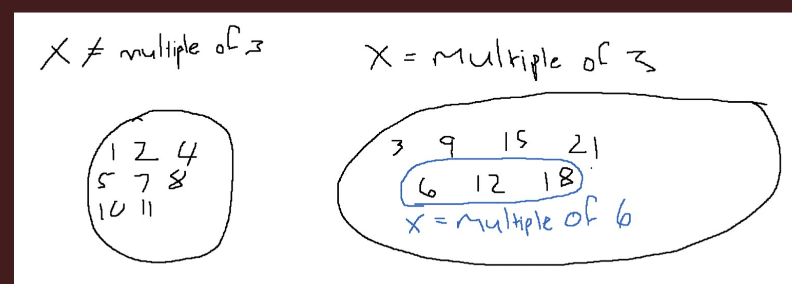
What is shared among all of these statements?

What is a truth condition (and falsehood condition) that allows students to see them all as “the same?”

Our working answers:

Truth condition – A conditional is true whenever the truth set of the if-condition is a subset of the truth set of the then-condition.

Falsehood condition – A conditional is false whenever there is an object that makes the if-condition true and the then-condition false.



Reasoning about truth sets

Important way of reasoning: Students need to develop the propensity to associate to any mathematical condition the set of all objects that exhibit it.

We focus on use of set-builder notation before we focus on arbitrary collections.

Sets of triangles

$\alpha = \{triangles\ ABC\colon ABC\ is\ isosceles\}$ $\beta = \{triangles\ XYZ\colon XYZ\ is\ equilateral\}$

$\alpha = \{triangles\ ABC\colon ABC\ is\ isosceles\}$ $\gamma = \{triangles\ RST\colon \angle R \cong \angle S\}$

$\alpha = \{triangles\ ABC\colon ABC\ is\ isosceles\}$ $\delta = \{triangles\ DEF\colon \angle D\ is\ a\ right\ angle\}$

$\alpha = \{triangles\ ABC\colon ABC\ is\ isosceles\}$ $\epsilon = \{triangles\ MNO\colon MNO\ is\ scalene\}$

Reasoning about truth sets

Important way of reasoning: Students need to develop the propensity to associate to any mathematical condition the set of all objects that exhibit it.

We focus on use of set-builder notation before we focus on arbitrary collections.

Students need chances to think through how an implication between properties relates the two truth sets.

Which set contains the other?

Reasoning about truth sets

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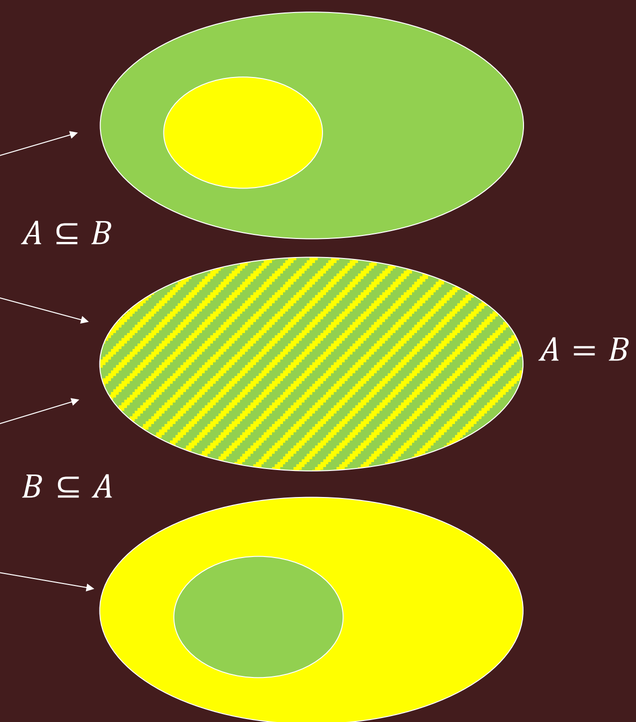
Students need chances to think through how an implication between properties relates the two truth sets.

Students especially need chances to reason about truth sets with unfamiliar conditions, compound conditions, and negative conditions.

Adopting technical language

- Given any x in S, if x has property P, then x has property Q.
- Given any x in S, if x has property Q, then x has property P.



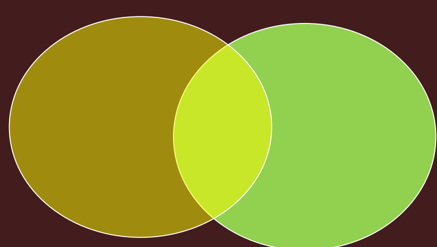
Adopting technical language

- Given any x in S, if x has property P, then x might have property Q.

(Descriptive statement)

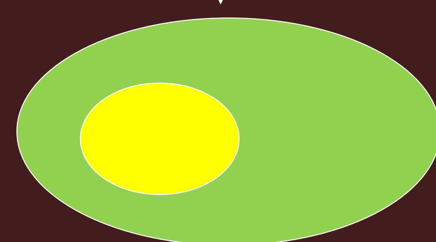
- Given any x in S, if x has property P, then x has property Q.

(False statement)



- Actual state of affairs

- Truth-conditions for the statement



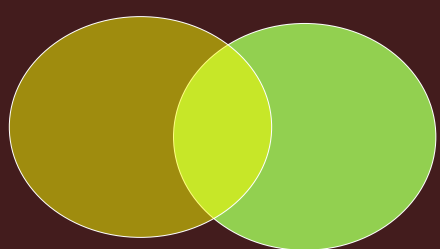
Adopting technical language

- There exist x in S such that x has property P and x has property Q.
- There exist x in S such that x has property P and x has property not Q

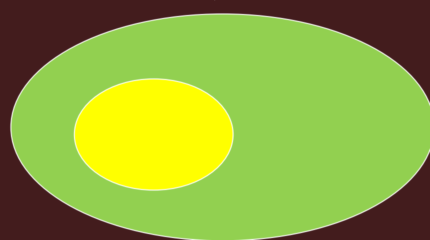
(Descriptive statements)

- Given any x in S, if x has property P, then x has property Q.

(False statement)

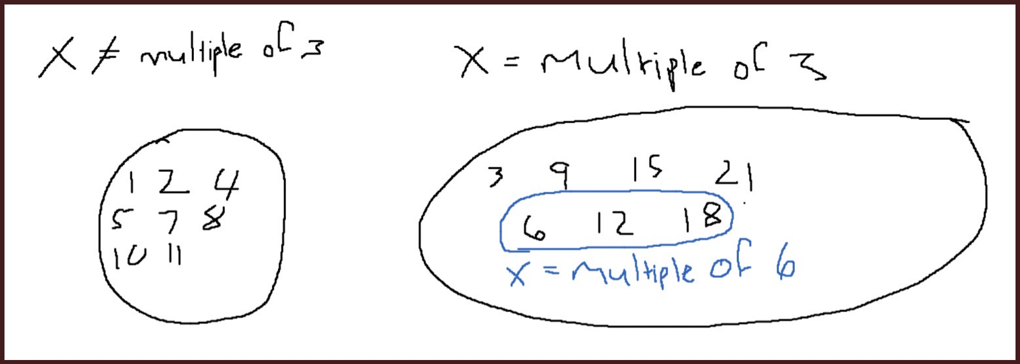


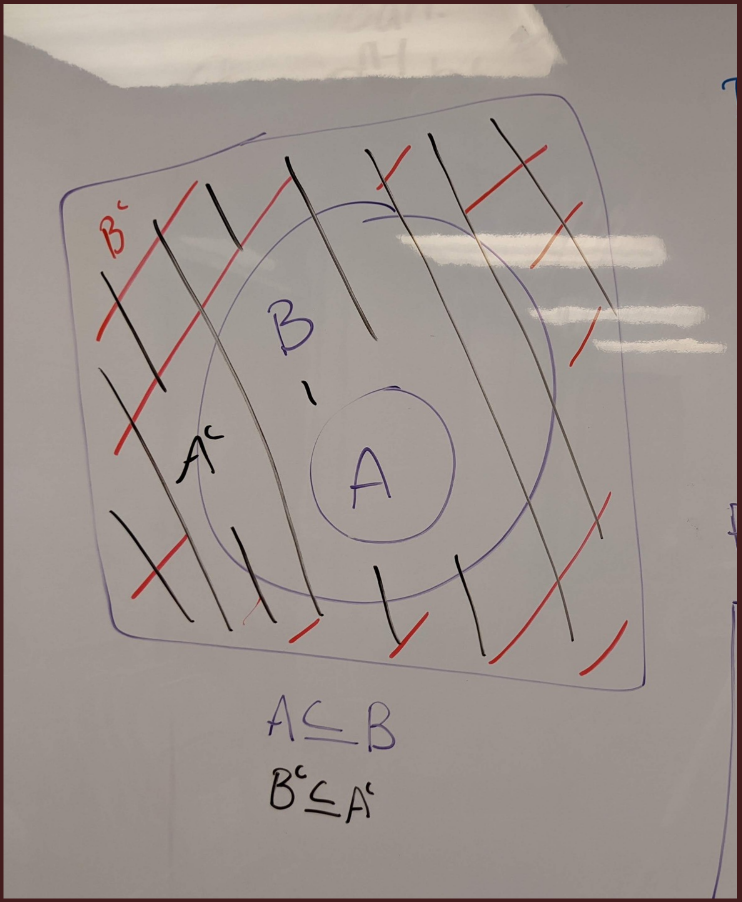
- Actual state of affairs



- Truth-conditions for the statement

Reasoning about reference sets can be formative





What is logic in our approach

- Logic entails students reasoning reflectively about how they use language and how language refers to (sets of) objects.
- As students construct a shared structure across the statements (and proofs), we consider that unifying structure their understanding of logic.
- Aristotelean modes of talking about groups of objects seem more accessible than modern modes of discussing arbitrary elements (but we need students to learn the modern approach).

What is logic in our approach

- Our historical conjecture is that logic arose among mathematicians as they reflected on their mathematical language use and reasoning.
- Logic answers questions many students have not yet asked.
- We want students to have similar experiences to reflect upon.