Dialectical Paradigms in the Philosophy of Mathematics: Epistemology and Axiology

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Preview of my Talk

I.) Dialectic as a General Method or Lens

- Interpreting something as arising out of "conflict" dynamics.
- For example: between "individual" and "group" (collective).
- Alternatively: between "parts" and "wholes".

II.) Dialectic in Mathematics (Examples of dialectic lens)

- Epistemology (Theory of Knowledge)
- Axiology (Theory of Value)
- Additionally: What is the relationship between these two?

In this Talk I shall Argue:

There are dialectical features to mathematical epistemology and axiology.

Rule of Saint Benedict: "Do not speak vainly or in ways that provoke laughter." I'll try not to!!!

Mathematical Epistemology & Axiology

Epistemological Questions

- What does it mean to understand mathematics (either individually or collectively) or possess mathematical knowledge?
- How does mathematical knowledge advance?
- Are there different ways of "mathematical knowing"?

Axiological Questions

- How is mathematical value determined?
- How can we understand what makes a mathematical question, inquiry, theorem or result "interesting" or "significant" or "beautiful"?
- Hardy's canon for publishability: "Is it true? Is it new? Is it interesting?" (BUT: Does this resolve the axiological question?)

Here Dialectic means:

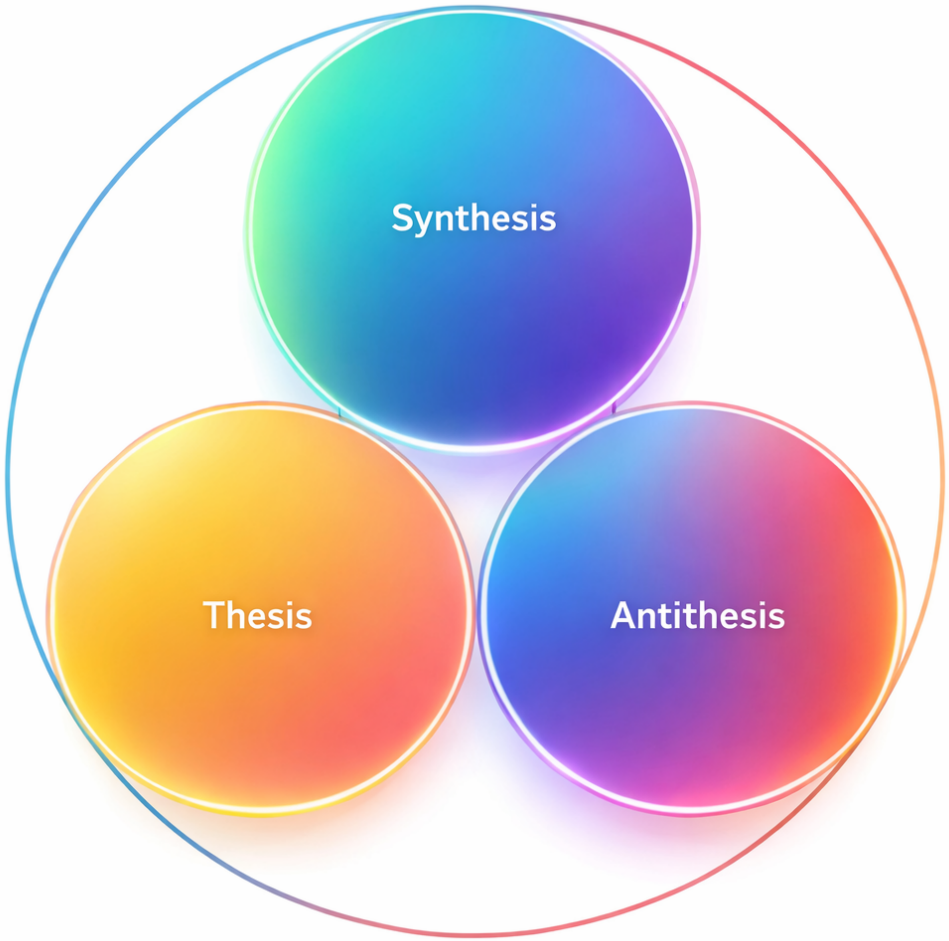
Viewing something as a Process (progression of states) that Evolves in Stages driven by Tensions, Contradictions, and their Resolutions, and whose Ultimate Trajectory is toward an ever increasing Underlying Unity.

- Can the development of mathematical thought be understood as such a process? (History of Mathematics)
- Can the way humans understand and value mathematics be so understood? (Philosophy of Mathematics)

As with Hegel, Dialectic is both:

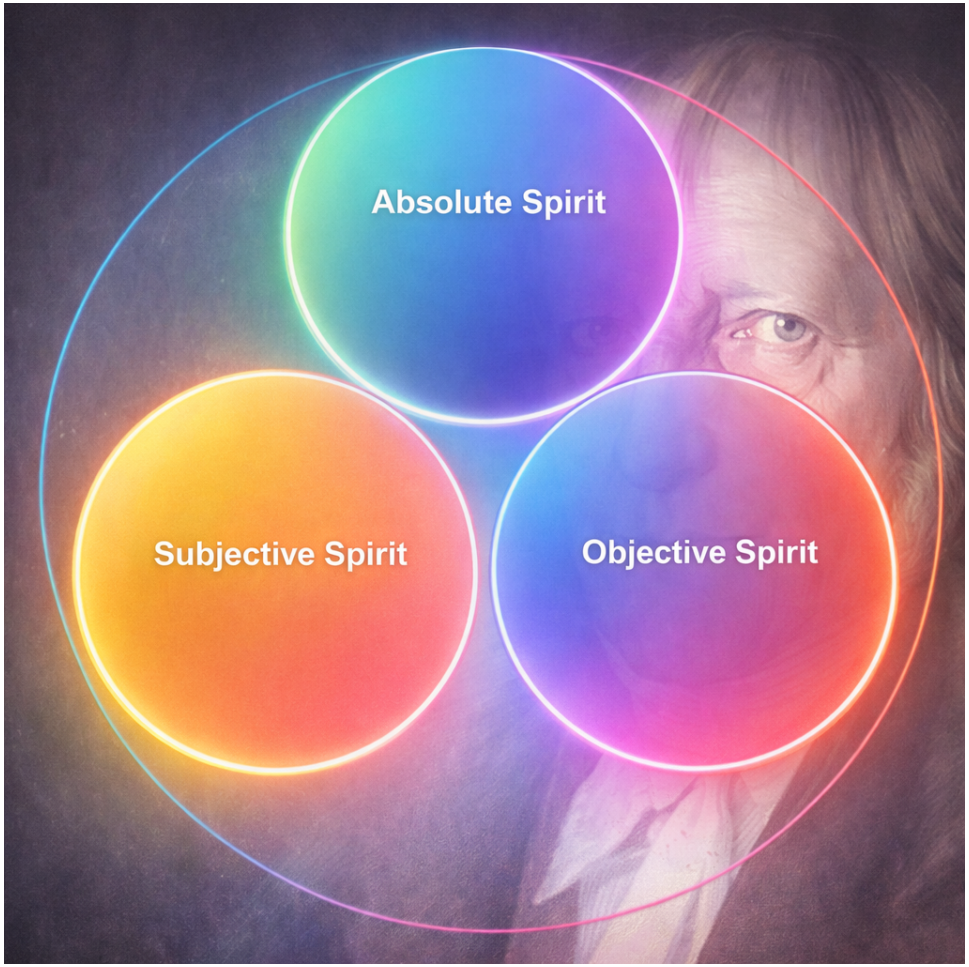
- A framework for conceptualizing the History of Mathematics.
- A framework for conceptualizing the Philosophy of Mathematics.

The Dialectic as Triad



Processes are conceptualized as advancing through triadic movements powered by the dialectic.

A Classical Hegelian Dialectic



As a first approximation we can think of Spirit as meaning something like human consciousness together with its cultural products. It "unfolds" through triadic movements powered by dialectical dynamcs, the rational dialectic.

Hegel's Concept of "Unfolding"

What do I mean by "Dialectical Dynamic"?

- **An Sich** meaning "in itself", i.e. potential
- **Für Sich** meaning "for itself", i.e. developing or development
- **An und Für Sich** meaning "in and for itself", i.e. actualized development

Example: What is A Mathematician?

- A person realizes he/she has an interest and possible aptitude for mathematics. (realization of potential, but undeveloped)
- A person receives formal education, perhaps a degree, but is not perhaps a full-fledged mathematician. (development of previous potential)
- A person becomes a practioner of the discipline of mathematics, perhaps teaches, writes, or even creates mathematics. (Actualized potential through development.)

Some Examples of Dialectic Triads

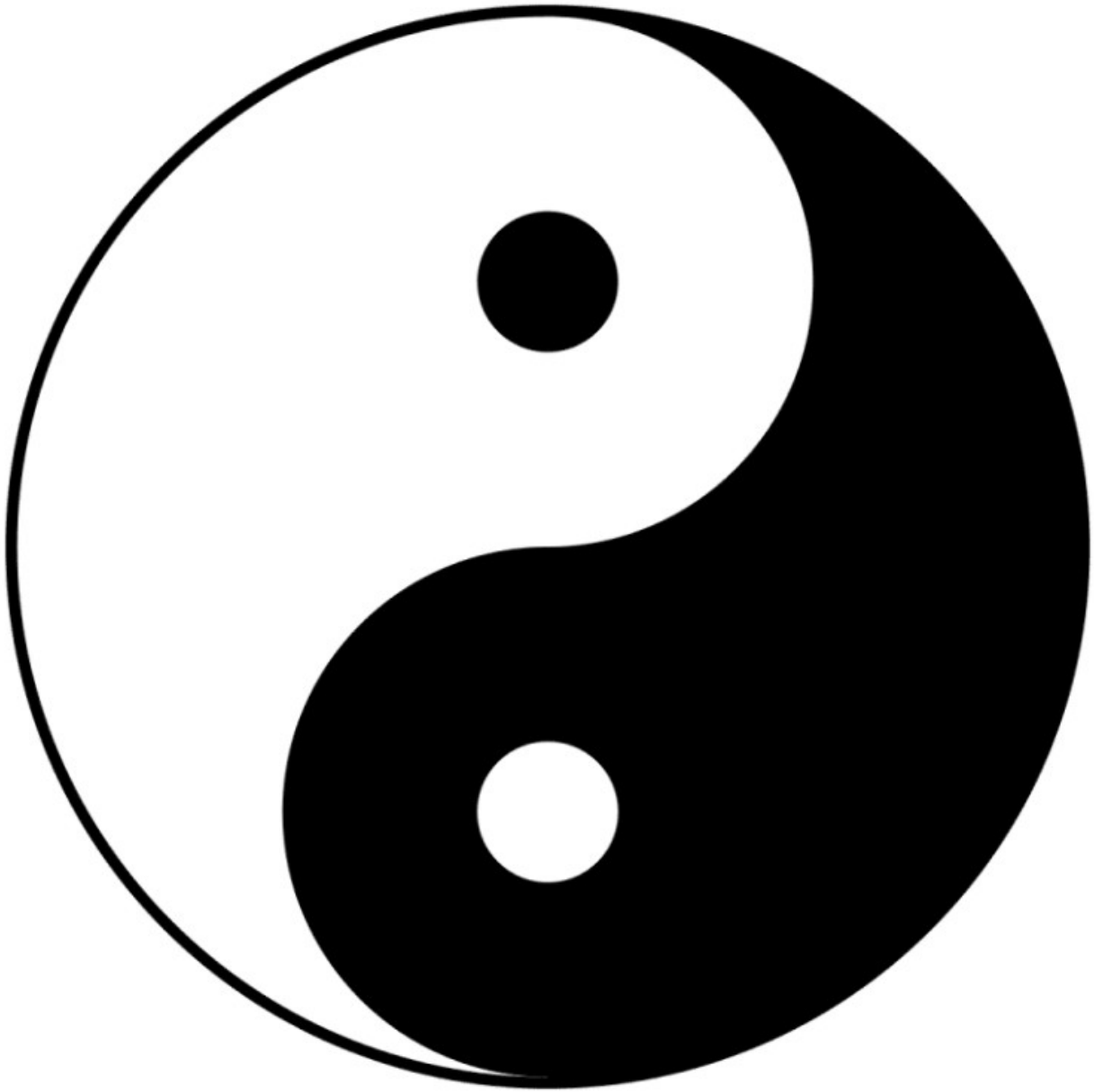
Philosophy

- Hegel: Absolute Idealism, (Subjective, Objective, Absolute Spirit)
- Kierkegaard: Existentialism/Moral Development, (Aesthetic Stage, Ethical Stage, Religious Stage)
- Nietzsche: Existentialism/Moral Development, 3 Metamorphoses, (The Camel, The Lion, The Child)

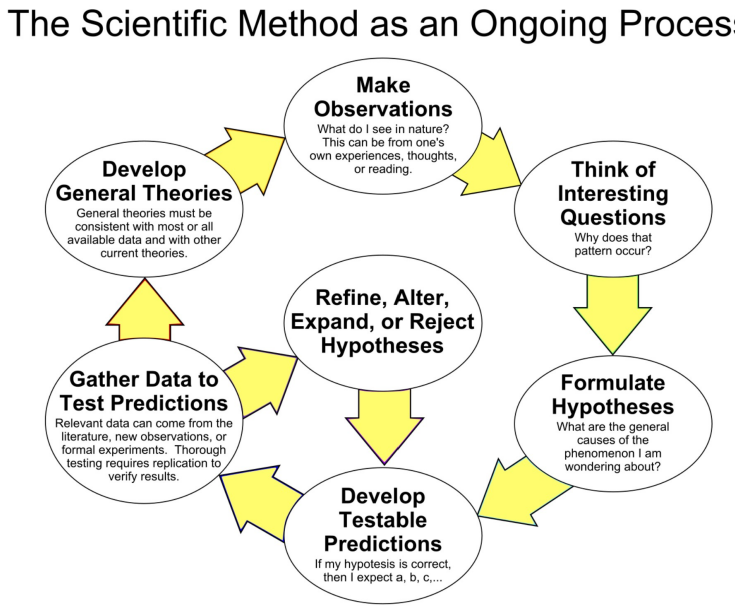
History/Economics/Psychology/Sociology

- Marx-Engels: Dialectical Materialism (Conditions of Production, Means of Production, Production Relations)
- Freud: Psychoanalytic Theory, (Id, Ego, Superego)
- William Perry: College Student Critical Thinking/Moral Development, (Dualism, Multiplicity, Relativism/Commitment in Relativism)

Taoism has Dialectical Features



The Scientific Method has Dialectical Features



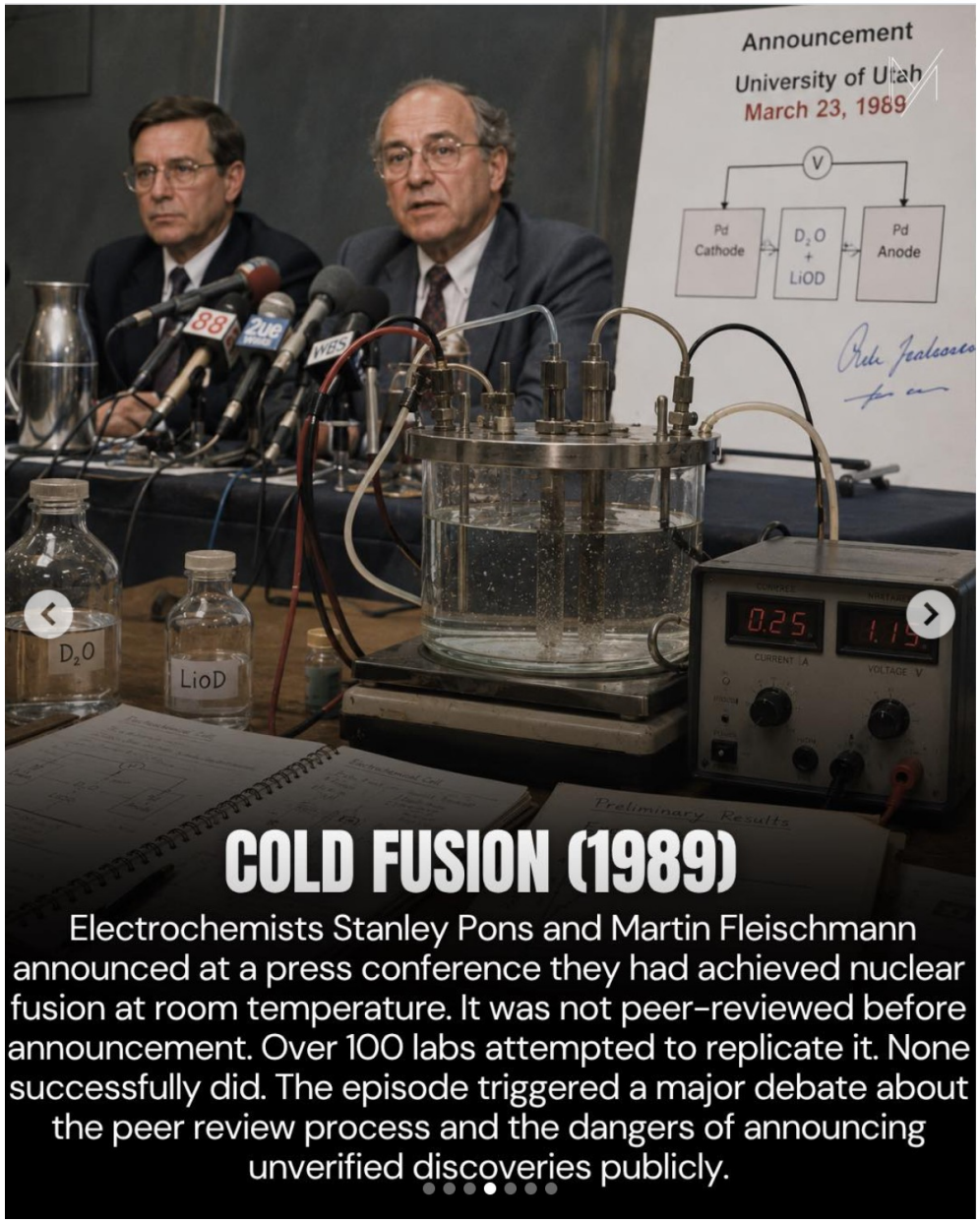
Scientific Understanding is Continually Challenged by Confirmation/Refutation through Testing.

Falsifiable hypotheses are proposed, tested, confirmed or refuted and ultimately replaced as understanding advances.

Knowledge is an Approximation to Realty

Scientific knowledge advances through an iterative process of corrections to previous understanding that is dialectic in character.

1989 Cold Fusion Controversy



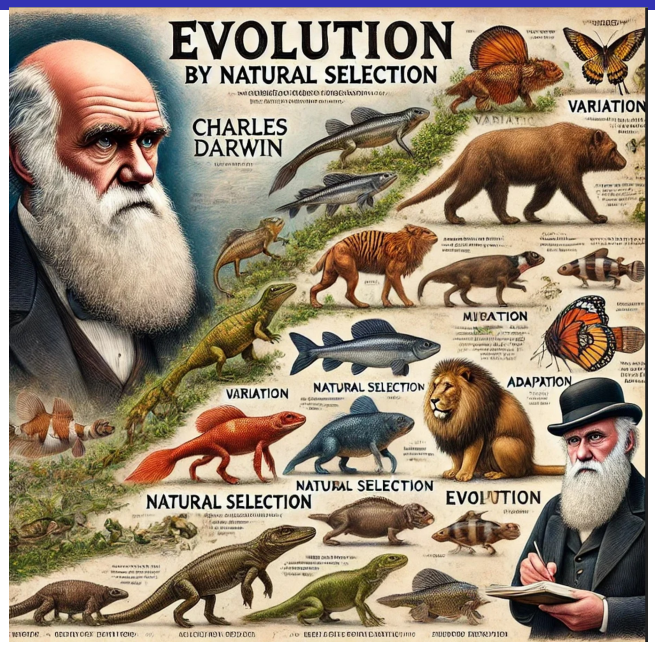
Dialectical Paradigms in Mathematics: Epistemology

Development of Mathematical Concepts

Mathematical Concepts Evolve through a Process of Proposal, Testing and Acceptance/Rejection.

" The emergence of fundamental mathematical concepts is almost always a long and intricate process... this concept (point-set topology), as understood today, is at first defined in a clear-cut and plain way, from here on it "exists"-but by that time the concepts had always passed through numerous preliminary stages, it was already known in important special cases, variants of it had been considered and discarded, etc., and it is often difficult, and sometimes impossible, to determine which mathematicians supplied the decisive contribution and should be considered the originator of the concept in question." -Klaus Jänich, Topology

Biological Evolution has Dialectical Features



Dialectical Dramatis Personae: The Individual and The Collective

- Individual species compete for survival based on adaptation to the environment. Species plays role of "individual."
- Environment comprises the panoply of other species as well as physical conditions. Environment plays role of "collective".
- Individual-Collective interactions result in Evolutionary Trajectory. This is a dialectical dynamic, called Natural Selection.

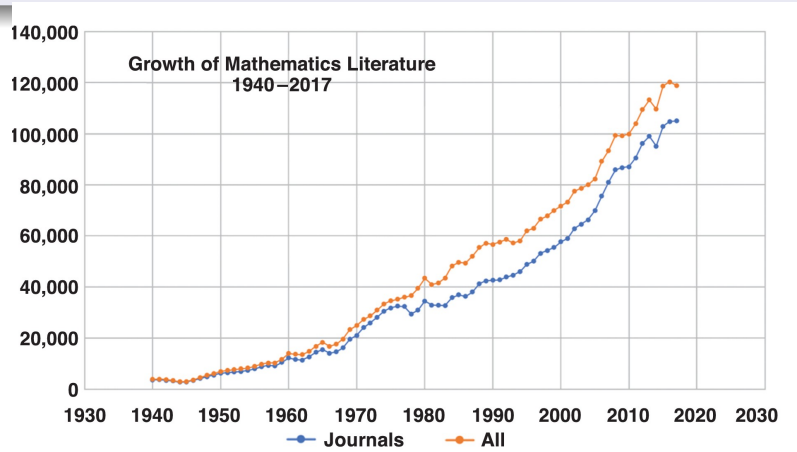
Dialectical Paradigms in Mathematics: Axiology

Mathematics as an Individual and Team Sport

- Individual mathematicians choose to research questions based on personal preference (analogous to Hegel's Subjective Spirit).
- Communities/Groups of mathematicians pursue a research agenda or programme defined by the collective (analogous to Hegel's Objective Spirit). The community of mathematicians may accept(as satisfactory) or reject (as unsatisfactory) work of lone mathematicians.
  It is this community or collective that determines the Value of mathematical knowledge. (Note nexus to mathematical epistemology.)
- The "limit" of these processes, their "synthesis" , is the contemporary incarnation of mathematical knowledge (analogous to Hegel's Absolute Spirit).

Publishability as a Proxy for Value

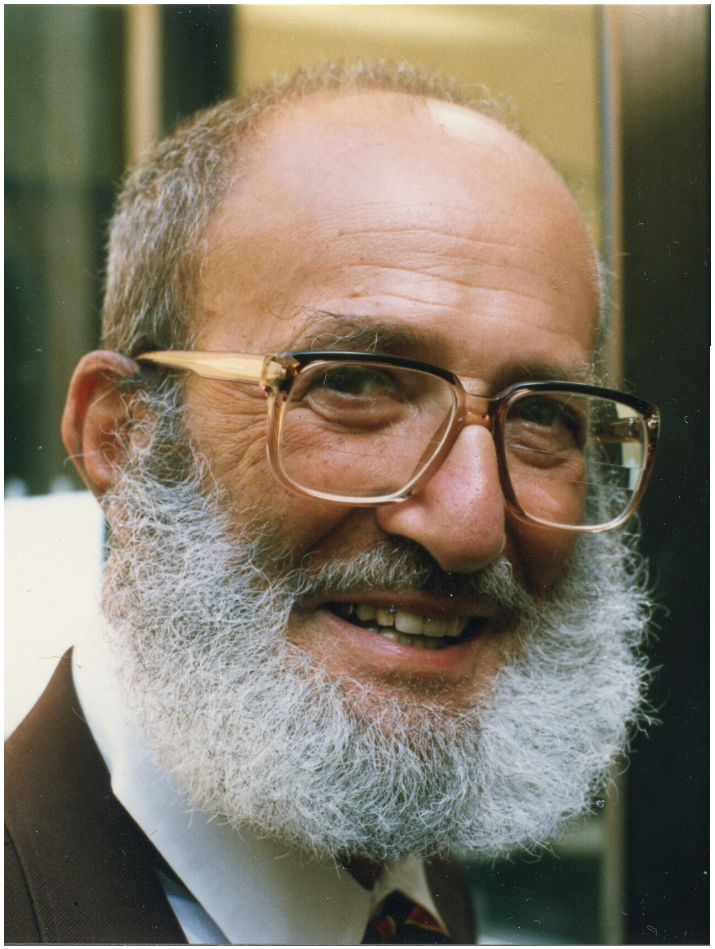
- Referees and journals make axiological judgements about publication.
- Mathematical value appears to be at least somewhat socially constructed: What is valuable is what the mathematical community says is valuable.
- Dialectical interaction between individual authors and the community of mathematicians, mediated through journal publication acceptance.
- Current volume of papers published per year is increasing exponentially (AMS Notices, 2019)



Axiological Riddle

"As I went around asking people's opinion about what to publish, the view that I heard expressed most often is that too much is now being published. The active, talented mathematicians, both the beginners and the established ones, complain about the flood of junk in the journals; their estimates of how much of it should have been published vary from a generous 50 % to a stringent 20%. The less active, less motivated members of the community complain (in my opinion rightly so) about the great pressure that is used on them to publish, publish, publish. They are frequently people of good taste, and they do not want to contribute to the flood of junk. Almost everybody's answer to "What to publish?" can be expressed in either one word - "less" - or two words - "good stuff". The trouble, of course, is how to define "good stuff" - how to establish canons of taste. G. H. Hardy's criteria are easy to say (is it true?, is it new?, is it interesting ?), but are they easy to apply?"

⇑ Said by Paul Halmos, 1975



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MATHEMATICAL ASSOCIATION OF AMERICA

Four Panel Talks on Publishing
Author(s): Leonard Gillman, P. R. Halmos, Harley Flanders and Beatrice
Source: *The American Mathematical Monthly*, Vol. 82, No. 1 (Jan., 1975),
Published by: Mathematical Association of America
Stable URL: http://www.jstor.org/stable/2319128
Accessed: 17-12-2017 21:48 UTC

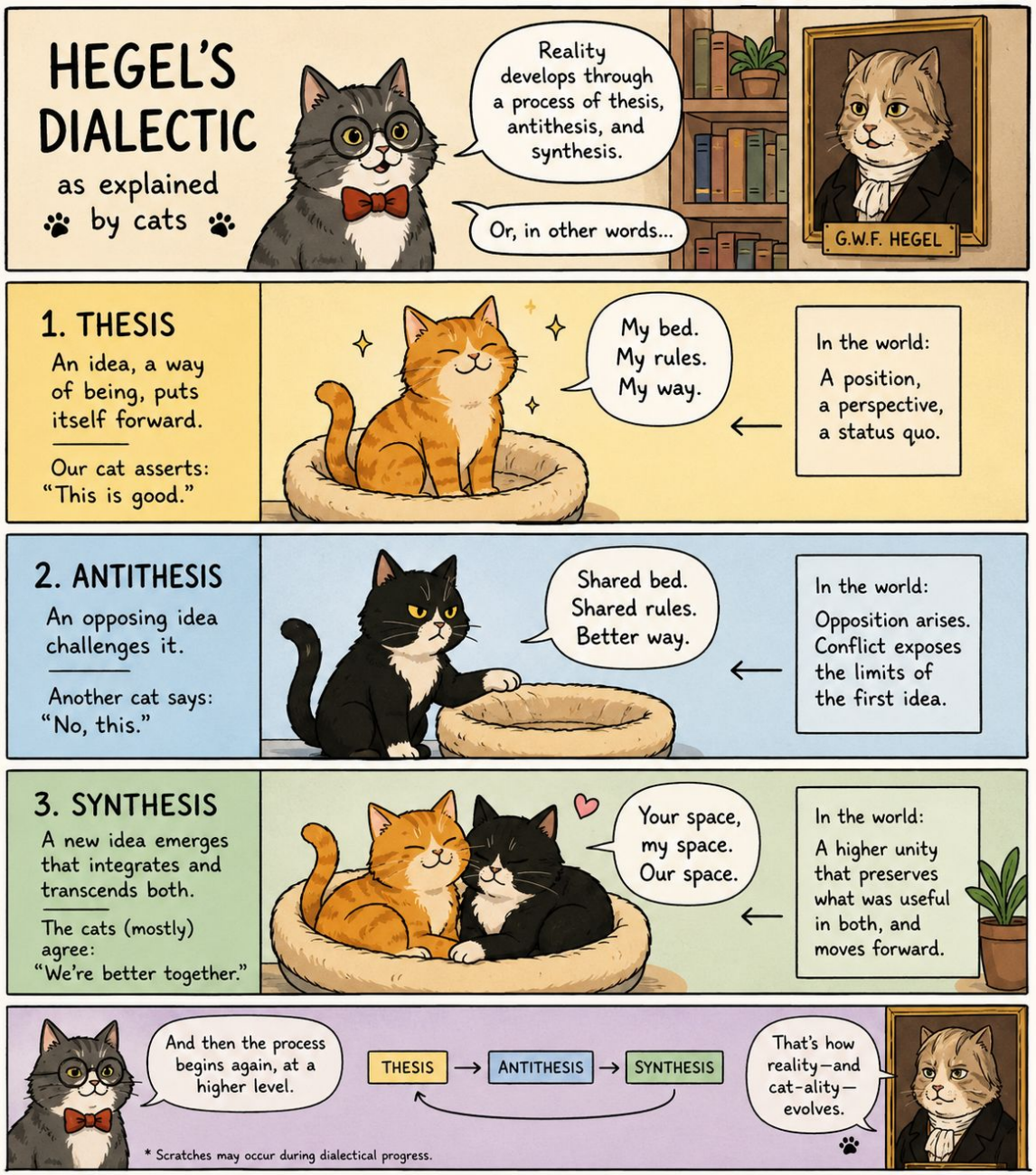
A More Recent Observation

"Dear Editors, . . .
As a result, many scientific papers today (certainly many of those I encounter as editor and referee) may be summarized as 'we investigated a slight variation of a well-studied problem, applied the usual techniques and obtained the expected results.' There is nothing fraudulent about this, the results are technically both new and true. For beginning mathematicians, this is often how we learn to do research, but more experienced authors would never write such papers if they truly had the freedom to study interesting problems! This applies especially to our younger, more creative but more precariously employed colleagues. I fear that, as the volume of publication balloons and dilutes the literature, a generation of talent goes to waste while genuine progress slows to a trickle."

AMS Notices, February 2026, Letter to the Editor by Florian Breuer, University of Newcastle, Australia

Master-Slave Dialectic between Referees and Authors

To Sum Up:

- A common dialectical feature of both mathematical epistemology and mathematical axiology is the interaction between the individual (the part) and the broader community or collective (the whole).
- Both mathematical thought and value begins with the individual but the community acts as the gate keeper of what becomes accepted knowledge and value.
- Since "individuals" and the "collective" consist of human beings, this dialectical perspective is consistent with social constructivism.
- Final Note: Although mathematical knowledge is not easily quantifiable, mathematical value, as reflected through journal publication rates, is materially impacted by dialectical dynamics between individual authors and journals.

Thank You All! Questions?