How should we understand the practice of Euclidean proof? I will argue that:

(a) Euclidean proof is not a purely formal system of deductive logic, but one in which the content of specifically geometrical concepts plays a central role.
(b) Our grasp of this geometrical content is not derived from experience.

1. Euclidean Proof as a Formal Deduction System

In order for a proof to be valid in a formal deduction system, each step must be derivable from the preceding steps via inferences that take account only of the form of the propositions in the earlier steps, without relying on the content of the non-logical terms.

- Famously, many of Euclid’s proofs fail to satisfy this constraint:

\[ \text{Euclid I.1} \]

\[ A(-1, 0); B(1, 0); C(0, \sqrt{3}) \]

The Gap: What licenses the assumption that the point of intersection, C, exists?

The Intended Model (continuous real plane): \( A(-1, 0); B(1, 0); C(0, \sqrt{3}) \)

The Defective Model (merely-dense rational plane): \( A(-1, 0); B(1, 0); C \) does not exist

We need a way to rule out the Defective Model:

- The formal features of the axioms, as stated by Euclid, fail to do so.
- The additional logical structure needed is only developed in the 19th c.

Conclusion: Euclid’s inferences rely on more than just the form of the axioms; the contents of our geometrical concepts play a crucial role.

- In particular, our concept of a circle as a genuinely continuous curve rules out the merely-dense Defective Model of Elements I.1.
- What is the source of this non-formal content? Is it derived from experience?

2. Strawson’s “Picturable Meanings”

Strawson: Euclid’s axioms concern our “phenomenal geometry,” – the geometry of our visual experience and imagination. Within this phenomenal geometry, “picturable meanings” supply the content needed to plug the gaps in Euclid's proofs.

- In 1.1, the “picture of the sense of the description rules out” any models in which the circles fail to intersect.

But there is a picture of the construction in which the circles fail to intersect: the picture of the Defective Model, which is visually identical to the picture of the Intended Model.
A possible empiricist response: Can the needed distinction be derived from experience via dynamic imagination?

- Can distinguish Descartes’ chiliagon and myriagon in this way.
- But can’t distinguish the Defective Model from the Intended Model.

Conclusion: No “picturable meaning” can fill the gap in Euclid’s proof.

3. Visual Limits

Wright: “The kind of idealization involved in the notion of perfect circularity… corresponds to a movement to the limit of a scale, as it were, whose intermediate values are ordered by a comparative – ‘is more circular than’.”

- But what is this limit, if “circular” is a perceptual concept?

Proposal: To determine the limit to which a perceptual concept approaches:

1) Abstract from contingent imperfections of visual capacities and circumstances.
2) Ask: Given such an idealization of visual experience, what kinds of properties could make a difference to visual experience?

We can thus distinguish, via imagination, a 1 nm-gapped curve from a continuous one.

- But can we distinguish continuous curves from merely dense ones?

Perceiving or imagining a gap involves perceiving a point, p, and the next point, p’, along a curve, together with the empty space separating them.

- The gaps in a merely dense curve are not like this; they do not fall between any two points in particular.
- Thus, the gaps in merely dense curves are not something we can grasp via experience or imagination, however idealized.

The distinction between dense and non-dense curves thus marks off a kind of visual limit, allowing us to derive from experience a separation between:

- Curves we can visually experience or imagine as gappy, because they have gaps falling between two specific points (i.e., non-dense curves)
- Curves we can’t visually experience or imagine as gappy, either because they are genuinely continuous, and so lack any gaps; or because they are merely dense, and so have gaps that don’t fall between any two specific points (i.e., the full range of dense curves)

But the distinction needed to ground Elements I.1 is not this distinction – it is a distinction within the category of dense curves, between genuine continuity and mere denseness.

Conclusion: The content that plugs the gap in Euclid’s proof—our concept of a circle as a genuinely continuous curve—cannot be derived from (even fully idealized) experience.

4. Conclusion

1) Euclidean proof utilizes a set of contentful, specifically geometrical concepts.

2) The content in question is a priori - our concept of a circle as a continuous curve can’t be derived from experience.