On the Autonomy of Mathematics

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Main Questions

1. Was mathematics ever truly "autonomous" – and if so to what degree?
2. Is "autonomy" still achievable and possible for mathematics given the recent intensive and growing encroachment of technology and the AI commercial sector into the discipline?
3. What is the endgame of this encroachment, and its significance in the wider culture?

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- Andy Arana (Philosophy, Université de Lorraine; Poincaré Archives)
- Walter Dean (Philosophy, Warwick)
- Marco Gaboardi (CS, BU)
- Juliette Kennedy (Math, Helsinki)
- Assaf Kfoury (C.S. BU)
- Kati Kish (STS, Sloan Postdoc BU)
- Nathan Mull (C.S. BU).

Backgrounds, Work

- A. Arana: purity of proof; history and philosophy of mathematics from the 16th century onwards; mathematical "hygienization" of culture; aesthetics and mathematics; political ramifications of the settings of mathematics.
- W. Dean: philosopher of computability; rule-following; mathematical difficulty; CE on Skolem; recent complexity measures on "autonomously proved" results from a year ago; history of the *Entscheidungsproblem*; co-author of the Leiden Declaration.
- J. Kennedy: set theory and history of foundations, Gödel and Turing; program in extended constructability and categoricity ("formalism freeness"); theory of art (20th century), curator, founder of Math/Art (MA) Collective.

Backgrounds, Work (Continued)

- Marco Gaboardi: Logician; Programming Languages, Differential Privacy; Teaching with LEAN4, Coq/Roq and other automated proving systems.
- Assaf Kfoury: mathematician/logician, type theories, LEAN4, Coq/Roq, etc.; co-organized "Mathematics with a Human Face" with JF at BU in March 2024 (Harris, Mumford).
- Kati Kish: STS, history of intuitionism; big science, AI "agency" and "intuition".
- Nathan Mull: logic and proof theory
- J. Floyd: History and philosophy of logic and language, analytic philosophy; Kant, pragmatism, philosophy of art and emerging media, Wittgenstein, Turing; Director BUCH.

"Mathematics With a Human Face" (BUCH, 4/2024)

How important are creativity and the human element to mathematics? In an age of AI and progress in automating proofs, these question arise. To ask philosophically, we need to include a characterization of actual mathematical practice, and not exclude the cultural, linguistic, pedagogical, computational and conceptual development of mathematics within wider historical and social contexts. The question of the human as mathematician is emblematic for our time, when larger philosophical and cultural questions about the automation of human labor become increasingly central. Is human bias something to be celebrated or eradicated? What is the relation of mathematics to cultural concerns and values? Can we learn from confronting its history in terms of the present?

(Co-sponsored with University of Bergen Group, S. Bangu.)

Turing 1947
Lecture to the London Mathematical Society

The Masters [i.e., mathematicians] are liable to get replaced because as soon as any technique becomes at all stereotyped it becomes possible to devise a system of instruction tables which will enable the electronic computer to do it for itself. It may happen however that the masters will refuse to do this. They may be unwilling to let their jobs be stolen from them in this way. In that case they would surround the whole of their work with mystery and make excuses, couched in well-chosen gibberish, whenever any dangerous suggestions were made. I think that a reaction of this kind is a very real danger.

Floyd on Turing

There are many philosophical questions embedded in Turing's remark, which was not simply a throwaway, but a prescient observation about what he elsewhere called "The Cultural Search", which he believed would become increasingly important over time, and include "the human community as a whole" (1948, "Intelligent Machinery").

Turing suggests (like Wittgenstein?) that intelligence consists in *appreciating the differences among different kinds of searching*. He strongly suggests that the scientific spirit is a democratic spirit and must include *all* peoples (humans).

We will return to the ideas of *democracy* and *autonomy* below.

An AI Aside

I very much doubt that any present AI system could have devised the very idea of a "Turing Machine". Turing framed an informal analogy in order to provide a satisfactory analysis of the *general* notion of taking a *step* in a formal system (or algorithm). Wittgenstein (1947): "Turing's 'Machines': these are *humans* that calculate". He couldn't have simply written down another formal system to accomplish this (Floyd 2012, 2023b).

To take another example, the "Turing Test" needn't be regarded as simply providing an operational test for "intelligence". It is also a language-game (in fact a piece of popular culture) designed to allow anyone to investigate and discuss their and others' responses (including emotional responses) to certain technologies. It highlights the importance of shared human-to-human interactions in language in the presence of machines (Floyd 2021, 2023a).

Michael Harris, "Silicon Reckoner" 4/24/2024: a Human Face for Mathematics

There is a small but growing community of mathematicians, historians, and philosophers (at least) who are committed to seeking insight about the nature of our vocation in whatever is ignored by *the Central Dogma* [the assumption that an informal mathematical proof is valid if and only if it corresponds to a formal derivation in a symbolic language]. (Question: *Is* this a dogma?)

All [this community] needs is a memorable name in order to leave its mark on the history of history of mathematics.

Articulation

The very idea of *mathematics with a human face* and related subjects are in early stages, without clear results. The future of mathematical and other forms of research in the next decade – the very structure of the research university – is difficult to foresee, and we expect the evolution of the situation in mathematics to be rapid over the next few years. How should we (re-)conceive the evolution of graduate education and a lifetime handshake with research mathematics? Or philosophy? Or history? –Today's talk is designed to stimulate discussion and lead, hopefully, to the posing and/or precisification of certain questions.

Articulation

At the moment, we lack appropriate terms of criticism – of mathematics, of technology, of politics and of philosophy.

This is not *simply* a matter of "metamathematics". With Wittgenstein: metamathematics involves us in more mathematics.

Carnap argued for *autonomy*, i.e., researcher's freedom from constraint in choosing or setting up formalized linguistic frameworks, or pieces of "conceptual engineering". Yet we need discussion of ongoing desiderata and problems, which are underdescribed. The situation is not one of *mere* conventions, or solutions to problems, but rather putting our conformity to practices and conventions into philosophical perspective. The design of formal languages can help here, but does not exhaust what needs investigating.

Autonomy

Our idea: investigate the concept of *autonomy* to gain clarity, with fine-grained attention to its evolving uses.

There is very little systematic literature investigating shifts in the concept over the last century in the sciences, and especially mathematics – whereas there is a substantial literature concerning autonomy in ethics, political philosophy and modernist art criticism ("medium specificity" and the "autonomy of taste", Greenberg) – including philosophical criticism emphasizing the dangers of the very idea of *autonomy*. A focus on autonomy may allow us to place the present situation in AI and mathematics into better cultural, historical, political and philosophical perspective.

Autonomy: Kant

In the history of "modern" philosophy, the notion of *autonomy* is linked with Kant and the notions of *freedom* and *human dignity* – although our capacities of choice, responsibility and the possibility of error ("free will") were also central to Augustine, Descartes, Locke, Hume and many others before Kant.

Descartes' insistence upon the point that algebra gives us a *method* for approaching problems, in contrast to ancient geometrical methods, is also associated with the importance of *ordering* discovery and experience, in this case by means of mathematical *methods*. (Newton resisted, emphasizing "genius".) Arana and Barnett (2023) have stressed historical processes in which the *hygienization* of language (normative ideas about best language uses) is a natural outgrowth of mathematics in culture.

Autonomy: Kant

For Kant autonomy has to do with the *sovereignty* and *dignity* of our human capacities to understand, think, reason, value and act independently: we are not merely passive perceivers and processors of experience given through external objects and internal psychological desires, pleasures, and interests, but also *agents* who structure and *order* what we perceive and know in accordance with necessary conditions (laws) of reason, thought, desire and feeling. We are free (individually and democratically) in being able to regard ourselves *as* self-legislating beings.

Autonomy allows us to rise above empirical and psychological ("heteronomous") needs and interests and exercise *responsibility* through our capacities of self-criticism, objectivity and enlightenment. E.g., the rule of law expresses a determination to order society's commitments to, e.g., justice rather than selfish or capricious rulers' desires.

Autonomy: Disciplines

Ordering experience involves imposing *discipline* on it by determining procedures and rules that systematically organize subject matters and systems of concepts. This helps with *surveyability*, which involves intelligibility, repeatability, and communicability (see Hilbert, Wittgenstein on *proof*).

For every discipline there is a systematic organization of laws, fundamental concepts, terms, modes of procedure. In modern mathematics the axiomatic method realizes this ideal of formation. In departments of the modern university we are looser about divisional boundaries; in fact interdisciplinary work seems the hallmark of the day, a great promiscuity of methods.

Is this a danger for pure mathematics and logic?

Autonomy

For Kant there are *realms* (*Reiche*) of human activity that autonomously self-legislate according to *a priori* rules that constitute them (geometry and arithmetic, cognition, ethics, physics, politics, law, sovereign nation states – for taste there is no rule, but a new form of autonomy, *heautonomy*). Humans are capable of acting in accordance with and recognizing (acknowledging) such autonomous law-constituted subjects; this expresses their capacity to achieve autonomy in their own selves, i.e., the capacity for pure thought and reflection, dignity, freedom, and responsible action.

Autonomy of Logic in Frege

In “Der Gedanke” (1918) Frege, writing in the wake of Gauss’s insistence that the science of number should be treated independently of Kantian “intuitions” of space, time, and physics, argues that arithmetic, the science of number, is part of logic, and thus belongs to a “third realm [*Reich*]” consisting of the most general laws of thought.

(See Arana (2024), *Elements of Purity*, on the complex history of purity of method in mathematics.)

Franks 2009

The Autonomy of Mathematical Knowledge

Hilbert's deepest and most central insight was that mathematical techniques and practices do not need grounding in any philosophical principles.

—JF: This has a Kantian flavor, i.e., it does not mean there are not ideals of ordering mathematics involved (e.g., axiomatization). Hilbert uses Kantian language when he writes, in aspiring to knowledge of the infinite, of our intellectual *dignity*, and he transposes Kant's call for Enlightenment, "Dare to Know!" into "We will know, we must know." For him philosophical problems about mathematics must be transformed into mathematical problems.

Technocracy?

This raises the issue of *technocracy*. Technocratic thinking extends the ideal of mathematization across societal problems, lodging decision-making in apparently neutral expertise-driven methods. Examples would include game-theoretic solutions to questions about economically just distributions, or Arrow's theorem concerning the possibility of satisfying voting procedures with ordered preferences.

Dean: There should be reverse philosophy done on certain mathematical arguments to measure their complexity and gauge their ability to satisfactorily analyze political situations (Dean and Neibo 2025, work in progress on mathematical difficulty).

Rawls on Dreben against Sacks (2001)

Dreben's point [about the fundamental notions of logic] was that no precise mathematical result can help us to understand any basic philosophical problem. This is because no mathematical argument can force us to accept a particular interpretation of a basic intuitive notion such as that of *logical validity*.

Rawls on Dreben against Sacks (2001): The Autonomy of Philosophy

This does *not* mean that, once we accept a mathematical argument, we can't go on to do various illuminating things with it. But we cannot begin with mathematical structures, relying on them to replace the basic logical notions. Whether we accept a mathematical structure as helping us to understand an intuitive logical notion like *validity* is up to us. It depends on our judgment, upon critical reflection.

It is one of Quine's great achievements, said Dreben, to have shown how little we can say about basic logical notions. I myself would interpret Dreben's view as similar to Kant's, who viewed reason as self-originating, self-authenticating, and alone competent to settle questions about its own authority.

Autonomy and Emersonian Perfectionism

Alongside our pointing out the dangers of technocracy and rationalized treatments of autonomy that provide only the illusion of responsibility may be set, in Emerson's voice, the dangers of conformity in democratic forms of life. The press toward individuality, resistance to conformity, and distinctness of voice and point of view are necessary ideals and forms of autonomy within democratic forms of life. Perhaps we need to rethink the legally-inspired conception of Kant, beginning from ordinary experience with human (and other) beings with whom we share our forms of life. None of us are fully spontaneous or fully determined; but each of us is *vulnerable* to the actions of others and this must be understood as part of the fragility and importance of fine-grained ways of observing and seeing to autonomy (see Murdoch, Carol Gilligan, Nussbaum, Diamond, Laugier, Shuster).

“Autonomy”

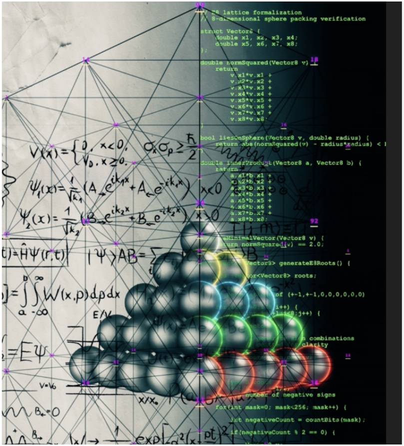
Though mathematics has long relied on developing computing devices and algorithms, and on clarifying the very *notion* of an “algorithm” or “formal system” (and its limits) (Frege 1879, Hilbert, Gödel 1931, Turing 1936), we live in a time when the notion of “autonomy” is re-emerging with newfound intensity in the face of new and surprising results with LLM’s and other combined technologies. Some mathematicians and scientists feel they face an existential crisis in the face of the encroachment of technology – especially AI – on their all-too-human forms of mathematical life.

NYT

The New York Times

They Spent Years on a Math Problem. Then They Were Scooped by A.I.

Artificial intelligence is mastering the kinds of projects that have long helped to build the careers of young mathematicians. What does that mean for their future?



"Autonomous"

We increasingly speak of "autonomous" algorithms, "autonomous" (even "FPV") vehicles and weapons, "autonomous" social structures, and in particular, in mathematics research, "automated" theorem proving, conjecturing, discovery, and verification, while retaining what some conceive as a *fundamental opposition* between human choice, freedom, agency, wisdom, beauty, insight, simplicity and intuitiveness, and the "mechanical" or "purely formal".

“Solving Mathematics”?

Within mathematics experts are being wooed into working with corporations, who pay them in a context where funding is scarce. One feels and hears that the CEO’s of many AI corporations desire the destruction of the modern research university, with its presumed autonomy from direct political and economic demands for efficiency (“time is money and money is time”, and “there’s never enough time”).

Their claim, at its most ambitious, is that human activity is being automated altogether out of the loop of activity, because ”solving math” means resolving (big and small) theorems.

"Solving Mathematics"? The Mathematical Singularity

Math, Inc.

Dedicated to verified superintelligence via autoformalization.

Solve math, solve everything.

Introducing Gauss, an agent for autoformalization

The Mathematical Singularity. Soon, a mathematical superintelligence will be able to reinvent and understand all of today's existing mathematics. Then, it will push beyond the frontier: learning, discovering, and verifying new knowledge. This is the mathematical singularity, where mathematics becomes a self-sustaining, self-improving and endlessly growing body of knowledge.

Optimism (Kfoury 2026)

- The distinction between *help* and *replacement* is crucial.
- Discovery for a mathematician – developing an *intuition* for a problem, or an *aesthetic* view of it, or an *alternative perspective* from another area of math on the same problem, or a *new mathematical ability* is more than *verifying* a conjecture or *certifying* a proof.
- LLM's, ATPs and IPA's have core mechanisms that depend upon many branches of mathematics, including mathematical logic and proof theory, whose evolution as branches of mathematics provides better designs and implementations. *Mathematics appears at both the object level and the metalevel.*

Optimism (Kfoury 2026)

- Neither LLMs or Automated Theorem Provers or Interactive Proof Assistants stand out for promoting the above dimensions. Mathematics is not just problem solving and delivering theorems to publishers, it is far more than that: it is about intellectual understanding, pleasure and beauty, and many other fulfilling things for its community of human practitioners.
- Mathematicians judge proofs and concepts not only by correctness, but also by clarity, depth, inevitability, economy, fruitfulness, surprise, and more, and these judgments are not [merely] decorative: they influence which definitions are adopted, which proofs are taught, and which problems become central. (See Kish 2025 on the social context of the history of intuitionism; Kish and Friedman on Big Science.)

Optimism (Kfoury 2026)

Even if mathematicians increasingly work with AI systems that *suggest*, *search*, *translate*, *check* and *formalize*, the decisive acts remain human-centered: formulating problems, choosing the formalism, judging the explanation, deciding what counts as progress.

Question: Are there certain areas of physics and mathematics (or kinds of conjectures within each field) that will yield more or less easily to AI methods? (See Dean and Naibo 2025). This is partly an empirical question...

The Leiden Declaration, 2026

A first pass at formulating desiderata and standards for the profession of mathematics in the newly emerging world of AI. Emphasized are: the importance of proving and verifying, of crediting authorship, of transparency and the avoidance of proprietary mechanisms that obfuscate, as well as the *autonomous* shaping of the direction of research by the professional mathematicians.

Also discussed are present challegnes of AI incursions into the field: informal and oversimplifying communication of results, press channels, blog posts, black boxing with programs that are proprietary, market timelines outpacing professional methods of review, and other important points.

A Question

Can we *make more foundational mathematics* to better understand the limits of the use of mathematics in mathematics?

Gödel 1964

- "The precise and unquestionably adequate definition of the general concept of formal system [made possible by Turing's work allows the incompleteness theorems to be] proved rigorously for every consistent formal system containing a certain amount of finitary number theory."
- "With Turing's analysis of computability one has for the first time succeeded in giving an absolute definition of an interesting epistemological notion, i.e., one not depending on the formalism chosen."

Gödel 1946 Praises Turing's Analysis

In all other cases treated previously, such as demonstrability or definability, one has been able only to define them relative to a given language, and for each individual language it is clear that the one thus obtained is not the one looked for. For the concept of computability, however, although it is merely a special kind of demonstrability or definability, the situation is different. By a kind of miracle it is not necessary to distinguish orders, and the diagonal procedure does not lead outside the defined notion.

Formalism Freeness (Kennedy 2020)

Idea: We can explore “absoluteness” with respect to *definability* in set theory. A notion of *autonomy* is embedded in the formalism freeness paradigm. One can test a mathematical object (and/or a theory and/or simply an intuitively given mathematical object such as the natural numbers) against variations in the formal framework over which the object is/can be built. Some objects do not change at all, even with very substantial perturbations in the formal framework—they exhibit a degree of *formalism freeness* or *autonomy* with respect to the class of formal systems in play. On the other hand some canonical objects (theories) are very sensitive to even very small variations of this kind. They are *entangled*; it is hard to see them as autonomous w.r.t. the formal systems in play.

Translation, Meaning, Context/Locality

First-order logic is "softly canonical" for many purposes. Yet the assignment of a recursively specified language or syntax involves serious conceptual issues. How do mathematical objects fare when they are translated into languages like LEAN and we explore variations in these languages? Does the mathematician lose autonomy in the transfer of mathematical labor to e.g. LEAN? Whereas the mathematician was (formerly?) free to regard their object as built from an array of formal systems, or not make any choice at all and simply work in natural language, what happens when the mathematician lets herself be drawn into collaborating with the machine? A certain price is paid when she commits herself to the language on which the agent is based. How might this be mathematically or logically measured?

Casilli 2025: Obscuring Forces

The workings of LLM's depend heavily on human labor and activity – in the form of prior publications and research efforts (violation of copyright), the ongoing work of curators and code-checkers, and tech-laborers whose labor is skilled, crucial and often quite arduous, but rendered invisible by platforms (Casilli 2025).

Moreover, the uses of the technologies are themselves vulnerable to how people *regard* and *feel about* them. (Cambridge Analytica...).

Translating Mathematics Globally and Intergenerationally

Kfoury is engaged in translating into Arabic for an Encyclopedia of mathematics logic materials. He is forced to choose modes of translation that may twist or creolify certain forms of language. E.g., writing equations left to right, when writing in Arabic is right to left.

Although the power of LLM's to aid in translation of ordinary information and informal discussion between languages is clear, finer points of translation, e.g. for philosophy, poetry – or even mathematics – outstrip what AI can decide for us.

To a certain extent (as teachers of mathematics know) proofs construct *narratives*. Is this an aspect that will be increased or decreased with the use of AI? Is it desirable? What is a satisfying narrative?

Wittgenstein: 3 Suggestions

1. The meaning is the *use*, and uses are constantly reimbedded in ongoing, evolving *forms of life*. We cannot expect in our world that there will be just one or even just a few forms of life with AI.
2. Philosophical questions often rest upon modes of formulation or positioning of questions [*Fragestellungen*] that are rooted in misunderstandings of the very forms of language in which they are posed.
3. "Progress always looks greater than it is" (Nestroy).

Thank You!

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