# Possible & Impossible Infinities

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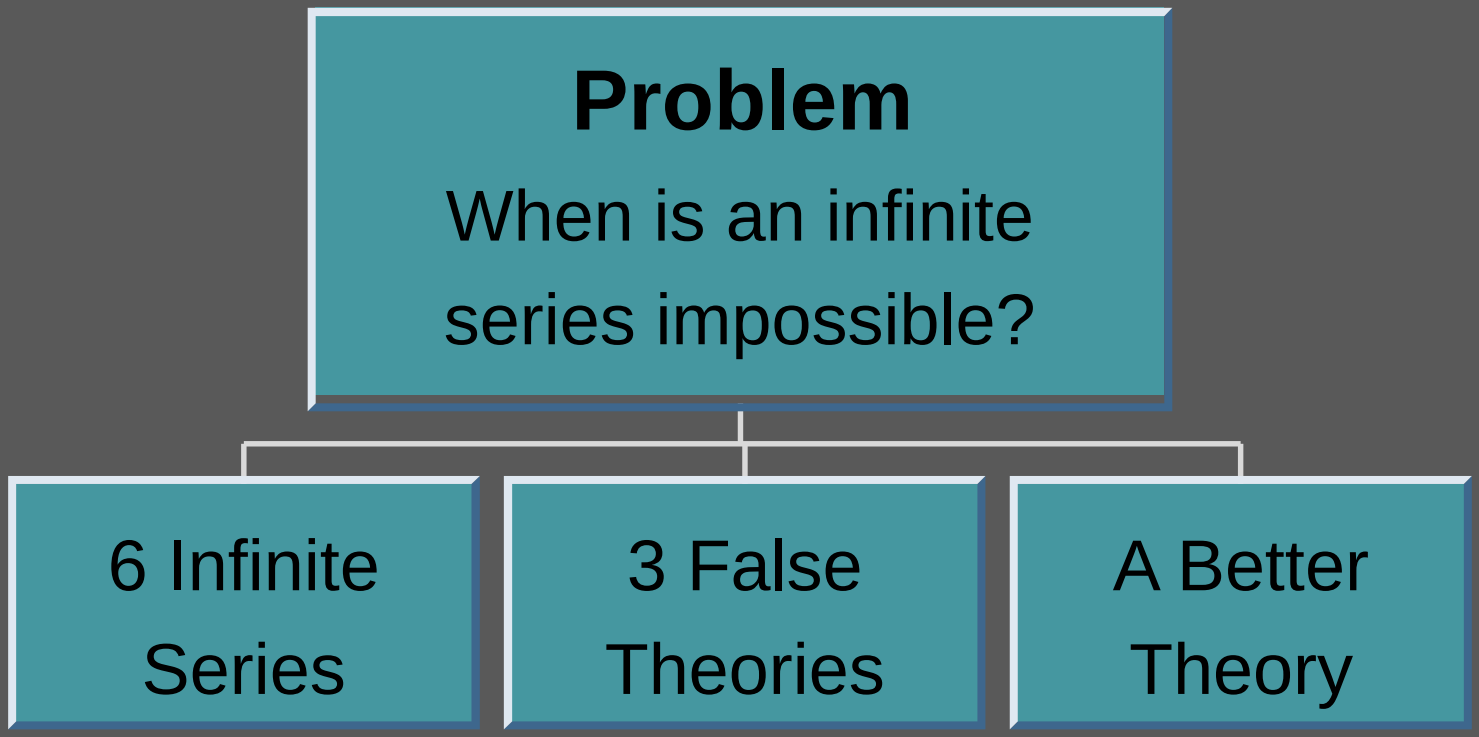

Problem
When is an infinite series impossible?
6 Infinite Series
3 False Theories
A Better Theory

# Six Infinities

The Truth Regress

Series: P.

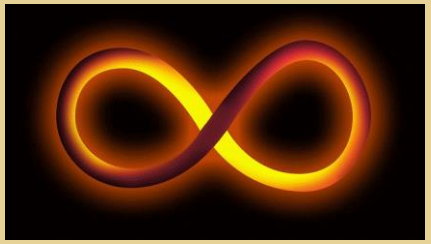
It's true that P.

It's true that it's true that P.

. . .

Generated by principle: P → It is true that P.

Verdict: Benign.



# Six Infinities

The Endless Zeno Series

Series: 1/2

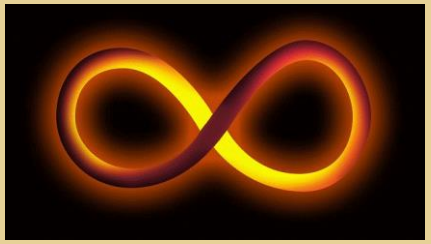
3/4

7/8

. . .

Generated by principle: To travel some distance, one must first travel half the distance.

Verdict: Benign.



# Six Infinities

Variant: The beginningless Zeno Series

Series: …

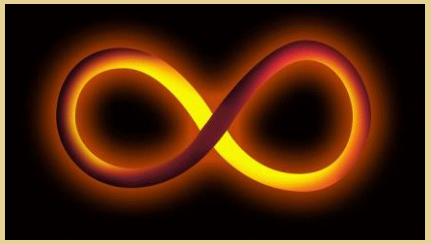
1/8

1/4

1/2

Generated by principle: To travel some distance, one must first travel half the distance.

Verdict: Benign.



# Six Infinities

Thomson's Lamp

| | | |
|---|---|---|
| Series: | Off | t=0 |
| | On | t=1/2 |
| | Off | t=3/4 |
| | . . . | |

Puzzle: At the end of the series, is it on or off?

Verdict: Impossible.

# Six Infinities

The Littlewood-Ross Banker

Start: infinite pile of $1 bills, bills # 1, 2, 3, ...

Series:
- $9 bills 2-10
- $18 bills 3-20
- $27 bills 4-30
- . . .

Puzzle: At the end of the series, how much money do you have?

Verdict: Impossible.

# Six Infinities

## The Regress of Causes

Series: A

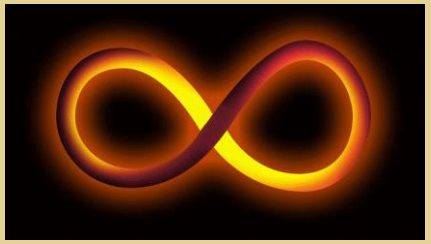
The cause of A

The cause of the cause of A

. . .

Generated by principle: Every event has a cause; every cause is an event.

Verdict: Controversial.



# Six Infinities

The Regress of Reasons

Series: P

The reason for P

The reason for the reason for P

. . .

Generated by principle: Every justified belief has a reason; every reason is a justified belief.

Verdict: Controversial.

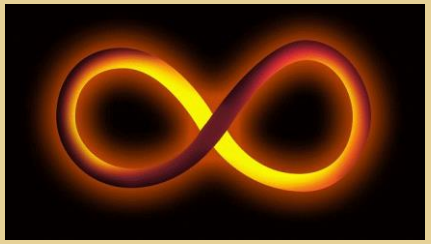
# Six Infinities

Desired:

A criterion of the impossible infinite that

a) Has some plausible theoretical rationale
b) Classifies the Truth Regress and Zeno's Series as benign
c) Classifies Thomson's Lamp and the Littlewood-Ross Banker as impossible

Regresses of causes and epistemic reasons are negotiable.



# Three Failed Criteria

Theory #1: There can be potential infinities, but no actual infinities.

Counter-examples

- Truth regress
- The Zeno series

# Three Failed Criteria

Theory #2: An infinite series cannot be completed by successive addition.

Counter-example*:*

- The endless Zeno series

# Three Failed Criteria

Theory #3: An infinite series is vicious when each member *depends on* the next.

Counter-example*:*

- The beginningless Zeno series

# A New Theory of the Vicious Infinite

Theory: There cannot be an infinite, natural, intensive magnitude.

Explanation: Two kinds of quantities:

- Cardinal numbers: 0, 1, 2, ... , $\aleph_0$, ...
- Magnitudes: (represented by real #s)

# A New Theory of the Vicious Infinite

Theory: There cannot be an infinite, natural, intensive magnitude.

Two kinds of magnitudes:

- Extensive: Additive across parts. Length, duration, volume, ...
- Intensive: Not additive across parts. Temperature, density, velocity, ...

# A New Theory of the Vicious Infinite

Theory: There cannot be an infinite, natural, intensive magnitude.

Two kinds of quantities:

- Natural: Real properties of objects. (Usually) causally efficacious.
- Artificial: Stipulative, may involve mathematical operations that lack physical significance. Non-causal.

# Classifying Cases

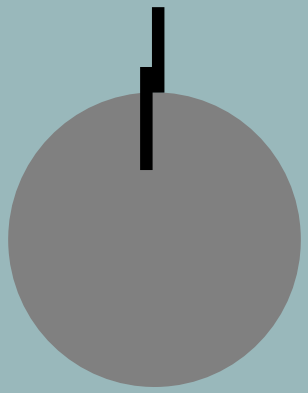
Thomson's Lamp:

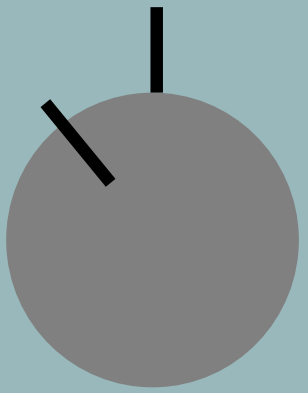
- Forces/accelerations increase without bound; hence...
- Infinite material strength
- Infinite energy density
- Infinite speed

# Classifying Cases

Variation on Thomson's Lamp:

The switch is a dial. At 0 degrees, lamp is on. All other positions are off.

On: 

Off: 

Diagnosis: This series is possible, but not paradoxical.

– In the end, the lamp is on.

# Classifying Cases

Littlewood-Ross Banker:

Impossible for similar reasons as Thomson's Lamp.

# Classifying Cases

Zeno:

- Infinite # of stages.
- All real physical magnitudes finite. (length, time, velocity, energy, …

# Classifying Cases

Truth regress:

- Infinite # of propositions: no problem.
  - p
  - T(p)
  - T(T(p))
  - …

# Theoretical Rationale

Fundamentally, there are only finite numbers.

- There is not a number larger than every real number.
- Claims about the infinite must be restatable in terms of finite numbers.

# Theoretical Rationale

Why are infinite cardinalities allowed?

- S has infinite cardinality $=_{df}$ for any natural number *n*, S contains a subset with more than *n* members.
- Conceptually, this only commits us to the existence of natural numbers.

# Theoretical Rationale

Why are infinite extensive magnitudes allowed?

*x* has an infinite amount of extensive magnitude *M* $=_{df}$ For any real magnitude *m* of *M*, *x* possesses a part whose quantity of *M* is greater than *m*.

Example: Space is infinite = for any chosen volume *v*, there is a part of space with volume greater than *v*.

Conceptually, this only commits us to the existence of finite magnitudes.

# Theoretical Rationale

Why can there *not* be an infinite intensive magnitude?

- The preceding analytical strategy fails for intensive magnitudes, since intensive magnitudes do not compound over parts.
- An ascription of infinite intensive magnitude requires that there be a number greater than all real numbers.

# Theoretical Rationale

Why consider only natural magnitudes?

Artificial magnitudes could do anything.

Ex.: define schmass to be $\frac{1}{3-\text{mass}}$. When mass=3, schmass=∞.

# The Controversial Cases

The regress of causes

- No reason why this could not exist.

The regress of reasons

- No reason why this could not exist.
- This is not to say that it’s plausible.

# Some Other Paradoxes

Hilbert’s Hotel

Benardete’s paradoxes

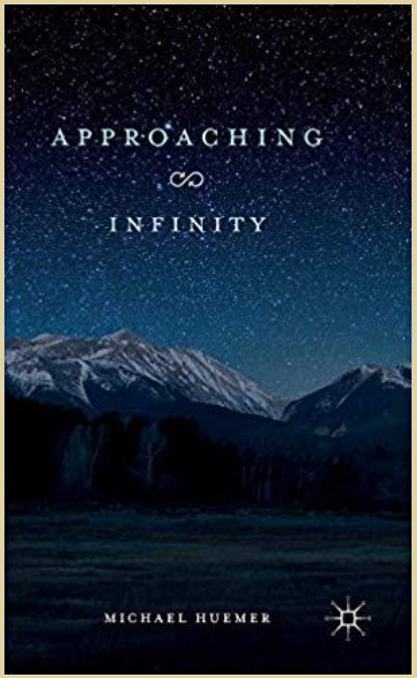
St. Petersburg

Laraudogoitia’s paradox

The spaceship

# Further Reading

Michael Huemer, *Approaching Infinity*
(New York: Palgrave Macmillan, 2016)



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