The Unreasonable Effectiveness of Mathematics: (Dis)solving Wigner's Applicability Problem

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Eugene P. Wigner

“The Unreasonable Effectiveness of Mathematics in the Natural Sciences” (1959/1960)

Nobel Prize winner in 1963 “for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles.”

Along with Weyl responsible for introducing group theory into physics.

Wigner’s Applicability problem

- The applicability problem is the problem of explaining why mathematics plays the fundamental role that it does in nature/ natural sciences.

Wigner calls this “the empirical law of epistemology”:

“the appropriateness and accuracy of the mathematical formulation of the laws of nature in terms of concepts chosen for their manipulability”.

Motivating Wigner's Applicability problem

- What is mathematics?

"mathematics is the science of skillful operations with concepts and rules invented just for this purpose."

The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colors or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics. (Hardy 1940, p. 85).

Motivating Wigner's Applicability problem

What is mathematics?

Mathematics, rightly viewed, possesses not only truth, but supreme beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as in poetry.

- BERTRAND RUSSELL, Study of Mathematics

Motivating Wigner's Applicability problem

. What is physics?

"The physicist, unlike the mathematician, is interested in inanimate nature and in discovering "regularities" that exist among the natural phenomena."

Physical objects are said to be causal and spatiotemporal. Mathematical objects, on the other hand, stand outside the flow of time and causality.

Wigner's Applicability problem

- Distinctness thesis (Ontological, methodological, epistemic..?)

Our situation in this case is like "a man who was provided with a bunch of keys and who, having to open several doors in succession, always hit on the right key on the first or second trial" (Wigner 1960, p. 223).

Wigner's Applicability problem

" [A] mathematician plays a game in which he himself invents the rules. While the physicist plays a game in which the rules are provided by Nature but as time goes on it becomes increasingly evident that the rules which the mathematician finds interesting are the same as those which Nature has chosen." (Dirac 1939)

"We are confronted with the peculiar fact that matter seems to comply well and truly to the formalism of mathematics. There arises an unforeseen unison of being and thinking, which for the present we have to accept like a miracle." (Hilbert 1919)

Wigner’s Applicability problem

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“ At this point an enigma presents itself which in all ages has agitated inquiring minds. How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality? Is human reason, then, without experience, merely by taking thought, able to fathom the properties of real things. ” (Einstein 1922)

Wigner's Applicability Problem

- "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning." (Wigner 1960, p. 237)

Wigner’s Applicability Problem

[T]he enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it. (Wigner 1960, p. 223)

It is difficult to avoid the impression that a miracle confronts us here, quite comparable in its striking nature to the miracle that the human mind can string a thousand arguments together without getting itself into contradictions, or to the Two miracles of the existence of laws of nature and of the human mind’s capacity to divine them. (Wigner 1960, p. 229)

Motivating Wigner's Applicability problem

1. Mathematical concepts arise from the aesthetic impulse in humans.
2. It is unreasonable to expect that what arises from the aesthetic impulse in humans should be significantly effective in physics.
3. Nevertheless, a significant number of these concepts are significantly effective in physics.
4. Hence, mathematical concepts are unreasonably effective in physics (Steiner 1998).

Reactions to Wigner

The unreasonable effectiveness of mathematics (Hamming 1980)
The unreasonable ineffectiveness of mathematics in economics (Velupillai 2005)
The reasonable ineffectiveness of mathematics in biology (Longo and Montévil 2016)
The reasonable (though perhaps limited) effectiveness of mathematics in the natural sciences (Grattan-Guinness 2008)

Reactions to Wigner

1. Mathematics is not so effective in other empirical sciences.
2. Mathematics didn't play a major role in Aristotelian physics.
3. Not all of mathematics is useful.
4. Not all of theories written using mathematical concepts are true.

I. To What Extent does Wigner acknowledge the limitations?

- While the title of the paper is misleading, Wigner only discusses modern physics. (Islami 2017)
- Wigner's question is, what is it about modern physics that makes it possible for mathematical concepts to play such a fundamental role?
- Answer: a peculiar structure (initial conditions, laws of nature, invariance principles)

Structure of Modern Physics

The objective of physics, defined in this way, is not to explain (inanimate) nature but to explain "regularities in the behavior of the object" (Wigner 1964, p. 39).

The scope of physicist's objective is, therefore, restricted to the domains in which these regularities can be found. And "the great success of physics is due", Wigner notes in his Nobel lecture, "to [this] restriction in its objective."

Structure of modern physics

Man, has, therefore devised an artifice that permits the complicated nature of the world to be blamed on something which is called accidental and thus permits him to abstract a domain in which simple laws can be found. The complications are called initial conditions, the domain of regularities, laws of nature. Unnatural as this division of the world structure may appear from a detached point of view, and probable though that the possibility of such a division has its own limits, the underlying abstraction is probably one of the most fruitful ones that human mind has made (Wigner 1949, p. 3).

Structure of Modern Physics

There is a strange hierarchy in our knowledge of the world around us. Every moment brings surprises and unforeseeable events…truly the future is uncertain. There is, nevertheless, a structure in the events around us, that is, correlations between the events of which we take cognizance. It is this structure, these correlations, which science wishes to discover, or at least the precise and sharply defined correlations. …We know many laws of nature and we hope and expect to discover more. Nobody can foresee the next such law that will be discovered. Nevertheless, there is a structure in the laws of nature which we call the laws of invariance. This structure is so far-reaching in some cases that laws of nature were guessed on the basis of the postulate that they fit into the invariance structure. …This then, the progression from events to laws of nature, and from laws of nature to symmetry or invariance principles, is what I meant by the hierarchy of our knowledge of the world around us. (Wigner 1963, pp. 28–29)

Structure of Modern Physics

"Indeed, classical physics teaches us that the size of planetary orbits is not the sort of thing we should aspire to predict. It makes a sharp distinction between the basic laws, which govern the evolution of systems in time, and are expected to be simple, and the initial conditions, which must be given from outside. The equations of classical physics can be applied to any number of different types of solar system, having different sizes and shapes. There is nothing in Newton's laws of gravity and mechanics, nor for that matter in the other pillar of classical physics, Maxwell's electrodynamics, that could serve to fix a definite size.

(Wilczek 1999, p. 303)

Structure of Modern Physics

In modern quantum mechanics as well as in Newtonian mechanics there is a clear separation between the conditions that tell us the initial state of a system (whether the system is the whole universe, or just a part of it), and the laws that govern its subsequent evolution. (Weinberg 1992, p. 34)

II. To What Extent does Wigner acknowledge the limitations?

Wigner gives a few examples of these theories such as Bohr's early theory of atom, Ptolemy's epicycles, and the free electron theory. In all of these cases there were amazing numerical agreements between the theory and experiment and were formulated in the language of mathematics (Wigner 1960, p. 236).

III. To What Extent does Wigner acknowledge the limitations?

It is true, of course, that physics chooses certain mathematical concepts for the formulation of the laws of nature, and surely only a fraction of all mathematical concepts is used in physics. It is true also that the concepts which were chosen were not selected arbitrarily from a listing of mathematical terms but were developed, in many if not most cases, independently by the physicist and recognized then as having been conceived before by the mathematician. (Wigner 1960, p. 229)

Limited Applicability Problem

So far, we learned:

- Wigner's Applicability problem is restricted to modern theoretical physics.
- Ask how question before why question.
- Pay attention to the structure of modern physics
- Consider mathematization before application (idealizations in physics)

How about mathematics?

What is it about mathematics…?

- Wigner as a Hilbert School Formalist (Ferreiros, Steingart) mixes abstract with formal…
- Wigner’s philosophy of mathematics, “creating miracles out of thin air” (Unger and Smolin)
- Origin of mathematical concepts in lifeworld (Husserl’s *Crisis*) (Islami & Wiltsche 2020)
- Other current philosophies of mathematics dont do better (Colyvan 2001)

What is it about mathematics…?

A defensible/sensible/ responsible philosophy of mathematics

- takes the evolving relationship between mathematics and physics as its starting point . e.g., "what did mathematics do to physics?" (Gingras 2001)

"I am acutely aware of the fact that the marriage between mathematics and physics, which was so enormously fruitful in past centuries, has recently ended in divorce." (Dyson 1972)

What is it about mathematics...?

A defensible/sensible/ responsible philosophy of mathematics

- takes the asymmetric relationship between mathematics/physics and mathematics/biology seriously. (Islami & Longo 2017)
- encourages attempts to create mathematical systems that fit better with evolutionary biology for instance.

Alessandra Sarti, Longo and Montevil,...

What is it about mathematics...?

A defensible/sensible/ responsible philosophy of mathematics

- through connecting mathematical concepts to their origins, highlights the cognitive and historical roots of mathematics, leading to an empowering didactic
- focuses on also the applicability of mathematics to mathematics itself

(“A Complex Problem for Formalists”, Islami 2018)

What is it about mathematics...?

A defensible/sensible/ responsible philosophy of mathematics

- is a result of close collaborations between mathematicians and philosophers, who are eager to think deeply about what mathematics is (descriptive) and perhaps also about the fruitful directions in which it can grow (normative)

Thanks...

- **From Wigner to Hamming and back Again**, *Foundations of Physics* (2022)
- **A Match Made on Earth. On the Applicability of Mathematics in Physics**, with Harald A. Wiltsche, Springer, *Phenomenological Approaches to Physics* (2020)
- **A Complex Problem for Formalists**, *Journal of Applied Logics* (2018)
- **Marriages of Mathematics and Physics: a Challenge for Biology**, with Giuseppe Longo, *Journal for Biophysics and Molecular Biology* (2017)
- **A Match Not Made in Heaven: on the Applicability of Mathematics in Physics**, *Synthese* (2017)

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