

The background is a light gray surface covered with faint, handwritten mathematical equations and diagrams. Some visible formulas include  $x^2 + y^2 = ab + 4c$ ,  $c(x, y)$ ,  $xy = z$ ,  $2\pi = c$ ,  $A_2 T B_3$ ,  $24 + \dots$ ,  $x = 9.20$ ,  $u = 14$ ,  $x \leq 549$ , and  $y = 4$ . There are also geometric shapes like a triangle and a circle with a shaded sector. Scattered across the page are several realistic water droplets of various sizes, some with highlights and shadows, giving a sense of depth and texture.

# AS-IFISM: MATHEMATICS AND METHOD WITHOUT METAPHYSICS.

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# AIMS

- CLAIM: WHEN WE SHIFT OUR FOCUS FROM SOLVING PHILOSOPHICAL PROBLEMS TO SOLVING MATHEMATICAL ONES, WE SEE THAT AN *AS-IF METHODOLOGICAL INTERPRETATION* OF MATHEMATICAL STRUCTURALISM CAN BE USED TO PROVIDE AN ACCOUNT OF BOTH THE PRACTICE AND THE APPLICABILITY OF MATHEMATICS WHILST AVOIDING THE CONFLATION OF *METHODOLOGICAL* AND *METAPHYSICAL* CONSIDERATIONS.
  - I BEGIN FIRST WITH PLATO TO SHOW THAT MUCH PHILOSOPHICAL MILK HAS BEEN SPILT OWING TO OUR *CONFLATING* THE METHOD OF MATHEMATICS WITH THE METHOD OF PHILOSOPHY.
  - I FURTHER USE MY READING OF PLATO TO DEVELOP WHAT I CALL *AS-IFISM*, THE VIEW THAT, IN MATHEMATICS, WE TREAT OUR HYPOTHESES *AS IF* THEY WERE FIRST PRINCIPLES AND WE DO THIS WITH THE PURPOSE OF SOLVING MATHEMATICAL PROBLEMS NOT PHILOSOPHICAL ONES.
  - I NEXT EXTEND *AS-IFISM* TO MODERN MATHEMATICS WHEREIN THE METHOD OF MATHEMATICS BECOMES THE AXIOMATIC METHOD, NOTING THAT THIS ENGENDERS A SHIFT FROM *AS-IF* HYPOTHESES TO *AS-IF* AXIOMS.
  - I PAUSE TO NOTE THAT THE CONFLATION OF THE METHOD OF MATHEMATICS WITH THE METHOD OF PHILOSOPHY, WITNESSED WELL BY THE FREGE-HILBERT DEBATE, HAS LED TO THE CONTINUED *CONFUSION* OF MATHEMATICS WITH METAPHYSICS.
  - FINALLY, I USE MY *METHODOLOGICALLY INTERPRETED STRUCTURAL AS-IFISM* TO BREAK BENACERRAF'S DILEMMA BY SHOWING THAT THERE ARE *TWO TYPES OF EXISTENCE* AT PLAY IN MATHEMATICS AND SCIENCE.

# PLATO

	Visible World (509d) Becoming (521d)		Intelligible World (509d) Being (521d)	
<b>Faculty</b>	Senses (510d) Perception (511d)		Soul (510b) Reason (510b)	
<b>Methodology</b>			<p><b>Mathematics</b></p> <p><b>Hypothetical (510b)</b> Hypotheses as first principles (511b) Down from hypotheses to conclusion (510b, 511a) ~Dialectic (531d, 534a) Dreaming about what is (533b) Dream about reality (534d)</p>	<p><b>Philosophy</b></p> <p><b>Dialectical (511b)</b> Hypotheses as hypotheses (510d) Up to unhypothetical first-principle from hypotheses (510b, 511b) Account of being (534b)</p>
	Opaque/untruth (519d, 511e)	Clear/truth (509d, 511e)	Opaque/truth (509d, 511e)	
	Copy (510a)	Original (510a)	Copy	
<b>Epistemology</b>	Opinion (510a) Belief (534a)		Knowledge (510a) Understanding (534a)	
	Imagination (511e, 534a)	Belief (511d,e) Opinion (534a)	Thought (511a,d,e, 534a) ~Knowledge (514d, 533c)	
<b>Ontology</b>	Images, shadows Reflections (509e-510a)	Animals, Plants, artifacts (510a)	Objects of thought (511c-d)	
		Currently practiced Incorrect use (523a) Images of physical objects (510b) visible forms (510d) Images drawn (510e) numbers attached to visible images (525d) Motion of ornaments	the odd, the even (510c) the figures and 3 angles as hypotheses (510c) square, diagonal itself (510e) things themselves (510e) numbers themselves (525d) Motions measured by numbers (529d)	

# PLATO

- PLATO KEPT A CLEAR DISTINCTION BETWEEN MATHEMATICS AND METAPHYSICS. AND THE KNIFE HE USED TO SLICE THE DIFFERENCE BETWEEN THE TWO WAS METHOD.
  - THE MATHEMATICAL METHOD REASONS *DOWN FROM AN HYPOTHESIS* TOWARDS A CONCLUSION, WITH THE PURPOSE OF SOLVING A *MATHEMATICAL* PROBLEM.
    - THE SOUL IS *FORCED TO USE HYPOTHESES* IN THE INVESTIGATIONS OF [A PROBLEM], NOT TRAVELING UP TO A FIRST PRINCIPLE, SINCE *IT CANNOT ESCAPE OR GET ABOVE ITS HYPOTHESES...* (511A)
  - THE PHILOSOPHICAL METHOD REASONS *UP FROM AN HYPOTHESIS* TOWARDS A FIRST PRINCIPLE WHICH TETHERS THE HYPOTHESIS, AND ONLY THEN CAN HE REASON DOWN TOWARDS A CONCLUSION, WITH THE PURPOSE OF SOLVING A *PHILOSOPHICAL* PROBLEM.
    - ALSO UNDERSTAND, THEN, THAT BY THE OTHER SUBSECTION OF THE INTELLIGIBLE I MEAN WHAT REASON ITSELF GRASPS BY THE POWER OF *DIALECTICAL DISCUSSION*, TREATING ITS HYPOTHESES, NOT AS FIRST PRINCIPLES, BUT AS *GENUINE HYPOTHESES*, THAT IS, STEPPING STONES AND LINKS IN A CHAIN, IN ORDER TO ARRIVE AT WHAT IS *UNHYPOTHETICAL* AND THE *FIRST PRINCIPLE* OF EVERYTHING. HAVING GRASPED THIS PRINCIPLE, IT REVERSES ITSELF AND, KEEPING HOLD OF WHAT FOLLOWS FROM IT, COMES DOWN TO A CONCLUSION...MOVING ON THROUGH FORMS TO FORMS, AND ENDING IN FORMS.(511B-C)

# THE CONFUSION

- THE MATHEMATICIAN'S HYPOTHESES, THEN, ARE TAKEN *AS IF* THEY WERE FIRST PRINCIPLES, *BUT THEY ARE NOT*, THE MATHEMATICIAN'S OBJECTS ARE TAKEN *AS IF* THEY WERE OBJECTS OF KNOWLEDGE, *BUT THEY ARE NOT*.
  - STUDENTS OF GEOMETRY, CALCULATION, AND THE LIKE *HYPOTHESIZE* THE ODD AND THE EVEN, THE VARIOUS FIGURES, THE THREE KINDS OF ANGLES, AND OTHER THINGS AKIN TO THESE IN EACH OF THEIR INVESTIGATIONS, *REGARDING THEM AS KNOWN*. THESE THEY TREAT THESE *AS IF* THEY WERE FIRST PRINCIPLES AND *DO NOT THINK IT NECESSARY TO GIVE ANY ACCOUNT OF THEM*, EITHER TO THEMSELVES OR TO OTHERS, *AS IF* THEY WERE EVIDENT TO EVERYONE. AND, CONSISTENTLY GOING FROM THESE FIRST PRINCIPLES THROUGH THE REMAINING STEPS, THEY CONCLUDE IN FULL AGREEMENT *AT THE POINT THEY SET OUT TO REACH IN THEIR INVESTIGATION*. (510C-D ITALICS ADDED.)
- THE PURPOSE OF THE MATHEMATICIANS' METHOD, WHICH *BEGINS WITH TAKING* HYPOTHESES *AS IF* THEY WERE FIRST PRINCIPLES, IS TO SOLVE A GIVEN MATHEMATICAL PROBLEM, *IT IS NOT* TO GIVE A PHILOSOPHICAL ACCOUNT OF THEM AS FIRST PRINCIPLES!
- AGAINST **METAPHYSICAL REALISM**: MATHEMATICS *DOES NOT NEED* A METAPHYSICS OF FORMS THAT, AS FIRST PRINCIPLES, ACCOUNTS FOR, OR TETHERS, ITS HYPOTHESES.

# THE CONFUSION

- THE *HYPOTHETICAL METHOD OF MATHEMATICS* IS DISTINCT FROM THE *METAPHYSICAL METHOD OF PHILOSOPHY*, AND, AS SUCH, SO IS ITS ONTOLOGY, AND ITS EPISTEMOLOGY.
  - MATHEMATICAL OBJECTS ARE NOT OBJECTS OF KNOWLEDGE, THEY NOT AS REAL AS PHILOSOPHICAL OBJECTS, BUT, AS *OBJECTS OF THOUGHT*, THEY ARE STILL “CONCERNED WITH BEING” (534A).
  - THE MATHEMATICAL METHOD YIELDS *A KIND OF UNDERSTANDING* BUT *NOT KNOWLEDGE*, THAT IS, IT YIELDS BELIEFS THAT ARE “RELIABLE GUIDES TO SOLVING PROBLEMS” (532B) BECAUSE THEY ARE BORN OUT OF *STABLE DEFINITIONS* AND A *RELIABLE METHOD*.
  - ONLY THE METAPHYSICAL METHOD OF PHILOSOPHY YIELDS TRUE UNDERSTANDING OR KNOWLEDGE, THAT IS, YIELDS TRUE BELIEFS THAT ARE THEMSELVES FIXED TO, OR TETHERED BY, A DOMAIN OF STABLE OBJECTS OR FORMS.
- EVEN GLAUCON IS SHOCKED TO HEAR THAT PHILOSOPHY AS A SCIENCE IS CLEARER THAN MATHEMATICS
  - I UNDERSTAND, THOUGH NOT ADEQUATELY ... YOU WANT TO DISTINGUISH THE PART OF WHAT IS AND WHAT IS INTELLIGIBLE, THE PART LOOKED AT BY THE SCIENCE OF DIALECTICAL DISCUSSION, AS *CLEARER THAN* THE PART LOOKED AT BY THE SO-CALLED SCIENCES – THOSE FOR WHICH HYPOTHESES ARE FIRST PRINCIPLES ... AND ALTHOUGH THOSE WHO LOOK AT THE LATTER PART ARE FORCED TO DO SO BY MEANS OF *THOUGHT* RATHER THAN SENSE PERCEPTION, STILL, BECAUSE THEY DO NOT GO BACK TO A GENUINE FIRST PRINCIPLE IN CONSIDERING IT, BUT PROCEED FROM HYPOTHESES, YOU DO NOT THINK THAT THEY HAVE TRUE UNDERSTANDING OF THEM, EVEN THOUGH ... THEY ARE INTELLIGIBLE. AND YOU SEEM TO ME TO CALL THE STATE OF MIND OF THE GEOMETERS – AND THE OTHERS OF THAT SORT – *THOUGHT* BUT *NOT UNDERSTANDING*; THOUGHT BEING INTERMEDIATE BETWEEN BELIEF AND UNDERSTANDING. (511C-D)

# CORRECTING THE CONFUSION

- *THE CONFUSION:* AS PHILOSOPHERS, WE HAVE CONTINUED TO CONFLATE THE HYPOTHETICAL METHOD OF MATHEMATICS WITH THE METAPHYSICAL METHOD OF PHILOSOPHY.
- *THE CORRECTION:* WHEN I SAY A MATHEMATICAL OBJECT EXISTS, WHAT I MEAN IS THAT I TREAT MY HYPOTHESIS *AS IF* IT WERE A FIRST PRINCIPLE AND, IN SO DOING, I ACT *AS IF* IT WERE TETHERED TO AN OBJECT.
- THUS, I TREAT MY OBJECT *AS IF* IT EXISTS FOR THE PURPOSE OF SOLVING A MATHEMATICAL PROBLEM.
  - FOR EXAMPLE, IN SOLVING THE *MENO* PROBLEM, I TREAT THE LENGTH OF LINE THAT DOUBLES THE AREA OF A 2 UNIT SQUARE AS IF WERE AN OBJECT, BUT IT IS NOT. MOREOVER, IT IS ONLY BECAUSE OF THE STABILITY OF THE DEFINITIONS OF SQUARE AND OF DIAGONAL, TOGETHER WITH THE PYTHAGOREAN THEOREM, THAT I CAN REASON DOWN TO THE CONCLUSION THAT THE LENGTH WILL BE THE LENGTH OF THE DIAGONAL OF THE 2 UNIT SQUARE, *BUT* I CANNOT KNOW THE LENGTH OF THIS LINE AS A STABLE OBJECT SINCE IT IS  $2\sqrt{2}!!$

# PLATONIC METHODOLOGICAL AS-IFISM

- IN MATHEMATICS, WE TREAT OUR HYPOTHESES *AS IF* THEY WERE FIRST PRINCIPLES, AND, CONSEQUENTLY, OUR OBJECTS *AS IF* THEY EXISTED, AND WE DO THIS WITH THE PURPOSE OF SOLVING MATHEMATICAL PROBLEMS.
- MATHEMATICS AS A SCIENCE IS FOUNDED ON THE THE HYPOTHETICAL *METHOD* AND THE STABILITY OF ITS DEFINITIONS; IT IS *NOT* FOUNDED ON THE DIALECTIC METHOD AND THE STABILITY OF ITS METAPHYSICAL OBJECTS.



# MODERN AS-IFISM

- I NOW EXTEND THIS *METHODOLOGICAL AS-IFISM* TO MODERN MATHEMATICS WHEREIN THE METHOD OF MATHEMATICS BECOMES THE AXIOMATIC METHOD.
- THIS ENGENDERS A SHIFT FROM STARTING WITH *AS IF* HYPOTHESES TO STARTING WITH *AS IF* AXIOMS.
- MATHEMATICS AS A SCIENCE IS FOUNDED ON THE AXIOMATIC METHOD AND THE STABILITY OF ITS DEFINITIONS, NOW IMPLICITLY EXPRESSED BY THE AXIOMS THEMSELVES.
- AGAIN, IT IS *NOT* FOUNDED ON THE STABILITY OF METAPHYSICAL OBJECTS.

# THE FREGE-HILBERT DEBATE

- FREGE, FOR EXAMPLE, CONFUSES THE METHOD OF MATHEMATICS WITH THE METHOD OF PHILOSOPHY (WITH THE METHOD OF CONCEPT CONSTRUCTION), THAT IS, HE TAKES AXIOMS AS FIRST PRINCIPLES AND SO PRESUMES THAT WE NEED A STABLE DOMAIN OF OBJECTS TO TETHER OR FIX THE TRUTH OF OUR AXIOMS
  - FOR THE FREGEAN AXIOMS-AS-FIRST-PRINCIPLES ACCOUNT, THE PRIMITIVE TERMS EMPLOYED BY THE AXIOMS MUST BE DEFINED OVER A *FIXED* DOMAIN *BEFORE* THE STATEMENT OF THE AXIOMS. THAT IS, THESE OBJECTS MUST BE LOGICALLY CONSTRUCTED IN THE CASE OF ARITHMETIC AND KANTIAN CONSTRUCTED IN THE CASE OF GEOMETRY.
- HILBERT, BY CONTRAST, TAKES AXIOMS AS *IF* THEY WERE FIRST PRINCIPLES THAT THEMSELVES IMPLICITLY *DEFINE* OBJECTS, SO WHATEVER SATISFIES THE AXIOMS IS TAKEN AS AN OBJECT THAT FIXES THE TRUTH OF THE AXIOMS.
  - HILBERT TOOK AXIOMS AS *IMPLICIT DEFINITIONS* OVER A *VARIABLE* DOMAIN, SO THAT THE AXIOMS SYSTEMS THEMSELVES ARE BUT A “SCHEMA” FOR DEFINING THOSE CONCEPTS THAT ORGANIZE WHAT WE SAY ABOUT THE PRIMITIVE TERMS AS VARIOUSLY INTERPRETED OBJECTS.

# META-MATHEMATICAL IF-THENISM

- FOR FREGE THE STABILITY OF MATHEMATICAL DEFINITIONS WAS TO BE JUSTIFIED BY ASSUMING THE *TRUTH* OF THE AXIOMS, TRUTH AS FIXED LOGICALLY, IN THE CASE OF ARITHMETIC, OR TRUTH AS FIXED PHILOSOPHICALLY BY KANTIAN INTUITION, IN THE CASE OF GEOMETRY.
  - FREGE'S META-MATHEMATICAL ACCOUNT OF THE METHOD OF MATHEMATICS WAS: *IF THE AXIOMS ARE TRUE, THEN THIS THEOREM CAN BE JUSTIFIED.*
- FOR HILBERT, HOWEVER, THE STABILITY OF DEFINITIONS WAS JUSTIFIED BY ASSUMING THE *CONSISTENCY* OF THE AXIOMS. HENCE HILBERT'S FAMOUS QUOTE:
  - *IF THE ARBITRARY POSTULATED AXIOMS DO NOT CONTRADICT EACH OTHER WITH THEIR COLLECTIVE CONSEQUENCES, THEN THEY ARE TRUE AND THE THINGS DEFINED BY MEANS OF THE AXIOMS EXIST. THAT, FOR ME, IS THE CRITERION OF TRUTH AND EXISTENCE.*
  - HILBERT'S META-MATHEMATICAL ACCOUNT OF THE METHOD OF MATHEMATICS WAS: *IF THE AXIOMS ARE CONSISTENT, THEN THIS THEOREM CAN BE JUSTIFIED.*

# IF-THENISM VERSUS AS-IFISM

- FEARING FORMALISM, BOTH FREGE AND HILBERT CAME TO REJECT *IF-THENISM* AS A META-MATHEMATICAL ACCOUNT OF THE METHOD OF MATHEMATICS.
- WHAT I WILL NOW CONSIDER IS WHETHER THESE *IF-THENIST* VIEWS CAN RECONSIDERED IN TERMS OF THE *METHODOLOGICAL STRUCTURAL AS-IFIST* VIEW THAT PLATO SEEMED TO BE OFFERING UP.

# LOGICAL IF-THENISM

- FREGE DEVELOPED TWO FORMS OF IF-THENISM.
- ACCORDING TO THE FIRST *DEDUCTIVE IF-THENIST* OPTION, MATHEMATICS IS IN THE BUSINESS OF ESTABLISHING RESULTS IN PURE LOGIC.
  - PRESUMING THAT
    - **A** STANDS FOR A QUANTIFICATIONAL SCHEMA DIAGRAMMING THE SUPPOSED AXIOMS AND **T** STANDS FOR A QUANTIFICATIONAL SCHEMA DIAGRAMMING THE SUPPOSED THEOREM OF THE THEORY. [SEE RESNIK, 1980, 117]
  - THIS FIRST OPTION CAN EITHER BE EXPRESSED AS “**A**  $\supset$  **T**” IS LOGICALLY VALID (LOGICALLY PROVABLE) OR AS THE CLAIM THAT **A**  $\vdash$  **T** (**T** IS LOGICALLY DERIVABLE FROM **A**).

# STRUCTURAL IF-THENISM

- ON THE SECOND *STRUCTURAL IF-THENIST OPTION*, FREGE
  - VIEWS A MATHEMATICAL THEORY AS STUDYING THE PROPERTIES OF ALL STRUCTURES *SATISFYING* CERTAIN DEFINING CONDITIONS, BUT HE NEVER MAKES USE OF THE ASSUMPTION THAT SUCH STRUCTURES EXIST [RESNIK, 1980, 117]. THIS OPTION IS EXPRESSED AS “ $\mathbf{A} \models \mathbf{T}$ ” ( $\mathbf{T}$  IS LOGICALLY ENTAILED BY  $\mathbf{A}$ ).
- RESNIK NOTES THAT THIS STRUCTURALIST OPTION OFFERS A STRAIGHTFORWARD ACCOUNT OF *APPLICABILITY*
  - WHEN ONE FINDS A PHYSICAL STRUCTURE SATISFYING THE AXIOMS OF A MATHEMATICAL THEORY, THE APPLICATION OF THAT THEORY IS IMMEDIATE [RESNIK, 1980, 118]
- RESNIK FURTHER NOTES A FINAL VIRTUE, VIZ., THAT
  - SUCH A STRUCTURAL IF-THENIST APPROACH IS IN-LINE WITH THE DEVELOPMENT OF ABSTRACT STRUCTURES, LIKE GROUP THEORY AND TOPOLOGY [RESNIK, 1980, 118] ... *AND CATEGORY THEORY!*

# PROBLEMS WITH STRUCTURAL IF-THENISM

- **THE STRUCTURE PROBLEM (HELLMAN'S "HOME ADDRESS" PROBLEM)**
  - WE NEED SET THEORY OR SOME OTHER THEORY AS A *BACKGROUND THEORY OF STRUCTURES THEMSELVES*.
    - *HELLMAN'S MODAL STRUCTURALIST ACCOUNT OF POSSIBLE STRUCTURES.*
    - *SHAPIRO'S PLATONIST ACCOUNT OF ACTUAL STRUCTURES.*
- **THE CONSISTENCY PROBLEM (SHAPIRO'S "TURN TO LOGIC OR PHILOSOPHY" PROBLEM)**
  - THE POSSIBLE INFINITE REGRESS OF RELATIVE CONSISTENCY PROOFS WILL ONLY BE STOPPED BY A TRUE THEORY. OTHERWISE, LIKE FREGE, WE NEED TO TURN TO LOGIC OR TO PHILOSOPHY TO ACCOUNT FOR TRUTH ITSELF.

# PROBLEMS WITH STRUCTURAL IF-THENISM

- FACED WITH THESE PROBLEMS, RESNIK PRESENTS US WITH TWO ALTERNATIVE ROUTES:
  - 1. WE CAN TAKE THE *FREGEAN ROUTE* OF TURNING TO PHILOSOPHY AND BASE THE ASSUMPTION OF CONSISTENCY ON
    - A BELIEF IN THE MATHEMATICAL REALITY AND TRUTH OF SOME THEORY WHICH WILL VOUCH SAFE THE CONSISTENCY OF MATHEMATICAL THEORIES [RESNIK, 1980, 119].
  - 2. WE TAKE THE *CARNAPIAN ROUTE OF TURNING TO LOGIC* AND OFFER UP A RELATIVE CONSISTENCY PROOF TO
    - ARGUE THAT SINCE CONSISTENCY IS A MATHEMATICAL QUESTION, IT, TOO MUST BE TREATED LOGICALLY...[SO] THE ASSERTION THAT A GIVEN AXIOM SET IS CONSISTENT MUST ITSELF BE CONSTRUED AS CONDITIONAL UPON A BACKGROUND THEORY WITH RESPECT TO WHOSE TRUTH THE DEDUCTIVIST CAN REMAIN AGNOSTIC [RESNIK, 1980, 119].
- RESNIK, SHAPIRO, HELLMAN, AND **FOUNDATIONALISTS**, TAKE THE FREGEAN ROUTE, I TAKE THE CARNAPIAN.



# METHODOLOGICALLY INTERPRETED STRUCTURAL AS-IFISM

- LET'S NOW COMPARE *METAPHYSICALLY INTERPRETED* STRUCTURALISM WITH *METHODOLOGICALLY INTERPRETED* STRUCTURAL AS-IFISM AND SEE IF WE CAN'T FORESTALL THESE PROBLEMS.
  - LET'S FIRST RECALL THAT THE BASIC PREMISE OF PLATO'S METHODOLOGICAL AS-IFISM IS THAT MATHEMATICS IS USED TO SOLVE MATHEMATICAL AND PHYSICAL PROBLEMS AND THAT IT IS IN VIRTUE OF THESE USES THAT WE ARE JUSTIFIED IN TAKING AN HYPOTHESIS AS *IF* IT WERE TRUE.
  - LIKEWISE, LET'S PRESUME THAT THE BASIC PREMISE OF METHODOLOGICALLY INTERPRETED STRUCTURAL AS-IFISM IS THAT MATHEMATICS IS USED TO SOLVE MATHEMATICAL AND PHYSICAL PROBLEMS AND THAT IT IS IN VIRTUE OF THESE USES THAT WE ARE JUSTIFIED IN TAKING A SET OF AXIOMS AS *IF* THEY WERE CONSISTENT AND THAT WE ARE META-MATHEMATICALLY JUSTIFIED IN TAKING OUR BACKGROUND THEORY AS *IF* IT WERE TRUE.
- TAKING THIS *METHODOLOGICAL AS-IFIST* ROUTE, BY PLACING OUR FOCUS ON WHAT IS NEEDED FOR THE *PRACTICE* AND *APPLICABILITY* OF MATHEMATICS, WE ARE NEITHER COMMITTED TO THE UNCONDITIONAL CONSISTENCY OF OUR MATHEMATICAL AXIOMS NOR THE UNCONDITIONAL TRUTH OF OUR META-MATHEMATICAL BACKGROUND THEORY.
- *IT IS METHODOLOGICAL* CONSIDERATIONS, AND NOT METAPHYSICAL ONES, THAT "CONDITION" OUR AS *IF* ASSUMPTIONS OF BOTH THE CONSISTENCY OF OUR MATHEMATICAL AXIOMS AND THE TRUTH OF OUR META-MATHEMATICAL BACKGROUND THEORY.

# METHODOLOGICALLY INTERPRETED STRUCTURAL AS-IFISM

- THE PROPOSED *METHODOLOGICALLY INTERPRETED STRUCTURAL AS-IFISM* POSITION HOLDS THAT
  - SOME OF OUR COMMITMENTS TO TAKING OUR AXIOMS AS IF THEY WERE FIRST PRINCIPLES, WILL BE MADE IN LIGHT OF *MATHEMATICAL PRACTICE*, WITH THE GOAL OF SOLVING MATHEMATICAL PROBLEMS,
  - SOME WILL BE MADE IN LIGHT OF *MATHEMATICAL APPLICABILITY*, WITH THE GOAL OF SOLVING PHYSICAL PROBLEMS, AND,
  - SOME WILL BE MADE IN LIGHT OF *LOGICAL/PHILOSOPHICAL CONSIDERATIONS*, WITH THE GOAL OF SOLVING META-MATHEMATICAL PROBLEMS.
- NONE OF THESE COMMITMENTS, HOWEVER, WILL BE MADE WITH THE GOAL OF SOLVING METAPHYSICAL PROBLEMS, I.E., PROBLEMS ABOUT WHAT “FIXES” CONSISTENCY OR “FIXES” TRUTH.

# METHODOLOGICALLY INTERPRETED STRUCTURAL AS-IFISM

- WITH RESPECT TO META-MATHEMATICAL CONSIDERATIONS, DOES THIS MEAN THAT WE WILL WE HAVE TO CALL IN A MODEL-THEORY TO SOLVE THE PROBLEM OF WHAT WE MEAN BY SATISFACTION? YES, IT DOES.
- DOES THAT MEAN THAT WE WILL HAVE TO TAKE MODELS AS NATURALISTICALLY CONSTRUCTED (MADDY) OR AS POSSIBLY EXISTING (PUTNAM)? NO, IT DOES NOT. WE ONLY NEED TAKE OUR MODEL THEORY AS IF IT WERE TRUE.
- DOES THAT MEAN THAT WE WILL HAVE TO TAKE STRUCTURES THEMSELVES AS ACTUALLY OR POSSIBILITY EXISTING (SHAPIRO, HELLMAN)? NO, IT DOES NOT. WE ONLY NEED TAKE OUR THEORY OF STRUCTURE AS IF IT WERE TRUE.
- WILL WE HAVE TO CALL IN SOME THEORY TO FRAME WHAT WE MEAN BY THE CONCEPTS “MODEL” AND “STRUCTURE”? YES, WE WILL.
- DOES THIS THEORY HAVE TO BE SET THEORY? NO, IT DOES NOT.
- CAN IT BE SET THEORY? YES IT CAN.
- CAN IT BE CATEGORY THEORY? YES, IT CAN!
- DOES IT HAVE TO BE CATEGORY THEORY? NO, IT DOES NOT.

# BREAKING BENACERRAF

- RECALL NOW BENACERRAF'S DILEMMA: EITHER WE HAVE A SHARED REFERENTIAL SEMANTICS OR WE HAVE A REASONABLE EPISTEMOLOGY.
  - WE HAVE A SHARED (TARSKIAN) SEMANTICS: OUR MATHEMATICAL STATEMENTS ARE MADE TRUE BY REFERENCE TO MATHEMATICAL OBJECTS.
  - WE HAVE A REASONABLE EPISTEMOLOGY IN MATHEMATICS: WE COME TO KNOW MATHEMATICAL OBJECTS VIA THE AXIOMATIC METHOD AS OBJECTS OF THOUGHT, WE TREAT THEM AS *IF* THEY EXIST BECAUSE WE TAKE OUR AXIOMS AS *IF* THEY WERE TRUE.
  - THE *DIFFERENCE* IS THAT, AS HILBERT NOTED, IN MATHEMATICS, EXISTENCE IS A CONSEQUENCE OF TRUTH AND IN SCIENCE TRUTH IS A CONSEQUENCE OF EXISTENCE.

# MATHEMATICAL REALISM WITHOUT METAPHYSICS

- IN MATHEMATICS THERE IS NOTHING MORE TO EXISTENCE THAN *AS IF EXISTENCE*, THAT IS, EXISTENCE ON THE BASIS OF WHAT WE SAY.
  - *AS IF MATHEMATICAL REALISM* IS NOT IDEALISM, NOMINALISM OR FICTIONALISM, BECAUSE THERE IS NO OTHER SENSE OF EXISTENCE IN MATHEMATICS.
- IN SCIENCE THERE IS MORE TO EXISTENCE THAN *AS IF EXISTENCE*, THERE IS EXISTENCE OF THE BASIS OF WHAT WE CAN SHOW.
  - *AS IF SCIENTIFIC REALISM* IS IDEALISM OR NOMINALISM OR FICTIONALISM, BECAUSE THERE IS ANOTHER SENSE OF EXISTENCE IN SCIENCE.

# MATHEMATICAL REALISM WITHOUT METAPHYSICS

- **THE PROBLEM:**
  - BY CONFLATING MATHEMATICAL ISSUES WITH METAPHYSICAL ONES, **METAPHYSICAL REALISTS** HAVE MADE MATHEMATICS INTO A SCIENCE AND SO HAVE PROCLAIMED THAT MATHEMATICS MUST *BE ABOUT OBJECTS*.
  - BY CONFLATING METAPHYSICAL ISSUES WITH MATHEMATICAL ONES, **STRUCTURAL REALISTS** HAVE MADE SCIENCE INTO A LANGUAGE AND SO HAVE PROCLAIMED THAT PHYSICS HAS *NO OBJECTS*.
- CLEARLY SOMETHING HAS GONE WRONG!
- **THE SOLUTION:**
  - WHEN WE AVOID THE CONFLATION OF MATHEMATICAL AND METAPHYSICAL ISSUES, WE SEE THAT **METHODOLOGICALLY INTERPRETED STRUCTURAL AS-IFISM** CAN BE USED TO PROVIDE AN ACCOUNT OF BOTH THE *PRACTICE* AND THE *APPLICABILITY* OF MATHEMATICS, BOTH *WITHOUT* REIFYING MATHEMATICAL OBJECTS AND *WITHOUT* ELIMINATING PHYSICAL ONES!