Transcendence and Emptiness

Agustín Rayo – MIT

Handout: tinyurl.com/522jx4r3

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A. Outline

1. Logical truths are more than just necessarily true: their truth requires *nothing* of the world.
2. More generally, "transcendental" truths are more than just necessarily true: their truth requires *nothing* of the world.
3. A picture of arithmetic on which arithmetical truths are transcendental allows us to address a stubborn problem in the philosophy of mathematics.

B. Logic

1. Possible-words semantics

   - A possible worlds semantics for a language L assigns to each sentence ϕ of L a set $[\![\phi]\!]$ of "possible worlds".
   - $[\![\phi]\!]$ is defined compositionally, from an assignment of semantic values to the basic lexical items of L.

Intuitively, a *possible world* is a (coherent) way for the world to be, and $[\![\phi]\!]$ is the set of worlds at which ϕ counts as true.

2. The standard treatment of Boolean logic

   - Consider a possible-worlds semantics based on a set of possible worlds, W.
   - The standard semantic clauses for negation and disjunction deliver the following result:

     $\ulcorner \psi \vee \neg\psi \urcorner$ is true at world $w \in W$ iff: either ψ is true at w or it is not the case that ψ is true at w.

   - But: the right hand side of this biconditional is itself a classical tautology.
   - So, we can use a classical metatheory to show that $\ulcorner \psi \vee \neg\psi \urcorner$ is true at $w \in W$ *without making any assumptions about w or W*.

These are the relevant clauses:
- $\ulcorner \phi \vee \psi \urcorner$ is true at w iff: either ϕ is true at w or ψ is true at w;
- $\ulcorner \neg\phi \urcorner$ is true at w iff: it is not the case that ϕ is true at w.

Contrast with: every $w \in W$ is such as to verify $\ulcorner \psi \vee \neg\psi \urcorner$.

3. How to think of this result

   - The truth of a sentence imposes a *requirement* on the world: that the world be as the sentence says it is.

     For instance, the truth of "Susan is happy" imposes the (non-empty) requirement that Susan be happy.
   - There are two ways for a requirement to be satisfied necessarily:

(a) the truth of ϕ imposes a requirement that turns out to be satisfied by every $w \in W$.

Example: Some philosophers think that God exists necessarily. On the most straightforward understanding of this picture, the truth of "God exists" imposes a non-empty requirement on the world. But, as it turns out, it is a requirement that is satisfied at every world.

(b) The truth of ϕ imposes an *empty* requirement on the world. (Informally: it requires *nothing* of the world.)

The biconditional in B.2 makes it natural to take the truth of $\ulcorner \psi \vee \neg\psi \urcorner$ to require *nothing* of the world.

All of this, against the background of a classical metatheory.

4. Why the difference matters

- Suppose you are a classical logician and are pressed (e.g. by an intuitionist) to justify your claim that $\ulcorner \phi \vee \neg\phi \urcorner$ is a necessary truth for any ϕ. Two cases:

  In each case, you might also be asked to explain why $\ulcorner \phi \vee \neg\phi \urcorner$ has the truth conditions that it has.

  (a) You think the truth of $\ulcorner \phi \vee \neg\phi \urcorner$ imposes a non-empty requirement R on the world. Then you need to explain why the space of possible worlds is such that R turns out to be satisfied at every world.

  (b) You think the truth of $\ulcorner \phi \vee \neg\phi \urcorner$ requires *nothing* of the word. Then you don't have to say anything about the space of worlds to explain why $\ulcorner \phi \vee \neg\phi \urcorner$ is necessarily true.

C. Transcendence

The basic idea comes from Fine (2005). Here I'll attempt to develop a notion of transcendence that steers clear of Fine's metaphysics.

1. The concept of transcendence

- A sentence ϕ of L is *transcendental* (by the lights of one's semantic theory M for L) iff the following is a theorem of M:

  ϕ is true at world $w \in W$ iff: Φ.

  where Φ is a formula in which neither "w" nor "W" occur free.

  More generally: an n-place predicate ψ of L is *transcendental* (by the lights of one's semantic theory M for L) iff the following is a theorem of M:

  $\ulcorner \psi(x_1, \ldots, x_n) \urcorner$ is true at world $w \in W$ (relative to a given variable assignment) iff: Φ.

  where Φ is a formula in which neither "w" nor "W" occur free.

- *Intuitively:* What the truth of a transcendental sentence requires of the world is either empty or absurd. So, whether a transcendental sentence is true at w does not depend on w or W.

- *A generalization:* whether a transcendental predicate applies to $x_1, \ldots, x_n$ at a world w does not depend on w or W. (And, in particular, it does not depend on whether $x_1, \ldots, x_n$ exist at w.)

2. Some examples of transcendental predicates:

- Every logical truth and every logical falsehood.

  Think of formulas as 0-place predicates.

- The "external" identity predicate "$=^e$":

  $\ulcorner x =^e y \urcorner$ is true at world $w \in W$ (relative to a variable assignment σ) iff: $\sigma(x) = \sigma(y)$.

  I assume, for simplicity, that we work within a necessitist metatheory.

  as opposed to an "internal" identity predicate:

  $\ulcorner x =^i y \urcorner$ is true at world $w \in W$ (relative to a variable assignment σ) iff: $\sigma(x) = \sigma(y)$ and $\sigma(x) \in D_w$.

- The "external" one-one correspondence predicate "$\approx^e$":

  $\ulcorner xx \approx^e yy \urcorner$ is true at world $w \in W$ (relative to a variable assignment σ) iff: $|\sigma(xx)| = |\sigma(yy)|$.

  $|\alpha|$ is the cardinality of α.

  as opposed to an "internal" one-one correspondence predicate:

  $\ulcorner xx \approx^i yy \urcorner$ is true at world $w \in W$ (relative to a variable assignment σ) iff: $|\sigma(xx)| = |\sigma(yy)|$, $\sigma(xx) \subseteq D_w$, and $\sigma(yy) \subseteq D_w$.

D. A philosophical obstacle to transcendent predicates

1. The Property View

- *Intuitively:* predicates express properties, and an object can't exemplify a property unless it exists.
- *Formally:*

  P1: A one-place predicate expresses a property.

  I assume, for simplicity, that the predicate in question contains no free variables.

  P2: A sentence $\ulcorner Fa \urcorner$ is true at a world w iff: at w, the property expressed by F is exemplified by the referent of a.

  P3: For an object to exemplify a property at a world w, the object must exist at w.

  And similarly for n-place predicates.

2. Motivation

- A property is a *way* for a thing to be. So, exemplifying a property is a matter of *being a certain way*.
- But how could an object be a certain way without being something? Doesn't an object have to *be* in order to be *somehow*?

3. Transcendental Predicates

- The Property View allows for transcendental sentences (e.g. tautologies), but not for n-place transcendental predicates ($n > 0$).
- In certain special cases, one can get around this expressive limitation. For instance, the formula "$s =^e t$" is model-theoretically equivalent to: "$\Diamond(s =^i t)$", assuming the denotations of "s" and "t" exist at some world.

For ϕ to be *model-theoretically equivalent* to ψ is for ϕ and ψ to be true at the same worlds with respect to any model and variable assignment.

- But not always. For example, one can prove that no formula of a second-order modal language is model-theoretically equivalent to "$ss \approx^e tt$".
- Also, it is not clear that one can give a compositional semantics for modal languages in which "all intermediate values in the composition are actual things." Stalnaker (2012, p. 125).

There is the option of paraphrasing "$tt \approx tt'$" as "$\exists n(\text{Num}(tt, n) = \text{Num}(tt', n))$". In order for this proposal to deliver the right results, one must assume that the domain of each world contains as many numbers as there might have been individuals (since otherwise "$\Box\forall xx\Box(xx \approx^e xx)$" would count as false). Such an assumption entails that there is a definite answer to the question of how many individuals there might have been—a claim that is presupposed by Williamson (2013) but is unlikely to be accepted by contingentists such as Stalnaker (2012).

E. The Propositional Function View

1. The driving thought

- One should try to minimize expressive restrictions driven by one's "metaphysics" of linguistic representation.
- All it takes for a semantics to count as legitimate is for it to assign a set of possible worlds to each <u>sentence</u>.
- In the case of a compositional semantics, the semantic values of sub-sentential expressions are determined by their role in specifying an assignment of truth-conditions to sentences.

2. The Propositional Function View

- *Intuitively:* predicates express propositional functions.

  (An n-place propositional function is a *condition* that delivers a proposition as output given a sequece of n objects as input.)

- *Formally:*

  F1: An n-place predicate expresses an n-place propositional function.

  F2: The sentence $\ulcorner Ft_1 \dots t_n \urcorner$ expresses the proposition $f(z_1, \dots, z_n)$, whenever the n-place predicate F expresses the n-place propositional function f and the singular terms $t_1, \dots, t_n$ refer, respectively, to $z_1, \dots, z_n$.

Propositions are 0-place propositional functions. (To keep things simple, I ignore open predicates. But it is easy to generalize the proposal.)

3. Motivation

- A (one-place) predicate can be thought of as a function that delivers a sentence as output given a singular term as input.
- Correspondingly, a the semantic value of a predicate can be thought of as a function that delivers the semantic value of a sentence as output given the semantic value of a singuler term as input.

4. Transcendental Predicates

- The Propositional Function View allows for transcendental sentences and for transcendental predicates like "$=^e$" and "$\approx^e$".
- This allows us to introduce a *cardinality* operator "#", which works as follows:

  $\ulcorner \# (F) =^e \# (G) \urcorner$ is true at w iff:

  (the number of possible individuals z such that: at w, z is F) =
  (the number of possible individuals z such that: at w, z is G).

  as contrasted with:

  $\ulcorner \# (F) =^i \# (G) \urcorner$ is true at w iff:

  (the number of possible individuals z such that, at w, z is F) =
  (the number of possible individuals z such that, at w, z is G);
  and that number is in the domain of w.

F. Transcendental Arithmetic

1. The basic idea

- In transcendental arithmetic, arithmetical truths are transcendentally true: their truth requires nothing of the world.
- The availability external identity statements involving # is the first step towards constructing a transcendental arithmetic.

  To complete the picture, we need transcendental quantifiers. (See below.)

  For instance, the following is a transcendental truth:

  $\#(F) =^e \#(F)$
  Read: the number of the Fs equals the number of the Fs.

2. An important qualification

- The claim that $\ulcorner \# (F) =^e \# (F) \urcorner$ is transcendentally true presupposes that one's metatheory includes substantial arithmetical principles.
- So: the resulting view is unlikely to win over an interlocutor who is unwilling to engage in mathematical practice.

  That doesn't mean that the proposal is pointless!

3. Benacerraf's Dilemma

- *The Dilemma:*
  (a) explain how one could come to know that the world turns out to contain mathematical objects (given that, e.g. such objects are causally inert); or
  (b) reject a standard semantics for mathematical discourse, according to which mathematical terms refer to mathematical objects.

- *A key presupposition*: the truth of arithmetical sentences, standardly interpreted, imposes non-empty demands on the world.
- So: a transcendental arithmetic would allow us to escape Benacerraf's Dilemma.

In particular: the non-empty demand that the world contain numbers.

G. Completing the Picture

1. Transcendental Quantifiers

   - A transcendental arithmetic requires more than just an external cardinality operator. It also requires an external arithmetical quantifier "$\exists^e$":

     $\ulcorner \exists^e n\ \phi \urcorner$ is true at w (relative to variable assignment σ) iff: there is a number $\mathfrak{n}$ such that ϕ is true at w relative to $\sigma[\mathfrak{n}/n]$.

     as opposed to the internal arithmetical quantifier "$\exists^i$":

     $\ulcorner \exists^i n\ \phi \urcorner$ is true at w (relative to variable assignment σ) iff: there is a number $\mathfrak{n}$ *in the domain of w* such that ϕ is true at w relative to $\sigma[\mathfrak{n}/n]$.

   - "$\exists^e n\,(n =^e n)$" is transcendental: its truth requires nothing of the world.
   - This is as it should be: "$\exists^e n\,(n =^e n)$" is entailed by "$\#(F) =^e \#(F)$", whose truth requires nothing of the world.

A (frst-order) existential quantifier $\exists x$ is *transcendental* iff the following is a theorem of one's semantic theory for L:

> $\ulcorner \exists x\, \phi \urcorner$ is true at world $w \in W$ (relative to σ) iff: there is a z such that Θ and such that ϕ is true at w (relative to $\sigma[z/x]$).

where Θ is a formula in which neither "w" nor "W" occur free.

2. A Transcendental Conception of Object.

   We need a conception of object that makes room for transcendental quantifiers. Here is a sketch of what such a conception might look like.

   (a) *The Bucket View*

   - There is a "metaphysically distinguished" domain—a domain of individuals that can be singled out on the basis of purely metaphysical considerations.
   - To be an object is to be a member of that domain.
   - A singular term can only refer if it refers to some object in the metaphysically distinguished domain.

   (b) *A Facts-First Conception of Object*

   - An object is an *aspect* of a way for the world to be—an aspect that might be rendered salient picking out the way for the world to be using a syntactically structured sentence.
   - We do not presuppose that a given feature of reality can be divided into "aspects" independently of a compositional language.

For details, see my "The Ultra Thin Conception of Object."

Think of an object as a *node* in a network of connections between ways for the world to be that is rendered salient by a compositional language.

Just like there is no sense to be made of the intersection between two roads independently of the roads themselves, so there is no sense to be made of a node in a network of connections independently of the connections themselves.

References

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